Final Exam for Math Preparation (with Solutions)

1. (10 points) If \$20,000 is invested now in a bank account for 5 years with an annual interest rate of 5% compounded monthly, at the end of the 5 years there is \$x in the account, find x.

$$x = \$20000\left(1+\frac{.05}{12}\right)^{5*12} = \$20000\left(1+\frac{.05}{12}\right)^{60} \approx \$25667.17$$

2. (6 points) For question 1, what is the effective annual rate of interest?

$$ER = \left(1+\frac{.05}{12}\right)^{12} - 1 \approx .0512$$

3. (10 points) If \$x is invested now in a bank account for 5 years with an annual interest rate of 4% compounded continuously, and at the end of the 5 years there is \$15,000 in the account, find x.

$$\$15000 = \$x\, e^{.04*5} = \$x\, e^{.2} \Rightarrow x = \frac{15000}{e^{.2}} = 15000e^{-.2} \approx 12280.96$$

4. (6 points) For question 3, what is the effective annual rate of interest?

$$ER = e^{.04} - 1 \approx .0408$$

5. (10 points) If $\ln(7x^2 + 4) = 2$, Find x.

$$ln(7x^2+4) = 2 \Longrightarrow 7\mathrm{x}^2 + 4 = \mathrm{e}^2 \Longrightarrow x = \pm\sqrt{\frac{e^2-4}{7}} \approx \pm.6958$$

6. (10 points) If $e^{5+4x} = 40$, Find x.

$$e^{5+4x} = 40 \Longrightarrow 5 + 4x = \ln(40) \Longrightarrow x = \frac{ln(40)-5}{4} \approx -.3278$$

7. (10 points) What is the derivative of $f(x) = (5x^6 + 12x^2 + 7x - 20)^3$? No simplification necessary.

$$\frac{df}{dx} = 3(5x^6 + 12x^2 + 7x - 20)^2(30x^5 + 24x + 7)$$

8. (10 points) What is the derivative of f(x) =$(6x^2+7)e^{3x}$? No simplification necessary.

$$\frac{df}{dx} = 12xe^{3x} + 3e^{3x}(6x^2+7) = 3e^{3x}(\,6x^2+4x+7)$$

9.

a. (8 points) What is the derivative of f(x) = $-7x^2 + 32x + 6$?

$$\frac{df}{dx} = -14x + 32$$

b. (8 points) What is/are the critical point(s) of f(x) = $-7x^2 + 32x + 6$?

The critical point is $x = \frac{32}{14} = \frac{16}{7}$.

10. (12 points) Does f(x) = $-6x^2 + 18x + 5$ have a maximum? If so, where is it? Does f(x) = $-6x^2 + 18x + 5$ have a minimum? If so, where is it?

The first derivative of f(x) is $-12x + 18$. The only critical point is at $= \frac{3}{2}$.
The second derivative of f(x) is -12. Since f(x) has only one critical point and it is always concave (second derivative is negative for all x), the critical point must be a local maximum and hence a global maximum as well. f(x) has no minimum.