The Risk-Adjusted Cost of Financial Distress*

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Abstract

In this paper we argue that systematic risk matters for the valuation of financial distress costs. Since financial distress is more likely to happen in bad times, the risk-adjusted probability of financial distress is larger than the historical probability. We propose a methodology for the valuation of distress costs, which uses observed corporate bond spreads to estimate risk-adjusted probabilities of financial distress. Because credit spreads are so large (the “credit spread puzzle”), the magnitude of the distress risk-adjustment can be substantial, suggesting that a valuation of distress costs that ignores systematic risk significantly underestimates the true value. For a firm whose bonds are rated BBB, our benchmark calculations suggest that the NPV of distress increases from 1.4% of pre-distress firm value if we use historical default probabilities, to 4.5% using risk-adjusted probabilities derived from bond spreads. Marginal distress costs also increase substantially. For example, a leverage increase that changes ratings from AA to BBB is associated with an increase in distress costs of 2.7% using risk-adjusted probabilities, but only 1.1% using historical probabilities. We argue that the magnitude of these marginal, risk-adjusted distress costs is similar to the magnitude of the marginal tax benefits of debt derived by Graham (2000), and thus that systematic distress risk can help explain why firms appear rather conservative in their use of debt.

Key words: Financial distress, corporate valuation, capital structure, default risk, credit spreads, debt conservatism.

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1 Introduction

There is a large literature that argues that financial distress can have both direct and indirect costs (Warner, 1977, Altman, 1984, Weiss, 1990, Ofek, 1993, Asquith, Gertner and Scharfstein, 1994, Opler and Titman, 1994, Sharpe, 1994, Gilson, 1997, and Andrade and Kaplan, 1998). However, there is much debate as to whether such costs are high enough to matter much for corporate valuation practices and capital structure decisions. Direct costs of distress, such as those entailed by litigation fees, are relatively small.\(^1\) Indirect costs, such as loss of market share (Opler and Titman, 1994) and inefficient asset sales (Shleifer and Vishny, 1992), are believed to be more important, but they are also much harder to quantify. Andrade and Kaplan (1998), for example, estimate losses of the order of 10% to 23% of pre-distress firm value for a sample of highly leveraged firms. However, they also argue that part of these costs might actually not be genuine financial distress costs, but rather consequences of the economic shocks that drove the firms into distress. They suggest that, from an ex-ante perspective, distress costs are probably small, specially in comparison to the potential tax benefits of debt.\(^2\) In contrast, Opler and Titman (1994) argue that distress costs can be large for certain types of firms, such as those that engage in substantial R&D activities.\(^3\)

While the previous literature has analyzed in detail the nature of distress costs, and has attempted to estimate the loss in value upon distress, it has devoted much less attention to the proper capitalization of financial distress costs. For example, Graham (2000) and Molina (2005) calculate the ex-ante cost of distress as the historical probability of default multiplied by Andrade and Kaplan’s (1998) estimates of the loss in firm value given default. This calculation ignores the capitalization and discounting of distress costs. Other papers

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\(^1\) Warner (1977) and Weiss (1990), for example, estimate costs of the order of 3%-5% of firm value at the time of distress.

\(^2\) Altman (1984) finds similar cost estimates of 11% to 17% of firm value on average, three years prior to bankruptcy. However, it is not clear that all such costs can be attributed to genuine financial distress (Opler and Titman, 1994, and Andrade and Kaplan, 1998).

\(^3\) Not all the literature agrees with the proposition that distress only has costs. Wruck (1990) argues that the organizational restructuring that accompanies distress might have benefits, and Ofek (1993) suggests that leverage might force firms to respond more quickly to poor performance. In addition, Eberhardt, Altman and Aggarwal (1997) find that firms appear to do unexpectedly well post-bankruptcy.
do incorporate some form of discounting. The usual approach in the literature is to assume risk-neutrality, and discount the product of historical probabilities and losses in value given default by a risk-free rate (e.g., Altman (1984)).

In this paper we develop a methodology to value financial distress costs. Like the existing literature, we take as given the estimates of losses in value given distress provided by Andrade and Kaplan (1998) and Altman (1984). We suggest a simple way to capitalize these losses into a NPV formula for (ex-ante) distress costs. Most importantly, we argue that the common practice of using both historical probabilities of distress and risk free rates to value distress costs is wrong.

The problem with the traditional approach is that the incidence of financial distress is correlated with macroeconomic shocks such as major recessions, generating a systematic component to distress risk. In fact, the asset pricing literature has provided substantial evidence for a systematic component in corporate default risk. It is well-known that the spread between corporate and government bonds is too high to be explained only by expected default (the “credit spread puzzle”). The literature also presents direct evidence for a large default risk premium implicit in corporate bond spreads (Elton, Gruber, Agrawal and Mann, 2001, Longstaff, Mittal, and Neis, 2005, Driessen, 2005, Chen, Collin-Dufresne, and Goldstein, 2005, and Cremers, Driessen, Maenhout and Weinbaum 2005). This systematic component of default risk raises the possibility that investors might care more about default (and thus financial distress) than what is implied by risk-free valuations. In particular, this insight suggests that in order to value distress costs correctly, either the discount rate or the probability of distress must be adjusted for risk. If historical probabilities are used to compute expected distress costs, then these costs must be discounted by a rate that is lower than the risk free rate. Alternatively, if the risk free rate is used in the valuation, then

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4Recent models of dynamic capital structure that incorporate distress costs also assume risk-neutrality, and thus implicitly discount the costs of financial distress by the risk free rate (e.g., Titman and Tsypaklov (2004), and Hennessy and Whited (2005)).

5See Denis and Denis (1995) for some evidence that the incidence of distress is related to macroeconomic conditions.

6Jones, Mason and Rosenfeld (1984) provide some early evidence on this.

7See also Collin-Dufresne, Goldstein and Martin (2001), who examine the determinants of movements in credit spreads.
the probability of distress must be *higher* than the historical one in order to account for distress risk. Either way, this insight suggests that the existing corporate finance research has underestimated the NPV of financial distress costs.

We propose a methodology to calculate the NPV of distress costs, which explicitly incorporates systematic distress risk. The basic idea is to use observed credit spreads to back out the market-implied probabilities of default, which will then incorporate any systematic default risk that is implicit in credit spreads. Such an approach is common in the credit risk literature (i.e., Duffie and Singleton, 1999, and Lando, 2004). Our calculations also incorporate recent insights of the literature on credit yield spreads, which suggests that one should not attribute the entire yield spread to default risk, because of tax and liquidity effects (Elton et al., 2001, Chen, Lesmond, and Wei, 2004). Specifically, in order to derive risk-adjusted probabilities we use only the fraction of bond yield spreads that is likely to be due to default (the “default component” of spreads). Our estimates imply that risk-adjusted probabilities of default and, consequently, the risk-adjusted NPV of distress costs, are considerably larger than, respectively, historical default probabilities and the non risk-adjusted NPV of distress.

To give an example of our findings, consider a firm whose bonds are rated BBB. The historical 10-year cumulative probability of default for BBB bonds in our data (Moody’s, average between 1970-2001) is 5.22%. However, our benchmark calculations suggest that the 10-year cumulative default probability that is implied in BBB spreads is 20.88%. This large difference between historical and market-implied probabilities translates into a substantial difference in NPVs of distress costs. To estimate the loss in value given default, our benchmark calculations use the midpoint of the range of 10%-23% suggested

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8 As we explain in section 3.2, in order to derive risk-adjusted probabilities from credit spreads we need to make specific assumptions regarding bond coupons and recoveries. However, we show in section 4.5 that the main results are robust to variations in these specific assumptions.

9 Our benchmark calculations assume that the tax and liquidity adjustment is given by the AAA spread (Chen et al., 2005). However, we verify the robustness of the results with respect to other ways to adjust spreads in section 4.2, such as using credit default swap premiums (Longstaff et al, 2005). With the exception of Huang and Huang (2003), all of these approaches generate similar default component of spreads.

10 We actually compute the entire term structure of default probabilities from year 1 to year 10 (see Section 4.3).
by Andrade and Kaplan (1998).\textsuperscript{11} Using this number in our NPV formula implies an NPV of 1.4\% of pre-distress firm value using historical probabilities, but 4.5\% using market-implied probabilities.

We believe that the main contribution of our paper is methodological: we show how one can compute the NPV of financial distress costs in the presence of systematic distress risk. The general procedure described in sections 3 and 4 incorporates both a term structure of risk free rates and a term structure of risk-adjusted default probabilities, which is derived from the term structure of credit spreads on coupon-paying bonds for all ratings between AAA and B. Because the formulas involved in this procedure are relatively complex, we start the paper with a much simpler example (section 2) that assumes flat term structures of both risk free rates and default probabilities, and derives the (constant) yearly default probability from the yield spread on a (fictitious) perpetual coupon bond issued by the firm. The formulas derived in section are extremely simple to implement, and provide a reasonable approximation to the more precise formulas derived in sections 3 and 4. For example, if we use the 10-year BBB spread to proxy for the spread on the perpetual bond, the implied NPV of distress for the BBB rating using the simple formula is 4.1\%.

Our results also have implications for capital structure theory. Graham (2000) estimates marginal tax benefits of debt, and conjectures that marginal distress costs are too small to overcome potential tax benefits of increased leverage, in the context of a static trade-off model of capital structure. He concludes that firms are probably too conservative in their use of debt. However, our results suggest that marginal, risk-adjusted distress costs can be of the same magnitude as marginal tax benefits of debt. For example, using our benchmark assumptions the increase in distress costs that is associated with a change in ratings from AA to BBB is 2.7\% of pre-distress firm value.\textsuperscript{12} To compare this number with marginal tax benefits of debt we derive the implicit marginal tax benefit of leverage that is implicit

\textsuperscript{11}We also present in section 4.5.5 valuation results for the endpoints of this range (10\% and 23\% of pre-distress firm value). The increase in distress costs is substantial, even if we use the lower end of this range (10\%).

\textsuperscript{12}For comparison purposes, the increase in marginal distress costs is only 1.11\% if we use historical default probabilities.
in Graham’s (2000) calculations, and use the relationship between leverage ratios and bond ratings recently estimated by Molina (2005). The implied gain in tax benefits as the firm moves from an AA to a BBB rating is 2.67% of firm value. Thus, it is not clear that the firm gains much by increasing leverage from AA to BBB levels.\textsuperscript{13} These results suggest that the large distress costs that we estimate can help explain why many US firms appear to be conservative in their use of debt.\textsuperscript{14}

The paper proceeds as follows. We start in the next section by presenting a simple example of how our valuation approach works. The general methodology is presented in section 3. In section 4 we present our empirical estimates of the NPV of distress costs. Section 5 discusses the capital structure implications of our results, and section 6 concludes.

2 Using Credit Spreads to Value Distress Costs: A Simple Example

In order to explain the procedure that we use to value financial distress costs using bond yield spreads, we first introduce an example that makes several simplifying assumptions. The purpose of the example is both to illustrate the intuition behind the general procedure that we explain in the next section, and to provide simple “back-of-the-envelope” formulas that can be used to value financial distress costs.

Suppose we want to value distress costs for a certain firm, which has issued a bond that matures in exactly one year and pays an annual coupon. The bond’s promised yield is equal to $r_D$, and the bond is priced at par, such that the promised coupon rate that is due in one year is also equal to $r_D$. Finally, let the recovery rate be equal to $\rho$, such that if the bond defaults creditors recover $\rho(1 + r_D)$. For simplicity, assume that the recovery rate is known with certainty. The bond valuation tree is depicted in Figure 1. The value of the bond

\textsuperscript{13}This conclusion generally holds for variations of the assumptions around those used in the benchmark valuations. The results are most sensitive to the estimate of losses given distress. For values close to 10%, and depending on the specific assumptions about leverage ratios, marginal tax benefits can become larger than marginal distress costs. See section 5.1.3.

\textsuperscript{14}Nevertheless, the full explanation for debt conservatism probably involves more than a static trade-off model, given Graham’s (2000) finding that firms that are likely to have the lowest costs of financial distress seem to be the most conservative in their use of debt.
must be equal to the present value of expected future cash flows, adjusted for systematic default risk. If we let \( q \) be equal to the risk-adjusted (or risk-neutral), one-year probability of default, we can express the bond’s value as:

\[
1 = \frac{(1 - q)(1 + r^D) + q\rho(1 + r^D)}{1 + r^F},
\]

where \( r^F \) is the one-year risk free rate. The idea behind this valuation formula is that the probability \( q \) incorporates the systematic risk-adjustment that is implicit in the bond’s promised yield to maturity \( r^D \). If investors were risk neutral, or if there was no systematic default risk, \( q \) would be equal to the expected probability of default (call it \( p \)). If systematic default risk is priced, then the implied \( q \) is higher than \( p \). Equation 1 can be solved for \( q \):

\[
q = \frac{r^D - r^F}{(1 + r^D)(1 - \rho)}.
\]

Notice that the extent of the risk adjustment implicit in \( q \) is a direct function of the yield spread \( r^D - r^F \). Large spreads translate directly into large risk neutral probabilities of default. Notice also that the higher the recovery rate, the higher the risk adjustment implied by equation (2). The intuition is that high recoveries reduce a creditor’s loss given default, and thus for a given spread the implied probability of default must be higher.

The basic idea of our paper is that we can use the risk-neutral probability of default \( q \) to do a risk-adjusted valuation of financial distress costs. Consider again Figure 1, which also depicts the valuation tree for distress costs. Let the loss in value given default be equal to \( \phi \), and the present value of distress costs be equal to \( \Phi \). For simplicity, suppose that \( \phi \) is non-stochastic. If we assume that financial distress can only happen at the end of one year, but never again in future years, then we can express the present value of financial distress costs as:

\[
\Phi = \frac{q\phi + (1 - q)0}{1 + r^F}.
\]

Formula 3 is similar to that used by Graham (2000) and Molina (2005) to value distress costs. The key difference is that while Graham (2000) and Molina (2005) use historical default probabilities, equation 3 uses a risk-adjusted probability of financial distress that is calculated from yield spreads and recovery rates using equation 2.
However, in order to provide a more precise estimate of the present value of financial distress costs, we must recognize that if financial distress does not happen at the end of the first year, it can still happen in future years. If we assume that the marginal risk-adjusted probability of default and the risk free rate do not change over time, then the valuation tree becomes a sequence of one-year trees that are identical to that depicted in Figure 1. This implies that if financial distress does not happen in year one (an event that happens with probability $1 - q$), the present value of future distress costs at the end of year one is again equal to $\Phi$. Replacing $0$ with $\Phi$ in the valuation equation 3 and solving for $\Phi$ we obtain:

$$\Phi = \frac{q}{q + r^F \phi}. \tag{4}$$

Given estimates for $q$, $r^F$ and $\phi$, equation 4 provides a better approximation to the present value of financial distress costs than equation 3. Notice also that for a given $q$ (that is, irrespective of the risk-adjustment), equation 3 substantially underestimates the present value of distress costs.

We note that the assumptions that $q$ and $r^F$ are constant over time are counter-factual. The general procedure that we describe later allows for time variation in $q$ and $r^F$. In particular, we use dynamic credit risk models to back out a term structure of risk-adjusted default probabilities ($q_1, q_2, \ldots$) from the observed term structures of bond yield spreads and risk free rates. For the purpose of illustration, however, suppose that $q$ and $r^F$ are indeed constant over time. In the appendix, we give conditions such that equation 2 can still be used to obtain the (constant) risk-adjusted probability of default $q$. The most important assumption is that $r^D - r^F$ is the promised spread on a perpetual bond issued by the firm, which is assumed to be constant over time.

To illustrate the impact of the risk-adjustment, take for example BBB-rated bonds. In our data, the historical 10-year spread on those bonds is approximately 1.90%,\footnote{Because formula 2 is based on the valuation of perpetual bonds (see the appendix), it seems more appropriate to use a long term yield spread for these calculations. The data refer to the period 1985-1995, but the average spread is almost identical if we use data from 1985-2004 (see the discussion in section 4.1).} and the average recovery rate is equal to 0.41.\footnote{The average recovery rate is from Moodys, 1970-2001. The recovery rate is an average across all bond ratings.} As we discuss in the next section, the credit risk
literature suggests that this spread cannot be attributed entirely to default losses, because it is also affected by tax and liquidity considerations. Essentially, our benchmark calculations remove 0.51% from this raw spread.\textsuperscript{17} The difference (1.39%) is what is usually called the “default component” of yield spreads. Using this default component, a recovery of 0.41, and a long term interest rate of 6.7% (the historical 10-year treasury rate in our data) in equation 2 gives an estimate for $q$ equal to 2.2%. For comparison, the historical marginal 10-year probability of default for BBB bonds is 0.86%, which is almost three times lower than the market-implied probability. These numbers suggest the existence of a substantial default risk premium, a conclusion that is consistent with the recent credit risk literature (i.e., Elton et al. (2001), Longstaff et al. (2005), and Chen et al. (2005)). The ratio of 3 between risk neutral and historical probabilities of default is also consistent with the literature (i.e., Driessen (2005)).

The full impact of systematic default risk on the NPV of distress costs is given by the term $\frac{q}{q+r_F}$ in equation 4. This term goes from 0.11, if we use the historical 10-year probability, to 0.25, using the risk-adjusted 10-year probability. As discussed in the introduction, the literature has estimated ex-post losses in value given default (the term $\phi$) of 10% to 23% of pre-distress firm value. If we use for example the mid-point between these estimates ($\phi = 16.5\%$), the NPV of distress for the BBB rating would go from 1.87% to 4.1%. Clearly, incorporating the risk-adjustment appears to make a big difference to the valuation of financial distress costs.\textsuperscript{18}

In the next section, we present a more general framework to estimate the implied term structure of risk neutral default probabilities from the yields on coupon bonds of different maturities, and we use these probabilities to provide a more precise valuation of financial distress costs.

\textsuperscript{17}This adjustment factor is the historical spread over treasuries on a one-year AAA bond. In section 4.2 we discuss alternative ways to adjust for taxes and liquidity, and we argue that most (but not all) of them imply similar default component of spreads.

\textsuperscript{18}As we show in the next section, these numbers are reasonably close to those derived using the general procedure. However, the difference between risk neutral and historical valuations for BBB bonds becomes even larger after incorporating a term structure of risk neutral probabilities (as opposed to assuming that $q$ is constant). The difference increases because the term structure is steeper for historical probabilities than for risk neutral ones, and so using the 10-year probability in a perpetuity formula underestimates this difference.
distress costs. In addition, we discuss the role of several important assumptions regarding bond recoveries, the calculation of default components of spreads, time-variation in default spreads, different assumptions about bond coupons, and losses given default. Our conclusions are similar to those in this section, in that the risk-adjusted NPV of financial distress increases substantially with the risk adjustment. Finally, in Section 5 we use our best estimates of distress costs to compare marginal costs of financial distress with marginal tax benefits of debt derived from Graham (2000), and show that these marginal costs and benefits are of similar magnitude.

3 The General Valuation Formula

Figure 2 illustrates the timing of the general model that we use to value financial distress costs. In the Figure, \( \phi_t \) is the deadweight loss that the firm incurs in case of default at time \( t \). We think of \( \phi_t \) as a one time cost paid in case of distress. After distress, the firm might reorganize, or it might be liquidated. In case it does not default, the firm moves to period \( t + 1 \), and so on. The assumption of no-arbitrage guarantees the existence of a pricing kernel, \( m_t \), and the general formula to compute the ex-ante costs of financial distress is

\[
\Phi = \mathbb{E} \left[ \sum_{t \geq 1} m_t d_t \phi_t \right],
\]

where \( d_t \) is an indicator of default at time \( t \). Note that we are making an ex-ante calculation. In particular, the operator \( \mathbb{E}(\cdot) \) represents an unconditional expectation, from the perspective of an initial date (date-0). In general, one would take conditional expectations, and obtain conditional net present values for the costs of distress. We come back to this issue in section 4.5.6.

Throughout the paper, we will maintain the assumption that \( \phi_t \) is idiosyncratic:

**Assumption A1:** The deadweight loss \( \phi_t \) in case of default is uncorrelated with the pricing kernel, \( \text{cov} (m_t d_t, \phi_t) = 0 \), and its unconditional mean is constant over time, \( \mathbb{E} [\phi_t] = \phi \).
There is much debate in the literature on how to estimate the actual cash flow losses that are exclusively due to financial distress. In particular, while the literature does provide some estimates of the average deadweight costs of distress (i.e., Andrade and Kaplan, 1998), no paper has attempted to estimate a time series of these deadweight costs that would allow us to estimate their covariance with the pricing kernel. Because of this difficulty, our estimates will be based only on the systematic risk in the probability that financial distress occurs. Assumption A1 could lead us to underestimate the risk adjustment if the dead-weight losses conditional on distress are higher in bad times, as suggested by Shleifer and Vishny (1992). However, it is also possible that deadweight losses are higher in good times, because financial distress might cause the firm to lose profitable growth options (Myers, 1977). While it would be theoretically straightforward to relax assumption A1, there is no data that would allow us to estimate the covariance between $m$ and $\phi$.

Under A1, we can rewrite equation (5) as

$$\Phi = \phi \sum_{t \geq 1} (E[m_t] E[d_t] + \text{cov}[m_t, d_t])$$  \hspace{1cm} (6)

$$= \phi \sum_{t \geq 1} (B_tE[d_t] + \text{cov}[m_t, d_t]),$$

where $B_t = E[m_t]$ is the price at time zero of a riskless zero-coupon bond paying one dollar at date $t$.

The first term in equation (6) is the compensation for default losses that a risk-neutral investor would demand. This term has been the focus of the literature so far. Our contribution is to estimate the second term of the equation. If default is more likely to happen when $m_t$ is high – in bad times – then the covariance is positive, and the ex-ante costs of financial distress are larger than suggested by the first term alone. The argument in the next section allows us to estimate this additional term from credit spread data.$^{19}$

$^{19}$In a previous version of the paper we have also attempted to estimate the term $\text{cov}[m_t, d_t]$ directly using standard pricing kernels and historical data on distress events.
3.1 Valuation Using Risk-Adjusted Default Probabilities

We start from the observation that the costs of distress tend to occur in states in which the firm’s debt is in default. Thus, as we explained in Section 2, financial distress costs can be valued using risk-adjusted default probabilities.

To proceed, we introduce some notation to describe default events and default rates. We let $q_t$ be the risk-adjusted, marginal probability of default in year $t$ (see Figure 2). Unlike in section 2, we now allow $q_t$ to vary over time. We also let $Q_t$ denote the cumulative, risk-adjusted probability of default up to time $t$, and thus $1 - Q_t = \prod_{s=1}^{t}(1 - q_s)$ is the cumulative, risk-adjusted survival rate, i.e., the probability of not defaulting between 0 and $t$. By convention, $Q_0 = 0$. The probability that default occurs exactly at date $t$ is thus equal to $(1 - Q_{t-1})q_t$. The credit risk literature uses risk-adjusted probabilities to estimate default risk premia. Equation (5), written with risk-adjusted probabilities, becomes

$$\Phi = \phi \sum_{t \geq 1} B_t (1 - Q_{t-1})q_t , \quad (7)$$

We now explain how to recover $Q_t$ and $q_t$ from corporate bond yields.

3.2 Credit Spreads and Risk Neutral Probabilities of Financial Distress

Suppose we observe an entire term structure of yields for the firm whose distress costs we want to value, that is, we know the sequence $\{r^D_t\}_{t=1,2...}$, where $r^D_t$ is the yield on the corporate bond of maturity $t$. In addition, we also know the coupons $\{c_t\}_{t=1,2...}$ associated with each maturity. For now, we assume that the entire spread between $r^D_t$ and the reference risk free rate is due to default losses. In section 4 we discuss adjustments for tax and liquidity effects, and other data related issues. The date-0 value of the bond of maturity $t$ can be expressed as:

$$V_t = \frac{c_t}{(1 + r^D_t)} + \frac{c_t}{(1 + r^D_t)^2} + ... + \frac{1 + c_t}{(1 + r^D_t)^t}, \quad (8)$$

For simplicity, we use a discrete model in which all payments (coupons, face value and recoveries) that refer to year $t$ happen exactly at the end of year $t$. 

The corporate bond can default at any point in time. We let $\rho_{\tau,t}$ be the dollar amount recovered if default occurs at date $\tau \leq t$, for a bond of maturity $t$. As discussed by Duffie and Singleton (1999), we need to make assumptions about bond recoveries in order to obtain risk neutral probabilities from the term structure of bond yields. The credit risk literature has used three alternative assumptions:

1. Recovery of treasury (RT): $E(\rho_{\tau,t}) = E(\rho)P_{\tau,t}$, where $P_{\tau,t}$ is the price at date $\tau$ of a default-free bond that is exactly equivalent to the corporate bond. In particular, it has the same maturity $t$, and pays the same coupons as the corporate bond. This assumption is due to Jarrow and Turnbull (1995). The idea behind this assumption is that default does not change the timing of the cash flows promised by the corporate bond. If default occurs, the creditor receives a fraction $E(\rho)$ of exactly the same cash flows that would be received in the event of no default.\footnote{Consistent with the framework above, $E(\rho)$ is an unconditional expectation for the recovery fraction. We use both objective (historical) and risk-adjusted expected recoveries in the calculations below.}

2. Recovery of face value (RFV): $E(\rho_{\tau,t}) = E(\rho)$. This assumption has been used by Brennan and Schwartz (1980), and Duffee (1998). In words, if default happens at time $\tau < t$, creditors receive a fraction of face value immediately upon default. There is zero recovery of coupons.

3. Recovery of market value (RMV): $E(\rho_{\tau,t}) = E(\rho V_{\tau,t})$, where $V_{\tau,t}$ is the market value prior to default at date $\tau$ of the corporate bond, contingent on survival up to date $\tau$. This assumption is due to Duffie and Singleton (1999).

Duffie and Singleton (1999) compare risk-neutral probabilities that are generated by assumptions RMV and RFV, and find that the two alternative assumptions generate very similar results unless corporate bonds trade at significant premiums or discounts, or if the term structure of interest rates is steeply increasing or decreasing. For simplicity, we focus only on assumptions RT and RFV. Notice that $E(\rho)$ is the date-0 expectation of future recoveries.
recovery rates. To economize notation, we let $E(\rho) = \rho$. In most of our analysis we also make the following assumption:

**Assumption A2:** *The recovery rate on defaulted bonds is uncorrelated with the pricing kernel. In other words, there is no systematic recovery risk.*

Assumption A2 is directly related to the previous assumption of no systematic risk on $\phi$, the deadweight losses associated with financial distress. Given that we assume no systematic risk on $\phi$, it seems natural to assume no systematic risk on $\rho$ as well. However, there is some evidence in the literature that recovery rates tend to be lower in bad times (Altman et al., 2003, and Allen and Saunders, 2004). We relax assumption A2 in section 4.5 to verify the robustness of our results to the introduction of recovery risk.$^{21}$

### 3.2.1 Deriving Risk-Neutral Probabilities Under Assumption RT

Using assumption RT, we can solve forward for the term structure of risk-neutral probabilities implicit in promised yields, or equivalently, in bond prices. To see this, start with a one-year bond with current value $V_1$. Notice that RT implies $E(\rho_{1,1}) = \rho(1 + c_1)$, and that the bond defaults with probability $Q_1$ in year 1. Under assumption RT, we can then express $V_1$ as:

$$V_1 = [(1 - Q_1) + Q_1 \rho] (1 + c_1) B_1.$$  

(9)

This equation implicitly gives $Q_1$ as a function of known quantities. As shown in the appendix, we can write down a similar valuation equation for all maturities. Given $\{Q_\tau\}_{\tau=1..t}$, and supposing that one observes a coupon paying bond with maturity $t + 1$, the valuation formula becomes

$$V_{t+1} = \sum_{\tau=1}^{t} [(1 - Q_\tau) + Q_\tau \rho] c_{t+1} B_\tau +$$

$$+ [(1 - Q_{t+1}) + Q_{t+1} \rho] (1 + c_{t+1}) B_{t+1}. $$

(10)

$^{21}$We note, however, that the evidence for systematic recovery risk is not uncontroversial. For example, Acharya, Bharath and Srinivasan (2004) relate recovery rates to Fama-French factors, GDP growth and the SP 500 return, and do not find significant relationships. See Allen and Saunders (2004) for a broader review of the literature.
This equation can be inverted to obtain $Q_{t+1}$. Therefore, we can derive recursively the sequence of risk-adjusted probabilities $\{Q_t\}_{t=1,\ldots}$ from $\{V_t\}_{t=1,\ldots}$, $\{q_t\}_{t=1,\ldots}$, $\{B_t\}_{t=1,\ldots}$, and $\rho$. This procedure is the generalization of the simple equation 2 that we derived in Section 2. The risk-adjusted probabilities $\{Q_t\}_{t=1,\ldots}$ can then be used to value distress costs using equation 7.

In the appendix, we derive the corresponding formulas for assumption RFV. In the empirical section we use assumption RT for our benchmark calculations, but in section 4.5 we verify that the results are robust to the specific recovery assumption.

4 Empirical Estimates

We start by describing the data that we use to implement formulas 10 and 7. We then present our estimates for the risk-neutral probabilities of default, for different ratings and maturities. Finally, we present our estimates of the NPV of distress costs, and provide some robustness checks.

4.1 Data on Yield Spreads, Recovery Rates and Default Rates

We obtain data on corporate yield spreads over treasury bonds from Citigroup’s yield book, which reports average spreads over the period 1985-2004. The data is available for bonds rated A and BBB, for maturities 1-3, 3-7, 7-10, and 10+ years. For bonds rated BB and below the data is available only as an average across all maturities. The yieldbook also reports data for AAA and AA bonds as a single category. Instead of using this single category, we use Huang and Huang’s (2003) yield spread data for these two ratings, from Lehman’s bond index (Table 1 in Huang and Huang). They report spread data for maturities 4 and 10. Because Huang and Huang’s data cover a different period (1985-1995), we calculate averages from the yield book data for the same time period (1985-1995). We note that the average spreads are very similar if we use the entire period (1985-2004) available in the yield book data. For example, the average 10+ spread for BBB bonds in the yieldbook data is 1.90\% for both time periods. Average B-bond spreads are 5.45\% if we use 1985-1995, and
5.63\% if we use 1985-2004. In addition, Huang and Huang’s spread data are very similar to those from the yield book, for similar ratings and maturities. For example, the 10 year spread for BBB bonds in Huang and Huang is 1.94\%. Thus, we believe that the average spreads that we use are representative estimates of typical spreads for different ratings and maturities.

We report our spread data in Table 1. For all ratings, we fill out the maturities between 1 and 10 years that are not available in the raw data by linearly interpolating the spreads. We assume constant spreads across maturities for BB and B bonds.

We also obtain data on average treasury yields and zero coupon yields on government bonds of different maturities, from FRED and from JP Morgan. Because high expected inflation in the 1980’s had a large effect on government yields, we use a broader time period (1985-2004) to calculate these yields. Some average treasury yields that we use are 5.74\% (1-year), 6.32\% (5-year), and 6.73\% (10-year). The data is also available for years 2, 3, 7 and 20. Again, we use a simple linear interpolation for missing maturities between 1 and 10. We use zero coupon yields because formulas 10 and 7 require prices of risk free zero coupon bonds, but the zero yields are very close to those on treasuries. For example, the average 10-year zero yield is 6.87\%.

In addition, we obtain historical cumulative default probabilities from Moodys, for the period 1970-2001. The cumulative default rates are available from one year following the issuance of the bonds, up to 17 years following issuance.\textsuperscript{22} While these data are not used directly for the risk-adjusted valuations, they are useful for comparison purposes. Moodys also reports a time series of bond recovery rates for the period 1982-2001. In most of our calculations we assume a constant recovery rate, which we set to the average value in the Moodys’ data (0.413). This value is lower than the one used by Huang and Huang (0.513). Below (see section 4.5.1), we use the time series of recovery rates to address the impact of recovery risk in our calculations.

\textsuperscript{22}The credit rating of the bonds refers to that of the time of issuance.
4.2 Estimating the Fraction of the Yield Spread That is Due to Default Risk

There is an ongoing debate in the literature about the role of default risk in explaining yield spreads such as those reported in Table 1, vis-a-vis other potential explanations. Because treasuries are more liquid than corporate bonds, part of the spread should reflect a liquidity premium (see Chen et al., 2004). Also, treasuries have a tax-advantage over corporate bonds because they are not subject to state and local taxes (Elton et al., 2001). These arguments suggest that we cannot attribute the entire spread reported in Table 1 to default risk. Thus, one should not compute formulas 10 and 7 using the entire spreads over treasuries.

While the literature agrees that not all the yield spread is due to default, there is some controversy as to the specific fraction that one should attribute to default losses (the default component of spreads). A number of papers have attempted to estimate the default component of spreads. Huang and Huang (2003) use a structural credit risk valuation model calibrated to historical default rates, and argue that credit risk accounts for only a small fraction of spreads, specially for investment-grade bonds. In contrast, Longstaff et al. (2005) and Chen et al. (2005) suggest that credit risk has much more explanatory power than Huang and Huang’s results suggest. Longstaff et al., for example, argue that credit default swap (CDS) premiums are a good approximation for the default component of yield spreads, and they show that the ratio between CDS premiums and spreads over treasuries is much larger than those suggested by Huang and Huang (2003), specially for investment-grade bonds. Chen et al. (2005) show that the entire spread between BBB and AAA bonds can be explained by credit risk, while assuming that the spread between AAA and treasury bonds is entirely due to tax and liquidity considerations.

In Table 2, we summarize the findings of these three recent papers.\footnote{Actually, Huang and Huang (2003) and Longstaff et al. (2005) report not only the fractions reported in Table 2, but also other fractions calculated under different assumptions. Because Huang and Huang provide the lowest fraction estimates, we chose the highest fractions suggested by their paper (from Table 7). The ratio of the default component to the total spread for Longstaff et al. (2005) comes from their Table IV, which, according to the authors, reports results for their preferred specification.} As one can see from Table 2, the results in Longstaff et al. and Chen et al. suggest a larger role for credit
risk in explaining the yield spread. However, notice that Chen et al. consider only BBB bonds in their analysis, while Longstaff et al. do not provide estimates for AAA and B bonds. In addition, Huang and Huang provide estimates for 4- and 10-year maturities only, while Longstaff et al. and Chen et al. consider only one maturity (5-years, and 4-years, respectively). In contrast, a full implementation of formulas 10 and 7 requires a complete term structure of the default component of spreads.

In order to estimate the default component of spreads for all ratings and maturities reported in Table 1, we use two alternative strategies that are motivated by this recent literature.

**4.2.1 Method 1: Using the 1-year AAA spread**

Following Chen et al. (2005), we assume that the component of the spread that is not given by default can be inferred from the spreads between AAA bonds and treasuries. As explained above, Chen et al. use a 4-year maturity in their calculations. However, the results in Huang and Huang suggest that at a 4-year horizon there is already a small default component for AAA bonds (see Table 2). In addition, our data on historical default probabilities suggest that while there has never been any default for AAA bonds up to a 3-year horizon, there is already a small probability of default at a 4-year horizon (0.04%). Thus, it seems appropriate to use a more short term spread to adjust for taxes and liquidity, such as the 1-year spread of 0.51% (see Table 1). In any case, the difference between 1-year and 4-year AAA spreads (0.04%) is negligible, so using the 4-year spread would produce virtually identical results. Specifically, we calculate average default components for rating \( i \) and maturity \( t \) using the following formula:

\[
(\text{Default component})_{i,t} = (\text{spread})_{i,t} - 0.51\% 
\]

\((11)\)

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24 Longstaff et al. (2005) use data from the credit default swap market to estimate the fraction of credit spreads that is due to default. In particular, they argue that the swap premium is free of tax and liquidity effects, and thus can be used as a direct measure of spreads that are due to default losses. The default swaps in their data have a typical maturity of 5 years.

25 Another advantage of deducting only the 1-year AAA spread from the other spreads to compute the default component is that since it is the smallest spread in Table 1 the resulting default components are all weakly greater than zero.
Table 2 also reports the fractions due to default that are implied by this procedure, for select maturities. However, notice that formula 11 allows us to construct spread default components for all ratings and maturities. By construction, the 4-year BBB fraction is virtually identical to that estimated by Chen et al. (2005). However, most of the other fractions are very close to those estimated by Longstaff et al., suggesting that method 1 produces default components that closely approximate CDS premiums. Not surprisingly, the only real discrepancy is with respect to Huang and Huang, who calculated lower fractions for investment-grade ratings.

4.2.2 Method 2: Using Spreads Over Swaps

As discussed above, Longstaff et al. (2005) (and also Cremers et al. (2005)) argue that credit default swap (CDS) premiums are a good approximation for the default component of yield spreads for the 5-year maturity. In addition, Blanco et al. (2005) show that, for the 5-year maturity, the yield spread over swaps tracks CDS premiums very closely. These results suggest that one can use spreads over swaps to estimate the default component of yield spreads. The advantage of using spreads over swaps (instead of CDS premiums directly), for our purposes, is that these are available for more maturities and ratings. However, data on swap rates start only in 2000. Therefore, we cannot work with Huang and Huang’s spread data (which refers to 1985-1995), and consequently can only provide fraction estimates for A, BBB, BB and B-rated bonds.

We collect swap data for 2000-2004, which allow us to calculate the 2000-2004 average fraction of the spread over treasury that is due to default for A to B bonds. Specifically, we use the following formula for 2000-2004 averages for rating \( i \) and maturity \( t \):\(^{26}\)

\[
(Fraction \ due \ to \ default)_{i,t} = \frac{(spread)_{i,t} - (swap_t - treasury_t)}{(spread)_{i,t}}
\]

As we show in Table 2, this alternative approach gives fractions due to default that are very close to those that we obtain using the AAA spread (method 1), and thus also very close

\(^{26}\)Because the spreads over swaps refer to a different time period, for comparison purposes we focus on fractions due to default as opposed to the level of the default component.
to Longstaff et al. (2005) and Chen et al. (2005). Given these results, we choose method 1 as our benchmark approach to calculate default components. An important advantage of this method given the data that we have is that it allows us to present valuations for all bond ratings, from AAA to B. Given the similarity of the fractions implied by method 1 and method 2, it makes little difference to use either method to value distress costs for the A, BBB, BB and B ratings.

On the other hand, these methods predict larger default components than those estimated by Huang and Huang (2003). To provide evidence on the importance of the default component for the valuation of distress costs, we will also provide valuation estimates that obtain using Huang and Huang's (2003) fractions due to default. Given that they have estimated fractions for two different maturities (4- and 10-year), we can calculate the default component of spreads for these two maturities and interpolate the default components directly.

4.3 Estimating Risk Neutral Probabilities

To recapitulate what we have done so far in the empirical section, we start from the partially interpolated spreads in Table 1, and then we apply the adjustment described in equation 11 for all ratings and maturities to calculate the default component of the spreads. We now use these default components to estimate a term structure of default probabilities.

The implied bond price for each rating and maturity can be found using equation 8. The promised yields $r_t^D$ are computed as the sum of the corresponding treasury rate and the default components of equation 11. Notice that we must make an assumption about coupon rates in order to use equation 8. The simplest assumption is that the corporate bonds trade at par, such that $c_t = r_t^D$, and $V_t = 1$ for all $t$. To show robustness with respect to the coupon assumption, we consider in section 4.5 the alternative assumptions that $c_t = (1 + k) * r_t^D$ (bonds trade at a premium), and $c_t = (1 - k) * r_t^D$ (bonds trade at a discount), for $k = 0.5$.

Given that $V_t = 1$ and $c_t = r_t^D$, equation 10 can be used to generate a sequence of
cumulative probabilities of default $\{Q_t\}_{t=1,2,...}$. Since we have spread data up to maturity 10, we can generate a term structure of probabilities up to year 10 as well.

Table 3 reports the risk-adjusted cumulative default probabilities for select maturities. For comparison purposes, we also report the historical cumulative probabilities of default using the Moody’s data that we described above. Clearly, the risk-adjusted, market-implied probabilities are substantially larger than the historical ones for all ratings and maturities, but specially for investment-grade bonds. For example, for BBB bonds, while the 5-year historical default probability is 1.95%, the risk-neutral one is 11.39%. The average (across maturities) ratio between risk-neutral and historical probabilities for each rating goes from 3.57 for AAA-rated bonds to 1.21 for B-rated bonds. These differences suggest a large credit risk premium implicit in bond yield spreads.

### 4.4 Valuation

We can now use the term structure of risk-neutral probabilities computed in Section 4.3 in the valuation equation 7. Because we only have cumulative default probabilities up to year 10, we compute a terminal value of financial distress costs at year 10 (details in the appendix).

Regarding $\phi$, the papers discussed in the introduction suggest that the term $\phi$ should be of the order of 10% to 23% of pre-distress firm value. Graham (2004) and Molina (2005) use numbers in this range to compare tax benefits of debt and costs of financial distress. As in Section 2, we use the mid-point between the two extreme values ($\phi = 16.5\%$), but we present robustness checks for changes in $\phi$ in the next section.

The second column of Table 4 shows our estimates of the risk-adjusted cost of financial distress, using equation 7 and the risk-adjusted probabilities reported in Table 3, for different bond ratings. For comparison purposes, in the first column we report valuations using the historical default probabilities (notice that equation 7 only requires default probabilities and risk free rates to translate $\phi$-estimates into NPV estimates).\footnote{We assume that the historical marginal default probability is fixed after year 10 for each rating. We have data on the complete term structure of historical default probabilities from year 1 until year 17. We}
If we use historical probabilities to value financial distress, the cost of distress goes from approximately 0.25% for AAA-rated bonds to up to 7.25% for B-rated bonds. The risk-adjustment has a substantial impact in these costs, confirming the results of Section 2. For example, the distress cost for the BBB-rating goes from 1.40% to 4.53%.28 In order to provide some evidence on the marginal increase in distress costs as the firm moves across ratings, we also report the difference in distress costs between the BBB and the AA rating. An increase in leverage that moves a firm from an AA to a BBB rating increases the cost of distress by 1.11% if we use historical probabilities to value distress costs. However, the corresponding increase in risk-adjusted distress costs is 2.7%, which is approximately 2.5 times higher. Thus, the marginal effect of a decrease in rating on the cost of distress can be quite large. In section 5 we compare these marginal costs with marginal tax benefits of debt.

4.5 Robustness checks

The valuation in column II of Table 4 is based on several specific assumptions about bond recoveries, coupon rates, and deadweight losses given distress. We now consider variations of these assumptions to check the robustness of the results.

4.5.1 Recovery Risk

Following assumption A2, the benchmark valuation in column II uses the average historical recovery rate (0.413) in equation 10. The use of an average historical recovery is common in the credit risk literature. Huang and Huang (2003) and Chen et al. (2005), for example, use average historical recoveries of 0.51 in their calibrations. However, there is some evidence in the literature of a systematic component of recovery risk (Altman et al., 2003, and Allen and Saunders, 2004). As discussed by Berndt et al. (2005) and Pan and Singleton (2005), a standard way to incorporate recovery risk into credit risk models is to use a constant risk-neutral (as opposed to average historical) recovery rate. Berndt et al. (2005) use a

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28 Notice this is a larger increase than that calculated in section 2.
risk-neutral recovery rate of 0.25, which is the lowest cross-sectional sample mean of recovery reported by Altman et al (2003).\textsuperscript{29} According to Pan and Singleton (2005), this is a common industry standard for the risk-neutral recovery rate. Pan and Singleton (2005) use the term structure of sovereign CDS spreads to separately estimate risk-neutral recoveries and default intensities, and estimate recovery rates that are larger than the commonly used value of 0.25.

As we discussed in section 2, the lower the recovery rate that we use in equation 10, the lower the implied risk-neutral probabilities. Nevertheless, it is not clear that a reduction in the risk-neutral probabilities is the only channel through which recovery risk will affect financial distress costs. Recall that we are also assuming that the risk associated with the deadweight losses given default is non-systematic (Assumption A1). Clearly, systematic recovery risk can be associated with systematic risk in $\phi$. This argument suggests that a reduction in the recovery rate $\rho$ should be associated with a corresponding increase in $\phi$, and that the use of historical recovery rates might be less problematic than it seems at a first glance.

In any case, in column III of Table 4 we report the results of decreasing the recovery rate to 0.25, without changing the estimate for $\phi$. Consistent with intuition, the risk-adjusted costs of financial distress decrease with lower bond recoveries. For example, the point estimate for the BBB rating goes from 4.53\% to 3.70\% if recovery goes from 0.41 to 0.25. However, the magnitude of the risk adjustment is considerable even at the most aggressive estimate for the risk neutral recovery. In addition, notice that the impact of lower recoveries on the marginal costs of distress is smaller. For example, the increase in distress costs for a firm that moves from AA to BBB goes down from 2.69\% to 2.23\%. We conclude that the results are robust to the introduction of recovery risk.

\textsuperscript{29}This is close to the recovery rate in our time-series data (1970-2001), which contains cross-sectional average recoveries from Moodys. The lowest recovery in our sample is 0.21 (2001).
4.5.2 Recovery of Face Value

Equation 10 is derived under the assumption that recovery is a fraction of a similar risk-free bond (RT assumption). As we discuss in section 3.2, another commonly used assumption is that recovery is a fraction of the face value of the bond, with zero recovery of coupons. In the appendix, we derive a formula that allows us to obtain the term structure of risk-neutral probabilities from the default component of the spreads. Column IV of Table 4 shows the valuation results that obtain with this alternative assumption. Because there is zero recovery of coupons, the implied risk neutral probabilities of default are lower, and thus the valuation results are slightly lower than those that obtain under RT. However, it is clear from column IV that the two assumptions generate very similar costs of financial distress. The AA minus BBB margin, for example, goes from 2.69% (under RT) to 2.47% (RFV). We conclude that the valuation is robust to alternative recovery assumptions.

4.5.3 Coupon Rates

The risk neutral probabilities in Table 3 are derived under the assumption that the bond coupons are equal to the adjusted bond yields (the default component of the yield plus the corresponding treasury rate). In order to show that the results are robust to different assumptions about coupons, we report in columns V and VI of Table 4 the valuation results that obtain if coupons are equal to 0.5 times the adjusted yields (on column V), or 1.5 times the adjusted yields (column VI). Risk-adjusted probabilities, and thus risk-adjusted distress costs, are higher with higher coupons. The effect of coupons is particularly pronounced for the highest bond ratings. However, it is clear from the Table that the results are relatively robust to variations in coupon rates. This is specially true of the marginal costs of distress. The BBB minus AA margin, for example, goes from 2.64% (when coupons are 0.5 times the yield) to 2.77% (when coupons are 1.5 times the yield). Thus, variation in coupons have little effect on the marginal costs of financial distress.
4.5.4 Using Huang and Huang’s (2003) fractions

As discussed above in Section 4.2, Huang and Huang (2003) have estimated default component of spreads that are substantially lower than those used in the estimates reported in Tables 3 and 4. Not surprisingly, using Huang and Huang’s fractions leads to lower costs of financial distress, as shown in column VII. The difference is specially pronounced for ratings between AAA and BBB. The BBB minus AA margin, for example, decreases approximately 1%, to 1.65%. This margin is close to that calculated using historical probabilities. These results show that if spreads are mostly due to factors other than default risk, then the impact of the risk-adjustment on distress costs is less substantial. While not surprising, these results highlight the importance of more recent papers such as Longstaff et al (2005) and Chen et al. (2005), who suggest that credit risk can indeed explain a large fraction of spreads. We focus on these latter set of results from now on.

4.5.5 Changes in $\phi$

Columns I and II assume a $\phi$ that is the midpoint (16.5%) of the range suggested by Andrade and Kaplan (1998) and other previous literature that estimates deadweight losses given distress (10-23%). In Panel B of Table 4 we report valuation results for the endpoints of this range, both for historical and risk-adjusted probabilities. Notice that unlike the robustness checks above, which only affect risk-adjusted probabilities, these variations also impact the valuation using historical probabilities.

Not surprisingly, direct changes in $\phi$ have a large impact on the valuations, both for historical and risk-adjusted probabilities. For example, the risk-adjusted BBB valuation goes from 1.95% (if $\phi = 10\%$), to 6.32% (if $\phi = 23\%$). Because the impact of changes in $\phi$ is higher if default probabilities are high, the effect on the margins is also large, specially when compared with the other assumptions in Table 4. The AA-BBB margin goes from 1.63% to 3.75% as $\phi$ goes from 10% to 23%. Thus, it is important to consider a range of values for $\phi$ in the capital structure exercises of the next section.

On the other hand, the results are robust in an important sense. The difference between
historical and risk-adjusted valuations remains important, irrespective of the specific value for $\phi$. For example, if $\phi = 10\%$, the increase in the BBB-valuation that can be attributed to the risk-adjustment is still equal to 1.90%. Thus, it appears to be the case that ignoring the risk-adjustment substantially undervalues the costs of distress, irrespective of the particular value for $\phi$.

4.5.6 Time Variation in Spreads

We have conducted our analysis from an ex-ante perspective, using average spreads for each rating and maturity. Conceptually, we have answered the question: what are the costs of financial distress for a company about to be created, assuming that aggregate business conditions are and will remain at historical averages?

In reality, however, the market price of credit risk (as captured by credit spreads) varies over time (see Berndt et al. (2005), and Pan and Singleton (2005)). This insight has two important implications for our paper. First, the NPV of future financial distress costs will change over time as credit spreads move around. Second, we might actually underestimate the size of the risk adjustment, because a risk-adjusted ex-ante valuation should put more probability weight on episodes of high spreads than on those of low spreads.

In order to understand these points more clearly, consider Figure 4. We want to compute the value of financial distress at time 0. At time 1, an aggregate shock is realized, which affects the price of credit risk, and thus changes credit spreads. We assume that the probability of each state, under the historical, objective measure (the “P-measure”) is 0.5. If spreads are high, the analysis above suggests that the risk-adjusted probability of financial distress will also be high. That is, $q_H$ (the risk-adjusted probability of distress conditional on high spreads) is higher than $q_L$ (the risk-adjusted probability of distress conditional on low spreads). These probabilities are depicted in Figure 4. At time 2, the firm learns whether financial distress occurs or not.

The risk-adjusted valuation that we performed in section 3 uses historical data to compute average spreads, and then uses those spreads to compute risk-adjusted probabilities.
of financial distress. In the context of Figure 4, the NPV of financial distress using the methodology of section 3 would be:

\[ \Phi_P = \frac{0.5q_H + 0.5q_L}{(1 + r_F)^2} \phi = \frac{E_P(q)\phi}{(1 + r_F)^2}, \]  

(13)

where \( E_P(q) = 0.5q_H + 0.5q_L \) is the date-0 expectation of the (time-varying, conditional) probability of financial distress, under the P-measure. This valuation formula incorporates systematic risk, because it uses the risk-adjusted conditional probabilities of distress \( q_H \) and \( q_L \). These probabilities are higher than their historical counterparts, reflecting the systematic risk that is incorporated in credit spreads.

Nevertheless, this valuation formula assumes that investors are risk-neutral towards the risk of variation over time in spreads (or risk-adjusted probabilities). However, it is likely that investors will also attribute a risk-premium to the uncertainty about spreads, or in other words, the risk-adjusted date-0 expectation of future default probabilities should be higher than that calculated under historical averages:\(^{30}\)

\[ E_Q(q) > E_P(q) \]  

(14)

Clearly, a valuation that incorporates the difference between \( E_Q(q) \) and \( E_P(q) \) will yield even higher NPVs of financial distress than those depicted in Table 4.

A complete, rigorous, adjustment for time-varying spreads is beyond the scope of this paper. However, we believe that incorporating time-variation in distress risk in dynamic capital structure models is an interesting avenue for future research (more on this in section 6). To get some sense of the impact of time-variation in spreads on conditional financial distress costs, we perform a simple exercise. As described in section 4.1, we have monthly time-series data between 1985-2004 for all ratings between A and B.\(^{31}\) We use these data to compute the standard deviation in spreads separately for each rating and maturity, as a fraction of average 1985-2004 spreads for that rating/maturity. These ratios range from

\(^{30}\)See Pan and Singleton (2005) for evidence on the risk premium associated with time-variation in default probabilities for sovereign bonds.

\(^{31}\)We do not have time-series data for AAA and AA bonds separately, so we restrict ourselves to A, BBB, BB and B ratings.
50% to 80% for A bonds (depending on maturity), 36% to 70% for BBB bonds, 38% for BB bonds and 33% for B bonds. We then scale our benchmark average spreads, which are calculated in 1985-1995 (see section 4.1), up and down uniformly using these ratios. Under the assumption that all spreads move together over time, these scaled spreads represent typical scenarios of high and low spreads (one standard deviation higher or lower than the mean).

Using these scaled spreads, we repeat the valuation exercises of sections 4.3 and 4.4.\textsuperscript{32} As in the benchmark valuation, these valuations assume that the scenarios of high, low and average spreads will last forever. The results are reported in Table 5. Clearly, the extent of time-variation in spreads is large enough to generate substantial fluctuations in the NPV of financial distress costs. For example, for BBB bonds the NPV of distress goes from 4.73\% (low spreads) to 8.38\% (high spreads). The impact of time-variation on margins, however, is less clear. The difference in distress costs between A and BBB bonds, for example, is highest when spreads are low. However, the difference between A and BB bonds shows the opposite pattern (highest when spreads are high). While these results show that time-variation in the price of credit risk can have significant effects on spreads, more research is required to establish its exact impact on marginal distress costs and capital structure choices.

5 Implications for Capital Structure

Existing literature suggests that distress costs are too small to overcome potential tax benefits of increased leverage, and thus corporations may be using debt too conservatively (Graham, 2000). This quote from Andrade and Kaplan (1998) captures well the standard belief in the literature:

“[..] from an ex-ante perspective that trades off expected costs of financial distress against the tax and incentive benefits of debt, the costs of financial

\textsuperscript{32}In these exercises, we keep all parameters fixed at their benchmark values, including recovery rates (0.41), losses given distress (0.165), and risk-free rates.
distress seem low [...]. If the costs are 10 percent, then the expected costs of distress [...] are modest because the probability of financial distress is very small for most public companies.” (Andrade and Kaplan, p. 1488-1489).

In other words, using estimates for $\phi$ that are in the same range as those used in Table 4 should not produce large NPVs of financial distress, because the probability of financial distress is very low. In this section, we attempt to verify whether this conclusion continues to hold if we use our valuation formulas to calculate marginal costs of financial distress.

Naturally, the calculations that we perform in this section are subject to the limitations of the static trade-off model of capital structure. Our point is not to argue that this model is the correct one, nor to provide a full characterization of firms’ optimal financial policies. We simply want to verify whether the magnitude of the costs of financial distress that we calculate is comparable to that of tax benefits of debt. We believe this is a worthy exercise, given the common belief that financial distress costs seem to be too small to matter for capital structure choices.

5.1 Estimating the Effect of Distress Risk on Capital Structure

In order to compare the financial distress costs displayed in Table 4 with tax benefits of debt, we need to estimate the typical tax benefits that the average firm can expect at each bond rating. To do this, we follow closely the analysis in Graham (2000), who estimates the marginal tax benefits of debt, and Molina (2005), who relates leverage ratios to bond ratings.

5.1.1 The Marginal Tax Benefit of Debt

Graham (2000) estimates the marginal tax benefit of debt as a function of the amount of interest deducted, and calculates total tax benefits of debt by integrating under this function. The marginal tax benefit is constant up to a certain amount of leverage, and then it starts declining because firms do not pay taxes in all states of nature, and because higher leverage decreases additional marginal benefits (as there is less income to shield).
Essentially, we can think of the tax benefits of debt in Graham (2000) as being equal to 
\( \tau^*D \) (where \( \tau^* \) takes into account both personal and corporate taxes) for leverage ratios 
that are low enough such that the firm has not reached the point at which marginal benefits 
start decreasing (see footnote 13 in Graham’s paper). If leverage is higher than this, then 
marginal benefits start decreasing. Graham calls this point the kink in the firm’s tax benefit 
function. Formally, the kink is defined as:

\[
kink = \frac{\text{amount of interest that causes marginal benefit to start decreasing}}{\text{actual interest expense}}, \quad (15)
\]

so that a firm with a kink of 2 can double its interest deductions, and still keep a constant 
marginal benefit of debt. Firms with high kinks use leverage more conservatively.

Graham calculates the amount of tax benefits that the average firm in his sample foregoes. The average firm in COMPUSTAT (in the time period 1980-1994) has a kink of 2.356, 
and a leverage ratio of approximately 0.34. Graham also estimates that the average firm 
could have gained 7.3% of their market value if it levers up to its kink. In addition, notice 
that the firm remains in the flat portion of the marginal benefit curve until its kink reaches 
one. Thus, these numbers allow us to compute the marginal benefit of increasing debt in 
the flat portion of the curve (\( \tau^* \)) implied in Graham’s data. If we assume that the typical 
firm needs to increase leverage by 2.356 times to move to a kink equal to one, we can back 
out the value of \( \tau^* \) as 0.157. Because we can use the formula \( \tau^*D \) in the flat portion of the 
curve, we can calculate tax benefits for each leverage ratio, assuming that kink is higher 
than one. Clearly, this approximation is no longer reasonable if leverage becomes too high. 
We use this approximation in the calculations below. To the extent that the approximation 
is not true for high leverage ratios, we are probably overestimating tax benefits of debt for 
these leverage values.\(^{33}\)

\(^{33}\)A related point is that these tax benefit calculations ignore systematic risk adjustments. We derive this 
adjustment in a previous version of the paper, assuming perpetual debt. If \( D \) is taken to be the market value 
of debt, the risk-adjustment does not have a substantial effect on Graham’s formula, because it is already 
incorporated in \( D \). In fact, with zero recovery rates the cash flows from tax benefits of debt are exactly a 
fraction \( \tau \) of the cash flows to bondholders in all states, and thus by arbitrage the value of tax benefits must 
be exactly equal to \( \tau D \). With non-zero recovery, there is a small risk-adjustment that reduces tax benefits, 
but its quantitative effect is small.
5.1.2 The Relation Between Leverage and Bond Ratings

To compute the tax benefits of debt at each bond rating, we need to assign a typical leverage ratio for each bond rating. A simple way of doing this is to collect average or median leverage ratios for each bond rating from COMPUSTAT. However, as discussed by Molina (2005), the relationship between leverage and ratings is affected by the endogeneity of the leverage decision. In particular, because less risky and more profitable firms can have higher leverage without increasing much the probability of financial distress, the impact of leverage on bond ratings might appear to be too small if we ignore this endogeneity problem.

The leverage data that we use is summarized in Table 6. Column I reports Molina’s predicted leverage values for each bond rating, from his Table VI (Molina (2005), p.1445). This table associates leverage ratios to each rating, using Molina’s regression model in Table V, and values of the control variables that are set equal to those of the average firm with a kink of approximately two in Graham’s (2000) sample. According to Molina, these values give an estimate of the impact of leverage on ratings for the average firm in Graham’s sample. In order to verify the robustness of our results, we also use the simple descriptive statistics in Molina’s (2005) Table IV (p. 1442). Molina’s data, which corresponds to the ratio of long term debt to book assets for each rating in the period 1998-2002, is reported in column II of Table 6. As discussed by Molina, despite the aforementioned endogeneity problem the rating changes in these summary statistics actually resemble those predicted by the model. We have also collected average (book) leverage ratios at each rating for manufacturing firms in a broader time period (1981-2004).34 The relation between leverage and ratings in this broader period is similar to that in Molina (see column III), with slightly lower leverage ratios at each rating. In the calculations below, we use both the leverage ratios in column I and the simple summary statistics in column II.

34 We use book leverage ratios to be consistent with Molina. Because market leverage ratios are lower, this choice will if anything overestimate tax benefits of debt.
5.1.3 Comparing Tax Benefits of Debt and Marginal Distress Costs

Table 7 depicts our estimates of the tax benefits of debt for each bond rating, using Molina’s (2005) leverage ratios (Table 6, columns I and II), and Graham’s (2000) marginal tax benefit of debt \( \tau^* = 0.157 \). If we use the leverage ratios from Molina’s (2005) regression model (Panel A), for example, the increase in tax benefits as the firm moves from the AA to the BBB rating is 2.67%. Under the benchmark valuation of distress costs (see Table 4), this marginal gain is clearly higher than the marginal cost calculated under historical probabilities (the latter number is 1.11%, according to Table 4). However, it is of a similar magnitude as marginal risk-adjusted distress costs (2.69% according to Table 4). The analysis of Table 4 also shows that the similarity between marginal tax benefits of debt and marginal financial distress costs holds irrespective of our specific assumptions about coupons and recoveries, as long as we use the benchmark assumption of \( \phi = 16.5\% \) (see Section 4.5). The comparison is even more favorable to distress costs if we use Molina’s summary statistics to compute marginal tax benefits of debt. In this case, Panel B of Table 7 shows that the BBB minus AA tax benefit margin is 1.73%.

In order to further compare marginal tax benefits and distress costs, Table 7 also reports the difference between the present value of tax benefits and the cost of distress for each bond rating. Under the static trade-off model of capital structure, the firm is assumed to maximize this difference. Because the specific assumption about \( \phi \) substantially affects marginal distress costs (see Panel B of Table 4), we report results that obtain for \( \phi = 10\% \) and \( \phi = 23\% \), as well as for the benchmark case of \( \phi = 16.5\% \). Panel A uses leverage ratios from Molina’s (2005) regression model, and Panel B uses Molina’s summary statistics.

The first conclusion that is obvious from the results in Table 7 is that the distress risk-adjustment substantially reduces the net gains that the average firm can expect from leveraging up. For example, if \( \phi = 16.5\% \) (columns 2 and 3), the firm can increase value by 3% to 4% (depending on the specific assumption about leverage ratios) if it leverages up from zero leverage to somewhere around a BBB bond rating. However, once we incorporate the distress risk-adjustment the net gain from leveraging up never goes above 1%. The small
differences between tax benefits of debt and risk-adjusted financial distress costs hold for almost all specifications, with the exception of column 5 in Panel A. However, notice that in Panel B the gains from levering up are modest even if $\phi = 10\%$.

The second, and related conclusion is that the distress risk-adjustment generally moves the optimal bond rating generated by these simple calculations towards the highest (investment-grade) ratings. For example, if $\phi = 16.5\%$, a firm that values distress costs using historical probabilities should increase leverage until it reaches a rating of A to BBB, which is associated with the largest differences between tax benefits and distress costs both in Panel A and in Panel B. However, after incorporating the distress risk adjustment the difference becomes essentially flat or decreasing for all ratings lower than AA (Panel A), or alternatively for all ratings (Panel B). Naturally, the result is even stronger for higher values of $\phi$. If $\phi$ becomes close to 10%, then the effect of the risk adjustment on the optimal bond rating depends somewhat on the particular assumption about leverage ratios (Panel A or Panel B).

Naturally, both conclusions are driven by the basic finding that marginal risk-adjusted distress costs are very close to marginal tax benefits of debt. Figure gives a visual picture of these results. In Figure 4 we plot the difference between tax benefits and distress costs for the benchmark case ($\phi = 16.5\%$, and leverage ratios from Panel A), both for non-risk-adjusted and risk-adjusted distress costs. Clearly, the marginal gains from increasing leverage are very flat for any rating above AA, if distress costs are risk-adjusted. The visual difference with the inverted U-shape generated by the non-risk adjusted valuation is very clear.

5.2 Interpretation and comparison with previous literature

Table 7 and Figure 4 shows that risk-adjusted costs of financial distress can counteract the marginal tax benefits of debt estimated by Graham (2000). Our results show mainly modest net gains in value of moving from the highest ratings such as AAA/AA. These results suggest that financial distress costs can help explain why firms use debt conservatively, as suggested by Graham (2000). We note, however, that Graham’s evidence for debt conservatism is not
based only on the observation that the average firm appears to use too little debt. It is also the case in his data that firms that appear to have low costs of financial distress have lower leverage (higher kinks). Our results do not address this cross-sectional aspect of debt conservatism.

Molina (2005) argues that the bigger impact of leverage on bond ratings and probabilities of distress that he finds after correcting for the endogeneity of the leverage decision can also help explain why firms use debt conservatively. However, Molina does not perform a full-fledged valuation of financial distress costs like we do in this paper. His calculations are based on the same approximation of marginal costs of financial distress used by Graham (2000), which is to write $\Phi = p\phi$, where $p$ is the 10-year cumulative historical default rate. As we discuss in Section 2, this formula underestimates the NPV of financial distress costs, even disregarding the risk-adjustment issue.35 Thus, the results on Table 7 and Figure 4 provide a more precise comparison between the NPV of distress costs and the capitalized tax benefits of debt, which incorporate Molina’s results through the relationship between leverage and bond ratings.

6 Final Remarks

We develop a methodology to estimate the present value of financial distress costs, which takes into account the systematic component in the risk of distress. Given the simplicity of our formulas and their easy implementation, we believe that they will be useful for practitioners and academics alike, for research and teaching purposes. In addition, our results show that the traditional practice of assuming risk-neutrality to value distress costs can result in severe underestimation of both the level of distress costs, and of the marginal effect of leverage on these costs. The marginal distress costs that we find can help explain the apparent reluctance of firms to increase their leverage, despite the existence of substantial

---

35 In addition, there are two differences between our calculations and those performed by Molina. First, his marginal tax benefits of debt are smaller than the ones we use, because he uses more recent data from Graham that implies a $t^*$ of around 13%. Second, when comparing marginal tax benefits with marginal costs of distress (Table VII) he uses the minimum change in leverage that induces a rating downgrade. In contrast, we use the average leverage values for each rating in Table 7.
tax benefits of debt. While large costs of financial distress are probably not the only reason why firms appear to be debt conservative, our results suggest that they can be part of the story.

In order to risk-adjust financial distress costs, we used information from average historical credit spreads. As we discussed in section 4.5.6, this exercise assumes that investors are risk-neutral towards the risk of variation over time in spreads (risk-adjusted default probabilities). Our capital structure calculations are also essentially static, in that they compare marginal financial distress costs implicit in historical average spreads with average marginal tax benefits of debt. However, given the extent of time variation in credit spreads (Berndt et al. (2005), Pan and Singleton (2005)), we believe that future research could introduce risk-adjusted (and time-varying) distress costs in dynamic capital structure models. It is possible, for example, that time variation in financial distress costs can yield predictions similar to those emphasized by the market timing literature, in that it might be optimal for firms to reduce their leverage in times when credit spreads are high (bond prices are low).

The large risk-adjusted NPVs of distress that we find are a direct consequence of the credit spread puzzle, namely the fact that bond yields spreads are too large to be explained by historical default rates. Thus, the fact that investors seem to require large risk premia to hold corporate bonds might justify firms’ aversion to leverage, if the firm’s goal is to maximize the wealth of these risk-averse investors. In other words, bond spreads and capital structure decisions appear to be consistent with each other. Recently, Cremers, Driessen, Maenhout and Weinbaum (2005) have shown that implied volatilities and jump risks, measured in option prices, can explain credit spreads across firms and over time. In other words, corporate bonds spreads and option prices are also consistent with each other. Taken together, these results suggest that risk aversion in financial markets may be high, but that it is not arbitrary. Market participants, from options and bonds traders to corporate managers, seem to respond similarly to the price of risk.
References


Huang, J. Z. and M. Huang, 2003, How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?, working paper, Penn State University and Stanford University.


Pan, J. and K. Singleton, 2005, Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads, working paper.


Appendix.

Proof of Equation 2 in the perpetuity example of Section 2: Suppose that the promised return on a perpetual bond is constant over time, and that given default creditors recover a fraction of the bond’s market value just prior to default, including the coupons that are due on the year that default occurs. This “recovery of market value” assumption is due to Duffie and Singleton (1999). In addition, we maintain the assumptions that the recovery rate is non-stochastic and that the risk free rate is constant (equal to \( r^F \)).

Let the bond’s promised yearly return be equal to \( r^D \). Without loss of generality, assume that the bond is priced at par such that the yearly coupon is also equal to \( r^D \). Next year, if the bond does not default, creditors receive a coupon equal to \( r^D \). The value of the remaining promised payments is constant over time, and equal to one. Thus, creditors receive \((1 + r^D)\) if there is no default. The recovery of market value assumption implies that creditors will receive \( \rho(1 + r^D) \) if there is default in year one. Thus, the bond valuation tree is identical to that presented in Figure 1, and the bond’s date zero value can be expressed by equation 1. Formula 2 then follows. Q.E.D.

Proof of equation 10: In order to see how the general term in equation 10 is going to look like it is enough to consider a two-year bond. If the bond defaults in year 1 we have:

\[
E(\rho_{1,2}) = \rho \times (c_1 + (1 + c_2)B_{1,2}),
\]

where \( B_{1,2} \) is the discount factor that brings cash flows from period 2 to period 1 (the date-1 price of a zero that pays one at date 2). If the bond defaults in year 2, we have \( E(\rho_{2,2}) = \rho(1 + c_2) \). We can then write the valuation equation for a two-period bond as (see Figure 3):

\[
V_2 = (1 - Q_1)c_2 B_1 + Q_1 \rho \times [c_2 + (1 + c_2)B_{1,2}] B_1 + \rho Q_2 \rho B_2,
\]

where, using the facts that \( B_2 = B_{1,2} B_1 \), \((1 - Q_1)(1 - q_2) = (1 - Q_2)\), and \( Q_1 + (1 - Q_1) q_2 = Q_2\), can be written as:

\[
V_2 = [(1 - Q_1) + Q_2 \rho] c_2 B_1 + [(1 - Q_2) + Q_2 \rho] (1 + c_2) B_2.
\]

Given \( Q_1 \), we can solve A3 for \( Q_2 \). The general term is clearly:

\[
V_{t+1} = \sum_{\tau = 1}^{t} [(1 - Q_\tau) + Q_{\tau} \rho] c_{t+1} B_\tau + \rho [(1 - Q_{t+1}) + Q_{t+1} \rho] (1 + c_{t+1}) B_{t+1}.
\]

Q.E.D.

Bond valuation formula using assumption RFV: Under assumption RFV, \( E(\rho_{t,t}) = \rho \) for all \( t \), and the valuation formula becomes:

\[
V_{t+1} = c_{t+1} \sum_{\tau = 1}^{t} (1 - Q_\tau) B_\tau + \rho \sum_{\tau = 1}^{t+1} (1 - Q_{\tau-1}) q_\tau B_\tau + (1 + c_{t+1}) (1 - Q_{t+1}) B_{t+1}
\]

Again, this formula can be easily inverted to obtain \( Q_{t+1} \) if one has the sequence \( \{Q_\tau\}_{\tau=1}^t \), and the yield on a coupon paying bond with maturity \( t + 1 \). Notice that \( Q_0 = 1 \), and that \( q_{t+1} = 1 - \frac{(1 - Q_{t+1})}{(1 - Q_t)} \).

Terminal Value calculation:

We assume that the marginal, risk-adjusted probability of default is constant after year 10, that is:

\[
q_\tau = q_{10} = 1 - \frac{(1 - Q_{10})}{(1 - Q_0)} \quad \text{for } t > 10.
\]

Similarly, we assume that the yearly zero coupon rate is constant after year 10, that is, the yearly risk free rate after year 10 is given by:

\[
r_{10}^F = \frac{B_0}{B_{10}} - 1
\]
Given these assumptions, we can compute a terminal cost of financial distress at year 10. We can expand equation 7 as:

$$
\Phi = \phi \left[ \sum_{t=1}^{10} B_t (1 - Q_{t-1}) q_t + (1 - Q_{10}) q_{11} B_{11} + (1 - Q_{11}) q_{12} B_{12} + \ldots \right].
$$

Using the assumptions that $q_t = q_{10}$ and $r^F_t = r^F_{10}$ for $t > 10$, we can write:

$$
\Phi = \phi \left[ \sum_{t=1}^{10} B_t (1 - Q_{t-1}) q_t + (1 - Q_{10}) q_{10} \frac{B_{10}}{(1 + r^F_{10})} + (1 - Q_{10}) (1 - q_{10}) q_{10} \frac{B_{10}}{(1 + r^F_{10})^2} + \ldots \right] = (A8)
$$

$$
= \phi \left[ \sum_{t=1}^{10} B_t (1 - Q_{t-1}) q_t + \frac{B_{10} (1 - Q_{10}) q_{10}}{(1 + r^F_{10})} \left( 1 + \frac{1 - q_{10}}{1 + r^F_{10}} + \frac{(1 - q_{10})^2}{(1 + r^F_{10})^2} + \ldots \right) \right] =
$$

$$
= \phi \left[ \sum_{t=1}^{10} B_t (1 - Q_{t-1}) q_t + \frac{B_{10} (1 - Q_{10}) q_{10}}{q_{10} + r^F_{10}} \right]
$$

Notice that the terminal value component (the second term in the equation above) is similar to equation 4 in Section 2, because both the marginal default probability and the risk free rate are assumed to be constant. It corresponds to the capitalized costs of financial distress from year 11 on, discounted back to year 0 (thus the term $B_{10}$), and conditional on survival up to year 10 (thus the term $1 - Q_{10}$).
<table>
<thead>
<tr>
<th>Maturity</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51%</td>
<td>0.52%</td>
<td>1.09%</td>
<td>1.57%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>2</td>
<td>0.52%</td>
<td>0.56%</td>
<td>1.16%</td>
<td>1.67%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>3</td>
<td>0.54%</td>
<td>0.61%</td>
<td>1.23%</td>
<td>1.76%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>4</td>
<td>0.55%</td>
<td>0.65%</td>
<td>1.30%</td>
<td>1.85%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>5</td>
<td>0.56%</td>
<td>0.69%</td>
<td>1.38%</td>
<td>1.94%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>6</td>
<td>0.58%</td>
<td>0.74%</td>
<td>1.28%</td>
<td>1.89%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>7</td>
<td>0.59%</td>
<td>0.78%</td>
<td>1.18%</td>
<td>1.84%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>8</td>
<td>0.60%</td>
<td>0.82%</td>
<td>1.08%</td>
<td>1.79%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>9</td>
<td>0.62%</td>
<td>0.87%</td>
<td>1.20%</td>
<td>1.84%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
<tr>
<td>10</td>
<td>0.63%</td>
<td>0.91%</td>
<td>1.32%</td>
<td>1.90%</td>
<td>3.32%</td>
<td>5.45%</td>
</tr>
</tbody>
</table>

The spread data for A, BBB, BB and B bonds come from Citigroup’s yieldbook, average corporate bond spreads over treasuries, for the period 1985-1995. Data for AAA and AA bonds comes from Huang and Huang (2003), and refer to averages over the period 1985-1995.
Table 2. Fraction of the Yield Spread Due to Default

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4-year spread</td>
<td>10-year spread</td>
<td>5-year spread</td>
<td>4-year spread</td>
<td>10-year spread</td>
</tr>
<tr>
<td>AAA</td>
<td>0.030</td>
<td>0.208</td>
<td>NA</td>
<td>0.000</td>
<td>0.190</td>
</tr>
<tr>
<td>AA</td>
<td>0.121</td>
<td>0.200</td>
<td>0.510</td>
<td>NA</td>
<td>0.215</td>
</tr>
<tr>
<td>A</td>
<td>0.134</td>
<td>0.234</td>
<td>0.560</td>
<td>NA</td>
<td>0.609</td>
</tr>
<tr>
<td>BBB</td>
<td>0.245</td>
<td>0.336</td>
<td>0.710</td>
<td>0.702</td>
<td>0.724</td>
</tr>
<tr>
<td>BB</td>
<td>0.581</td>
<td>0.633</td>
<td>0.830</td>
<td>NA</td>
<td>0.846</td>
</tr>
<tr>
<td>B</td>
<td>0.976</td>
<td>0.833</td>
<td>NA</td>
<td>NA</td>
<td>0.906</td>
</tr>
</tbody>
</table>

This Table reports the fractions due to default of the yield spread calculated over benchmark treasury bonds, for each credit rating and different maturities. The first two columns use Huang and Huang (2003)'s results from Table 7, which reports calibration results from their model under the assumption that market asset risk premia are counter-cyclically time varying. The third column uses Longstaff et al.'s (2005) Table IV, which reports model-based ratios of the default component to total corporate spread. The fourth column uses results from Chen et al. (2005). The fraction reported for BBB bonds is the ratio of the BBB minus AAA spread over the BBB minus treasury spread. The fifth and sixth columns report for each rating and maturity the ratio between the default component of the spread and the total spread, where the default component is calculated as the spread minus the 1-year AAA spread. The seventh and eight columns report for each rating and maturity the ratio between the default component of the spread and the total spread, where the default component is calculated as the spread minus the difference between swap and treasury rates, for the period 2000-2004. NA = not available.
<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>5-year Historical</th>
<th>5-year Risk-Neutral</th>
<th>Ratio</th>
<th>10-year Historical</th>
<th>10-year Risk-Neutral</th>
<th>Ratio</th>
<th>Average Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.14%</td>
<td>0.54%</td>
<td>3.83</td>
<td>0.80%</td>
<td>1.65%</td>
<td>2.07</td>
<td>3.57</td>
</tr>
<tr>
<td>AA</td>
<td>0.31%</td>
<td>1.65%</td>
<td>5.31</td>
<td>0.96%</td>
<td>6.75%</td>
<td>7.04</td>
<td>6.22</td>
</tr>
<tr>
<td>A</td>
<td>0.51%</td>
<td>7.07%</td>
<td>13.86</td>
<td>1.63%</td>
<td>12.72%</td>
<td>7.80</td>
<td>9.95</td>
</tr>
<tr>
<td>BBB</td>
<td>1.95%</td>
<td>11.39%</td>
<td>5.84</td>
<td>5.22%</td>
<td>20.88%</td>
<td>4.00</td>
<td>4.84</td>
</tr>
<tr>
<td>BB</td>
<td>11.42%</td>
<td>21.07%</td>
<td>1.85</td>
<td>21.48%</td>
<td>39.16%</td>
<td>1.82</td>
<td>1.86</td>
</tr>
<tr>
<td>B</td>
<td>31.00%</td>
<td>34.90%</td>
<td>1.13</td>
<td>46.52%</td>
<td>62.48%</td>
<td>1.34</td>
<td>1.21</td>
</tr>
</tbody>
</table>

This Table reports cumulative risk-neutral probabilities of default calculated from bond yield spreads, as explained in the text. The Table also reports historical cumulative probabilities of default (data from Moodys, averages 1970-2001), and ratios between the risk-neutral probabilities and the historical ones. In the last column, we report the average ratio between risk-neutral and historical probabilities across all maturities from 1 to 10.
Table 4. Risk-Adjusted Costs of Financial Distress, as a Percentage of Firm Value

Panel A

<table>
<thead>
<tr>
<th></th>
<th>Historical</th>
<th>Benchmark Recovery 0.25</th>
<th>RFV 0.5*Yield</th>
<th>Coupon 1.5*Yield</th>
<th>Huang and Huang (2003)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.25%</td>
<td>0.32%</td>
<td>0.25%</td>
<td>0.31%</td>
<td>0.50%</td>
</tr>
<tr>
<td>AA</td>
<td>0.29%</td>
<td>1.84%</td>
<td>1.47%</td>
<td>1.77%</td>
<td>2.07%</td>
</tr>
<tr>
<td>A</td>
<td>0.51%</td>
<td>3.83%</td>
<td>3.17%</td>
<td>3.66%</td>
<td>4.10%</td>
</tr>
<tr>
<td>BBB</td>
<td>1.40%</td>
<td>4.53%</td>
<td>3.70%</td>
<td>4.24%</td>
<td>4.71%</td>
</tr>
<tr>
<td>BB</td>
<td>4.21%</td>
<td>6.81%</td>
<td>5.59%</td>
<td>6.15%</td>
<td>6.88%</td>
</tr>
<tr>
<td>B</td>
<td>7.25%</td>
<td>9.54%</td>
<td>8.04%</td>
<td>8.44%</td>
<td>9.58%</td>
</tr>
</tbody>
</table>

| BBB minus AA | 1.11% | 2.69% | 2.23% | 2.47% | 2.77% | 2.64% | 1.65% |

This Table reports our estimates of the NPV of the costs of financial distress expressed as a percentage of pre-distress firm value, calculated using historical probabilities (first column), and risk-adjusted probabilities (remaining columns). We use an estimate for the loss in value given distress of 16.5%. The valuation in column 2 (benchmark valuation) assumes recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In column 3 we change the recovery rate to 0.25. In column 4 we use a recovery of face value (RFV) assumption. In column 5 we assume that coupons are one half times the default component of spreads, and in column 6 we assume that coupons are one and a half times the default component of spreads. In column 7 we use Huang and Huang’s (2003) fractions due to default to calculate the default component of spreads that we use in the valuation.
Table 4. Risk-Adjusted Costs of Financial Distress, as a Percentage of Firm Value

Panel B

<table>
<thead>
<tr>
<th></th>
<th>Phi = 0.16</th>
<th>Phi = 0.10</th>
<th>Phi = 0.23</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Historical</td>
<td>Risk-adjusted</td>
<td>Historical</td>
</tr>
<tr>
<td>AAA</td>
<td>0.25%</td>
<td>0.32%</td>
<td>0.15%</td>
</tr>
<tr>
<td>AA</td>
<td>0.29%</td>
<td>1.84%</td>
<td>0.17%</td>
</tr>
<tr>
<td>A</td>
<td>0.51%</td>
<td>3.83%</td>
<td>0.31%</td>
</tr>
<tr>
<td>BBB</td>
<td>1.40%</td>
<td>4.53%</td>
<td>0.85%</td>
</tr>
<tr>
<td>BB</td>
<td>4.21%</td>
<td>6.81%</td>
<td>2.55%</td>
</tr>
<tr>
<td>B</td>
<td>7.25%</td>
<td>9.54%</td>
<td>4.39%</td>
</tr>
<tr>
<td>BBB minus AA</td>
<td>1.11%</td>
<td>2.69%</td>
<td>0.67%</td>
</tr>
</tbody>
</table>

This Table reports our estimates of the NPV of the costs of financial distress expressed as a percentage of pre-distress firm value, calculated using historical probabilities (first, third and fifth columns), and risk-adjusted probabilities (remaining columns). The risk-adjusted valuations assume recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In columns 1 and 2 we assume a loss given default of 16.5%. In columns 3 and 4 we assume a loss given default of 10% and in columns 5 and 6 we assume a loss given default of 23%.
Table 5. Risk-Adjusted Costs of Financial Distress and Time-varying Credit Spreads

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Low spreads</th>
<th>Average spreads</th>
<th>High spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>NA</td>
<td>0.32%</td>
<td>NA</td>
</tr>
<tr>
<td>AA</td>
<td>NA</td>
<td>1.84%</td>
<td>NA</td>
</tr>
<tr>
<td>A</td>
<td>2.95%</td>
<td>3.83%</td>
<td>4.64%</td>
</tr>
<tr>
<td>BBB</td>
<td>3.84%</td>
<td>4.53%</td>
<td>5.18%</td>
</tr>
<tr>
<td>BB</td>
<td>4.73%</td>
<td>6.81%</td>
<td>8.38%</td>
</tr>
<tr>
<td>B</td>
<td>7.60%</td>
<td>9.54%</td>
<td>10.88%</td>
</tr>
<tr>
<td>BBB minus A</td>
<td>0.89%</td>
<td>0.70%</td>
<td>0.53%</td>
</tr>
<tr>
<td>BB minus A</td>
<td>1.78%</td>
<td>2.98%</td>
<td>3.73%</td>
</tr>
</tbody>
</table>

This Table reports estimates of the NPV of the costs of financial distress expressed as a percentage of pre-distress firm value, calculated using risk-adjusted default probabilities. The risk-adjusted valuations assume recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. The second column reports the results that obtain if we use average historical spreads (1985-1995) to compute risk-adjusted default probabilities. In column 1 we scale down the average spreads uniformly by one standard deviation, and in column 3 the spreads are scaled up by one standard deviation. The standard deviations of spreads are computed using 1985-2004 data. We compute the ratio between standard deviations and averages separately for each maturity and rating for which we have time series data (1985-2004), and then multiply the 1985-1995 average spreads by these ratios. NA = not available.
Table 6. Typical Book Leverage Ratios for Each Bond Rating

<table>
<thead>
<tr>
<th>Rating</th>
<th>Predicted Values</th>
<th>Summary Statistics</th>
<th>COMPUSTAT 1981-2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.03</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>AA</td>
<td>0.16</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>A</td>
<td>0.28</td>
<td>0.22</td>
<td>0.16</td>
</tr>
<tr>
<td>BBB</td>
<td>0.33</td>
<td>0.28</td>
<td>0.21</td>
</tr>
<tr>
<td>BB</td>
<td>0.46</td>
<td>0.34</td>
<td>0.27</td>
</tr>
<tr>
<td>B</td>
<td>0.57</td>
<td>0.42</td>
<td>0.36</td>
</tr>
</tbody>
</table>

This Table reports typical book leverage ratios calculated for separate bond ratings. The first two columns are drawn from Molina (2005). The first column shows predicted leverage ratios from Molina's Table VI. These values are calculated using Molina's regression model (Table V), with values of the control variables set equal to those of the average firm with a kink of approximately two in Graham's (2000) sample. Column II replicates the simple summary statistics in Molina's Table IV. Column III reports average leverage ratios at each rating for manufacturing firms in the period 1981-2004, from COMPUSTAT.
Table 7. Tax Benefits of Debt against Costs of Financial Distress

Panel A: Leverage Ratios from Molina’s (2005) regression model

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Tax benefits of debt</th>
<th>Historical</th>
<th>Risk-adjusted</th>
<th>Historical</th>
<th>Risk-adjusted</th>
<th>Historical</th>
<th>Risk-adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.47%</td>
<td>0.22%</td>
<td>0.15%</td>
<td>0.32%</td>
<td>0.28%</td>
<td>0.12%</td>
<td>0.02%</td>
</tr>
<tr>
<td>AA</td>
<td>2.51%</td>
<td>2.22%</td>
<td>0.67%</td>
<td>2.34%</td>
<td>1.40%</td>
<td>2.11%</td>
<td>-0.05%</td>
</tr>
<tr>
<td>A</td>
<td>4.40%</td>
<td>3.89%</td>
<td>0.56%</td>
<td>4.09%</td>
<td>2.07%</td>
<td>3.69%</td>
<td>-0.95%</td>
</tr>
<tr>
<td>BBB</td>
<td>5.18%</td>
<td>3.78%</td>
<td>0.65%</td>
<td>4.33%</td>
<td>2.43%</td>
<td>3.23%</td>
<td>-1.14%</td>
</tr>
<tr>
<td>BB</td>
<td>7.22%</td>
<td>3.01%</td>
<td>0.41%</td>
<td>4.67%</td>
<td>3.09%</td>
<td>1.35%</td>
<td>-2.28%</td>
</tr>
<tr>
<td>B</td>
<td>8.95%</td>
<td>1.70%</td>
<td>-0.59%</td>
<td>4.56%</td>
<td>3.17%</td>
<td>-1.15%</td>
<td>-4.35%</td>
</tr>
</tbody>
</table>

BBB minus AA  | 2.67%                |

This Table reports tax benefits of debt and the difference between tax benefits of debt and costs of financial distress computed using different assumptions about losses in value given default. The relation between ratings and leverage is estimated using Molina’s (2005) regression model. This relation is reported in our paper in column I of Table 6. The first column depicts tax benefits of debt calculated for each leverage ratio as explained in the text. The remaining columns show the difference between tax benefits and distress costs. The risk-adjusted valuations assume recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In the second and third columns we assume that losses given default are equal to 16.5%. In the fourth and fifth columns we assume that losses give default are equal to 10%, and in the sixth and seventh columns we assume a loss given default of 23%.
Table 7 (cont.) Tax Benefits of Debt against Costs of Financial Distress

Panel B: Leverage Ratios from Molina’s (2005) summary statistics

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Tax benefits of debt</th>
<th>Historical</th>
<th>Risk-adjusted</th>
<th>Historical</th>
<th>Risk-adjusted</th>
<th>Historical</th>
<th>Risk-adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>1.41%</td>
<td>1.16%</td>
<td>1.09%</td>
<td>1.26%</td>
<td>1.22%</td>
<td>1.06%</td>
<td>0.97%</td>
</tr>
<tr>
<td>AA</td>
<td>2.67%</td>
<td>2.38%</td>
<td>0.83%</td>
<td>2.49%</td>
<td>1.55%</td>
<td>2.27%</td>
<td>0.11%</td>
</tr>
<tr>
<td>A</td>
<td>3.45%</td>
<td>2.94%</td>
<td>-0.38%</td>
<td>3.15%</td>
<td>1.13%</td>
<td>2.74%</td>
<td>-1.89%</td>
</tr>
<tr>
<td>BBB</td>
<td>4.40%</td>
<td>3.00%</td>
<td>-0.14%</td>
<td>3.55%</td>
<td>1.65%</td>
<td>2.45%</td>
<td>-1.92%</td>
</tr>
<tr>
<td>BB</td>
<td>5.34%</td>
<td>1.13%</td>
<td>-1.48%</td>
<td>2.79%</td>
<td>1.21%</td>
<td>-0.53%</td>
<td>-4.16%</td>
</tr>
<tr>
<td>B</td>
<td>6.59%</td>
<td>-0.65%</td>
<td>-2.95%</td>
<td>2.20%</td>
<td>0.81%</td>
<td>-3.51%</td>
<td>-6.71%</td>
</tr>
</tbody>
</table>

BBB minus AA 1.73%

This Table reports tax benefits of debt and the difference between tax benefits of debt and costs of financial distress computed using different assumptions about losses in value given default. The relation between ratings and leverage is estimated using Molina’s (2005) summary statistics. This relation is reported in our paper in column II of Table 6. The first column depicts tax benefits of debt calculated for each leverage ratio as explained in the text. The remaining columns show the difference between tax benefits and distress costs. The risk-adjusted valuations assume recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In the second and third columns we assume that losses given default are equal to 16.5%. In the fourth and fifth columns we assume that losses given default are equal to 10%, and in the sixth and seventh columns we assume a loss given default of 23%.
Figure 1

One-year par bond valuation tree

\[
\begin{align*}
1 & \quad \text{q} \quad \rho (1+r^D) \\
1 - q & \quad \text{1 - q} \quad (1+r^D)
\end{align*}
\]

One-year valuation tree for distress costs

\[
\begin{align*}
\Phi & \quad \text{q} \quad \phi \\
1 - q & \quad \text{1 - q} \quad 0
\end{align*}
\]
Figure 2: Valuation tree for distress costs

\[
\text{Prob. default in year 2} = (1 - Q_1)q_2
\]

\[
\text{Prob. surviving beyond year 2} = (1 - Q_2) = (1 - Q_1)(1 - q_2)
\]
Figure 3: Valuation tree for a two-year bond under assumption RT

\[
V_2 \begin{cases} 
Q_1 \rightarrow \rho[c_2 + (1+c_2)B_{12}] \\
(1 - Q_1) \rightarrow c_2 \begin{cases} 
q_2 \rightarrow \rho(1+c_2) \\
(1 - q_2) \rightarrow (1+c_2)
\end{cases}
\end{cases}
\]
Figure 4: Time variation in the price of credit risk

Historical probability of high and low spreads