Decomposition of the Market-to-Book Ratio: Theory and Evidence

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Abstract: We decompose the Market-to-Book ratio into two additive component ratios: a Conservatism Correction factor and a Future-to-Book ratio. The Conservatism Correction factor exceeds the benchmark value of one whenever the accounting for past transactions has been subject to an (unconditional) conservatism bias. For the Future-to-Book ratio, the benchmark value is zero for firms that are not expected to make economic profits in the future. Our analysis derives a number of structural properties of the Conservatism Correction factor, including its sensitivity to growth in past investments, the percentage of investments in intangibles and the firm’s cost of capital. The observed history of these variables allows us to infer the magnitude of a firm’s Conservatism Correction factor, resulting in an average value for this factor that is about two-thirds of the overall Market-to-Book ratio. We test the hypothesized structural properties of the Conservatism Correction factor by forming an estimate of this variable which is obtained as the difference between the observed Market-to-Book ratio and an independent estimate of the Future-to-Book ratio.
1 Introduction

The Market-to-Book (M-to-B) ratio is commonly defined as the market value of a firm’s equity divided by the book value of equity. It is well understood that this ratio exhibits considerable variation not only over time, but also at any given point in time, across industries and even across firms within the same industry. For instance, Penman (2009, p. 43) shows plots for the Market-to-Book ratio for U.S. firms, with an average value near 2.5. Among the many factors that are believed to contribute to the premium expressed in a firm’s M-to-B ratio, earlier accounting and finance literature has focused on two related aspects. First, the accounting rules for financial reporting tend to understate the value of the firm’s assets on the balance sheet.\(^1\) Secondly, the market value of the firm’s equity incorporates investors’ assessment of the firm’s potential to earn abnormal economic profits in the future.\(^2\)

Our analysis seeks to provide theoretical and empirical insights into the M-to-B ratio by decomposing it into two components. The first of these takes the ratio of the replacement cost of a firm’s assets relative to its accounting book value. This component, which we refer to as the Conservatism Correction factor, is only a function of past investments undertaken by the firm and the historical cost accounting rules in place. The second component will be referred to as the Future-to-Book ratio. Its numerator represents investors’ expectations regarding future discounted economic profits. This ratio is determined by both past and future investments, with the latter expected to be made optimally in light of anticipated future revenue opportunities. The Future-to-Book ratio thus incorporates the anticipated “growth opportunities” frequently mentioned in connection with high Market-to-Book ratios. In contrast, for firms operating in a competitive environment, investors would not expect positive economic profits and therefore the M-to-B ratio reduces to the Conservatism Correction factor for those firms.

Our notion of unbiased (non-conservative) accounting for operating assets is grounded in a model framework that views firms as making investments in productive capacity which, in turn, drives sales revenues. Unbiased accounting amounts to a depreciation schedule that reflects the anticipated decline of the asset’s productive capacity over time. The resulting

\(^1\)Without attempting to summarize the extensive literature on accounting conservatism, we note that parts of the theoretical literature on unconditional conservatism takes a Market-to-Book ratio greater than one as a manifestation of conservative accounting; see, for example, Feltham and Ohlson (1995, 1996), Zhang (2000) and Ohlson and Gao (2006).

\(^2\)This second aspect is usually attributed to firms with a high Tobin’s $Q$; see, for example, Lindberg and Ross (1986), Landsman and Shapiro (1995) and Roll and Weston (2008). As stated in Ross et al. (2005, page 41): “Firms with high $q$ ratios tend to be those firms with attractive investment opportunities or a significant competitive advantage.”
book value of an asset then corresponds to its replacement cost insofar as it is equal to the value that the used asset would trade for in a hypothetical competitive market for rental capacity services. Conservative accounting in our set-up reflects that some investment expenditures are never capitalized, possibly because they correspond to intangibles, such as R&D or advertising. Conservative accounting also arises if the straight-line depreciation rules commonly applied for operating assets are accelerated relative to the underlying useful life of an asset and its anticipated pattern of productivity declines. Our principal decomposition of the M-to-B ratio exploits the fact that the well known residual income valuation formula (Edwards and Bell, 1961) applies to all accounting rules that satisfy comprehensive income measurement. For our purposes, it will be useful to invoke this identity for unbiased accounting.

From an empirical perspective, neither of the two components of the M-to-B ratio we focus on can be observed directly. However, the Conservatism Correction factor can be estimated based on the firm’s past investments and based on direct estimates of other parameters such as the equity cost of capital and the useful life of the firm’s investments. By subtracting the estimated Conservatism Correction factor from the observed M-to-B ratio, we obtain a measure of the implied Future-to-Book value. We find that on average the Conservatism Correction factor accounts for about two-thirds of the M-to-B ratio. Negative Future-to-Book values can (and do) arise because future value is partly driven by past investments that are “locked-in” irreversibly at the present date. The expected future economic profits associated with these investments may be negative if future revenue prospects are assessed less favorably at the present time compared to the time at which the investments were undertaken. One would not expect such shifts to occur on average, a prediction that is borne out by our empirical findings.

The overall Market-to-Book ratio and both of the components identified in our analysis are increasing in the degree of accounting conservatism. With regard to the percentage of investments directly expensed, the M-to-B is furthermore increasing at an increasing rate; that is, it is a convex function of the percentage of investments directly expensed. Another empirical indicator of unconditional conservatism is accelerated depreciation in the sense that the useful life of a firm’s operating assets is assessed conservatively relative to the assessments of the firm’s industry peers. We find that such accelerated depreciation is indeed associated with higher Market-to-Book ratios.

The extensive literature on intangibles has argued that the conservative accounting for intangibles expenditures is particularly deficient because intangibles are a source of innova-

\[ ^3 \text{Our model framework here builds on the earlier work of Rogerson (2008), Rajan and Reichelstein (2009) and Nezlobin (2010).} \]
tion and competitive advantage. Our decomposition approach allows us to examine whether firms with a high percentage of intangibles investments are indeed expected to earn higher economic profits in the future. To that end, we examine a modified version of the implied Future-to-Book ratio which corrects for the accounting bias (direct expensing) in the denominator of that ratio. Our findings show that a higher percentage of intangibles assets does not result in a higher modified Future-to-Book ratio. This finding is consistent with a perspective that views upfront investments in intangibles as a subsequent source of competitive advantage, as evidenced by higher subsequent accounting profitability. However, the combination of higher upfront investment expenditures and subsequent abnormal accounting profitability does appear to be consistent with normal economic profitability in the long run.

Higher rates of growth in past investments tend to increase both the numerator and the denominator of the Market-to-Book ratio. Therefore the net impact of higher past growth is ambiguous. However, it can be established analytically that provided reported book values are too small relative to their replacement cost values (i.e., accounting is conservative), the Conservatism Correction factor is decreasing in higher past growth. To test this prediction empirically, we form an estimate of the Future-to-Book ratio by capitalizing the firm’s current economic profits. For the resulting implied Conservatism Correction factor, given by the difference between the Market-to-Book and the estimated Future-to-Book ratios, we do indeed find a negative association with higher growth rates in past investments. Furthermore, this negative association is more pronounced for firms that exhibit a higher percentage of intangibles investments and therefore are more prone to conservative accounting biases. Taken together, our findings speak to the interaction of accounting conservatism, past growth and anticipated future growth opportunities in shaping the M-to-B ratio.

Holding all parameters fixed for a given firm, an increase in the cost of capital should lower its market value. Since book values are generally not affected by the cost of capital,

5Of course, our arguments here are predicated on the notion that market price conveys a correct valuation of the firm’s equity. Lev (2001, Ch.4) argues that the poor disclosures in connection with intangibles investments frequently leads to an undervaluation of intangibles-intensive firms.
6This approach is broadly consistent with the valuation model formulated in Nezlobin (2010), where the capitalization of current economic profits reflects both the discount rate and the rate of growth in the firm’s sales revenues.
7Our finding that the Conservatism Correction factor is decreasing in past growth implies that ceteris paribus higher growth rates tend to push a stock in the direction of a “value stock”, that is, a relatively high Book-to-Market ratio, contrary to the view in the ”value/glamour” literature that low Book-to-Market firms are growth stocks (Lakonishok, Shleifer and Vishny, 1994).
one might conjecture that a higher cost of capital translates ceteris paribus into a lower M-to-B ratio. Yet such reasoning is likely to be misleading. For instance, for firms operating in competitive industry a higher cost of capital would translate into higher revenue in the future in order for this firm and others (whose cost of capital presumably also increased) to be able to earn zero economic profits. In such settings, the question then becomes how the Conservatism Correction factor, which coincides with the M-to-B ratio for competitive firms, is affected by changes in the cost of capital. Intuitively, it makes sense that this correction factor is increasing in the cost of capital, because incumbent assets that were recorded at their effective replacement value now become more valuable. We demonstrate this analytically and obtain empirical support for our hypothesis by showing that the estimated Conservatism Correction factor has a positive association with the cost of capital.

The Market-to-Book ratio has featured prominently in earlier empirical literature in accounting and finance. One recurring theme in these studies has been the ability of the M-to-B ratio to predict future stock returns and future accounting rates of return. For instance, Penman (1996) examines how the Market-to-Book ratio and the Price-to-Earnings ratio jointly relate to a firm’s future return on equity. Beaver and Ryan (2000) hypothesize that the M-to-B ratio is affected by two accounting related components which they term bias and lag, respectively. Both of these factors are conjectured to be negatively related to future accounting rates of return and the authors find empirical support for this prediction. In contrast to our decomposition approach, however, both the bias and the lag component in the M-to-B ratio are extracted by a regression of the M-to-B ratio on both current and past annual security returns with fixed firm effects.

The positive association between the Book-to-Market ratio and future security returns has been documented robustly in a range of earlier studies. However, there appears to be no consensus for this relation. While Fama and French (1992, 2006) point to risk as an explanation, other authors have invoked mispricing arguments for this association; see, for instance, Rosenberg, Reid and Lanstein (1985) and Lakonishok, Shleifer and Vishny (1994). In much of the earlier finance literature, it appears that book value is merely viewed as a convenient normalization factor in the calculation of the B-to-M ratios, without recognition that the measurement bias in this variable may differ considerably across firms.

In contrast to the above mentioned studies, the objective of the present paper is not an

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8With regard to bias, this prediction is based on the steady state growth model in Ryan (1996).
9The addition of accounting information is, of course, the general motivation for studies like those in Piotroski (2000) and Mohanram (2005). By including firm-specific scores derived from financial statement analysis, these authors are able to refine the association between M-to-B ratios and stock returns by partitioning firms with similar B-to-M ratios into different subgroups.
improved understanding of the relation between the Market-to-Book ratio and future returns. Our analysis seeks to identify the share of the overall premium expressed in the M-to-B ratio that is attributable to accounting conservatism and past growth in operating assets. From that perspective, our focus on past growth complements some of the recent work that has emphasized the link between conservative accounting and future earnings growth (Ohlson, 2008, and Penman and Reggiani, 2009). These papers also seek to capture the link between future growth and risk.

The remainder of this paper is organized as follows. Section 2 contains the model framework. Based on our results in Propositions 1-5, we formulate a set of hypotheses for empirical testing in Section 3. Empirical proxies, our data set and the actual empirical results are reported in Section 4. We conclude in Section 5. Two separate Appendices contain the tables and the proofs of the propositions.

2 Model Framework

Our model examines an all equity firm that undertakes a sequence of investments in productive capacity. The assets recorded for these investments are the firm’s only operating assets. In particular, we abstract from any need for working capital and assume that any cash is paid out immediately to shareholders. Accordingly, the denominator in the firm’s market-to-book ratio is given by the book value of equity, which is equal to the book value of operating assets.

Capacity can be acquired at a constant unit cost. Without loss of generality, one unit of capacity require a cash outlay of one dollar. New investments come “on-line” with a lag of \( L \) periods and have an overall useful life of \( T \) periods. Specifically, an expenditure of \( I_t \) dollars at date \( t \) will add productive capacity to produce \( x_t \cdot I_t \) units of output at date \( t + \tau \), with \( x_1 = x_2 = \ldots = x_{L-1} = 0 \) and \( x_t > 0 \) for \( L \leq t \leq T \). At date \( T \), the total capacity currently available is thus determined by the investments \((I_0, \ldots, I_{T-L})\). To allow for the possibility of decaying capacity, possibly to reflect the need for increased maintenance and repair over time, we specify that \( 1 = x_L \geq x_{L+1} \geq \ldots \geq x_T > 0 \). To illustrate some of our results, it will be useful to consider particular decay patterns. In particular, we will consider scenarios where capacity declines linearly at some rate \( \beta \), with \( \beta = 0 \) reflecting the common one-hoss shay scenario.\(^{10}\) Alternatively, a pattern of geometric decline would set \( x_t = x^{t-L} \) for some \( x \leq 1.\(^{11}\)

\(^{10}\)Arrow (1964) also refers to this latter productivity pattern as “sudden death.”

\(^{11}\)In connection with solar power panels, for instance, it is commonly assumed that electricity yield is subject to a constant “systems degradation rate” which is modeled as a pattern of geometrically declining
For a given history of investments, \( I_T \equiv (I_0, \ldots, I_T) \), the overall productive capacity at date \( T \) becomes:

\[
K_T(I_T) = x_T \cdot I_0 + x_{T-1} \cdot I_1 + \ldots + x_{L+1} \cdot I_{T-L-1} + x_L \cdot I_{T-L}.
\]  

(1)

The first term in the above expression reflects the final period of productive capacity for the earliest investment made by the firm \((I_0)\). The last term represents the first period of productive use \((x_L = 1)\) for the investment made at time \( T - L \).

We now turn to the accounting for investments. One maintained assumption of our model is that capacity investments are the only source of accruals in the firm’s financial statements. In particular, all variable costs are incurred on a cash basis and can therefore, without loss of generality, be included in sales revenues. Investments comprise expenditures for tangible and intangible assets, e.g., expenditures for plant, property and equipment, as well as expenditures for process control, training and development. Our analysis takes the ratio of tangible to intangible assets as exogenous. Consistent with the external financial reporting rules employed in most countries, we assume that the proportion of intangible investments is fully expensed at the time the investment expenditure is incurred. Accordingly the initial book value per dollar of investment, \( bv_0 \), is given by:

\[
bv_0 = (1 - \alpha).
\]

The parameter \( \alpha \geq 0 \) reflects that an \( \alpha \)-portion of investment expenditures in each period correspond to items that are directly expensed. The entire depreciation schedule for investments will be denoted by \( d = (\alpha, d_1, d_2, \ldots d_T) \), with \( \sum_{t=1}^{T} d_t = 1 \). The depreciation charge in period \( t \) of the asset’s existence is given by:

\[
dep_t = bv_0 \cdot d_t,
\]

for \( 1 \leq t \leq T \). Since \( bv_t = bv_{t-1} - dep_t \), an asset acquired at date 0 will have a remaining book value of:

\[
bv_t = bv_0 \cdot (1 - \sum_{i=1}^{t} d_i).
\]  

(2)

at date \( t \). Given an investment history, \( I_T = (I_0, \ldots, I_T) \), the aggregate book value at date \( T \) is then given by:

\[
BV_T(I_T, d) = bv_{T-1} \cdot I_1 + bv_{T-2} \cdot I_2 + \ldots + bv_0 \cdot I_T.
\]  

(3)

capacity levels (Campbell, 2008).
Among the $T$ terms in the above representation, the first $T - L$ terms refer to investments that were in use during period $T$, in chronological order of their inception. The latter $L$ terms denote the more recent investments which, because of the $L$–period lag, have not yet come into productive use.$^{12}$

In addition to depreciation charges and book values, it will be conceptually convenient to identify the capital costs of an investment. At date $t$, the capital cost imputed to an investment undertaken at date 0 is the sum of current depreciation plus an interest charge on the remaining book value. Denoting the firm’s cost of (equity) capital by $r$, the depreciation schedule leads to the following sequence of capital cost charges: $z_0 = \alpha$ and

$$z_t \equiv \text{dep}_t + r \cdot \text{bv}_{t-1},$$

for $1 \leq t \leq T$. The Conservation Property of Residual Income (Preinreich, 1938) ensures that, irrespective of the accounting rules $d$:

$$\sum_{t=0}^{T} z_t(d) \cdot \gamma^t = 1$$

where $\gamma \equiv \frac{1}{1+r}$. The book value of an asset, per dollar of investment, can be represented as the discounted sum of future capital costs:

$$\text{bv}_t = \sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i-t}.$$  

Finally, the aggregate of all capital costs corresponding to past investments will be referred to as the historical cost. At date $T$, this cost is given by:

$$H_T(I_T, d) \equiv z_T(d) \cdot I_0 + \ldots + z_0(d) \cdot I_T.$$  

The first term in (6) captures the depreciation and interest charge for the oldest investment undertaken at date 0, while the final expression represents the immediate expensing associated with the current investment at date $T$, since $z_0(d) = \alpha$.

We next turn to the concept of unbiased accounting. Suppose hypothetically that the firm had access to a rental market for capacity services in which suppliers provide short-term (periodic) capacity services. If such a market were competitive, suppliers would charge a unit price at which they make zero economic profits on their own investments. Since new

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$^{12}$We carry the initial investment $I_0$ as part of the relevant history $I_T$, because $I_0$ still affects the capacity available at date $T$. 

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investments come online with a lag of \( L \) periods and productive capacity may decline between dates \( L \) and \( T \), the competitive rental price would be given by:

\[
c = \frac{\gamma^{-L}}{\sum_{t=L}^{T} x_t \cdot \gamma^{t-L}} = \frac{1}{\sum_{t=L}^{T} x_t \cdot \gamma^{t}}.
\]

(7)

We note that since \( x_t = 0 \) for \( 1 \leq t \leq L - 1 \), the competitive rental price \( c \) can also be written as \( c = \frac{1}{\sum_{t=1}^{T} x_t \cdot \gamma^t} \). Even without reference to a hypothetical rental market for capacity services, the unit cost \( c \) can be interpreted as the marginal cost of a unit of capacity that is available for one period of time. Provided the firm anticipates positive investments in all future periods, the value \( c \) emerges as the incremental cost of one unit capacity to be available at date \( T + L \) because all future investments can be adjusted so as to add one unit of capacity at date \( T + L \), while keeping the capacity in all other future periods unchanged.\(^{13}\)

The notion of unbiased accounting is that the capital costs charged for an incumbent asset in each period should equal the economic cost of the capacity consumed in the current period.\(^{14}\)

\[
z_t^* = x_t \cdot c
\]

(8)

for \( 1 \leq t \leq T \). The historical cost charges in (8) will be referred to as the Relative Practical Capacity (R.P.C.) rule.

Earlier literature has shown that there is a one-to-one relation between depreciation schedules \( d = (\alpha, d_1, ..., d_T) \) and the capital cost charges \( (z_0, z_1, ..., z_T) \) (Rogerson, 1997). Formally, the linear mapping defined by (4) is a one-to-one mapping: for any \( z \) with the property that \( \sum_{t=0}^{T} z_t \cdot \gamma^t = 1 \), there exists a unique depreciation schedule \( d \) such that (4) is

\(^{13}\)This claim has been formalized in the work of Arrow (1964) and Rogerson (2008). The argument is readily seen in the special case of \( T = 2 \) and \( L = 1 \). Suppose the firm seeks one more unit of capacity at date 1. At date 0 the firm then needs to acquire one more unit of capacity, but it may compensate by buying \( x_2 \) unit less at 1, buy \((x_2)^2\) more unit at date 2, and so on. The cost of this variation as of date 0, is given by:

\[
[1 - \gamma \cdot x_2 + \gamma^2 \cdot (x_2)^2 - \gamma^3 \cdot (x_2)^3 + \gamma^4 \cdot (x_2)^4 \ldots] = \frac{1}{1 + \gamma \cdot x_2} = c \cdot \gamma^{-1},
\]

Therefore the present value of this variation at date 1 is indeed \( c \).

\(^{14}\)Thus our notion of unbiased accounting differs from that in Feltham and Ohlson (1995, 1996), Zhang (2000) and Ohlson and Gao (2006). The two notions will coincide if the firm earns zero economic profits on all its investments. In the literature on ROI, the concept of unbiased accounting is operationalized by the criterion that for an individual project the accounting rate of return should be equal to the project’s internal rate of return; see, for instance, Beaver and Dukes (1974), Danielson and Press (2003), Gjesdal (2004), Rajan et al. (2007) and Lampenius and Staehle (2010). Thus the accruals must generally reflect the intrinsic profitability of the project. Again, this criterion coincides with our notion of unbiased accounting if all projects have zero NPV and therefore the internal rate of return coincides with the cost of capital.
satisfied. For future reference, we will denote by $d^*$ the depreciation schedule corresponding to the R.P.C. cost allocation rule.\footnote{The Relative Practical Capacity cost allocation rule is a close variant of the so-called Relative Benefit Depreciation rule which has played a central role in earlier studies on managerial incentives for investment decisions; see, for instance, Rogerson (1997) and Dutta and Reichelstein (2005), among others. As the name suggests, the charges under the relative benefit depreciation rule apportion the initial investment expenditure in proportion to the subsequent expected future cash inflows. In contrast, the R.P.C. charges are based on the anticipated decay in productive capacity.}

According to (8), the R.P.C. cost allocation rule requires that $z_0^* = 0$, that is, $\alpha^* = 0$. In addition, since the interest charges on investment accrue even when assets are not in productive use, the requirement $z_1^* = \ldots = z_{L-1}^* = 0$ can be met only by compounding the asset at the cost of capital $r$ during the lag period.\footnote{This is exactly the accounting treatment that Ehrbar (1998) recommends for so-called “strategic investments,” which are characterized by a long time lag between investments and subsequent cash returns.} Equivalently, the depreciation charges in the first $L - 1$ periods are negative with:

$$d_t^* = -r \cdot (1 + r)^{t-1}, \text{ for } 1 \leq t \leq L - 1.$$  

For periods subsequent to the lag, it is readily verified that in the one-hoss shay scenario ($x_t = 1$), the depreciation schedule corresponding to the R.P.C. cost allocation rule rule is simply the annuity depreciation method. These depreciation amounts are applied to the compounded book value $b v_{L-1}^* = b v_0 \cdot (1 + r)^{L-1}$. On the other hand, it can be shown that the R.P.C. rule coincides with straight-line depreciation if practical capacity declines linearly over time such that $x_t = 1 - \frac{r}{1 + r (T - L + 1)} \cdot (t - L)$ for $t \geq L$.

We denote the book values corresponding to the R.P.C. cost allocation rule by $b v_t^*$. From (5) and (8) it follows immediately that:

$$b v_t^* = c \cdot \sum_{i=t+1}^T x_t \cdot \gamma^{i-t}. \quad (9)$$

In particular, we obtain $b v_t^* = b v_0^* \cdot (1 + r)^t$ for $1 \leq t \leq L - 1$. One final observation about the R.P.C. rule, due to Rogerson (2008), is that the historical cost, i.e., the aggregate capital cost, is equal to the economic cost of the capacity used in the current period.

**Observation 1:** For any investment history $I_T$:

$$H_T(I_T, d^*) = c \cdot K_T(I_T).$$

Next, we turn to the valuation of the firm. Investors are assumed to expect future investment decisions to be made so as to maximize firm value. In particular, there are no
frictions due to agency problems. For reasons of parsimony, we also present the valuation problem as one of certainty, that is, investors have complete foresight of the firm’s future growth opportunities. As a consequence, they anticipate the stream of future free cash flows that the firm derives from past investments in productive capacity and optimally chosen future investments.\footnote{Conceptually, it would not be difficult to extend our model formulation so as to include uncertainty and investors’ expectations. Such an extension would, however, not serve any particular purpose for either our theoretical or our empirical analysis. We also note that our present model formulation is not suited to address issue of conditional conservatism, as considered, for instance, in Basu (1997), Watts (2003), Beaver and Ryan (2005) and Roychowdhury and Watts (2007).} Let $R_{T+t}(K_{T+t})$ denote the net revenue (operating cash flow) that the firm will obtain at date $T + t$ from having capacity level $K_{T+t}$ in place. The future capacity levels are a function of the investment history $I_T$ and the future investment levels $I_\infty \equiv (I_{T+1}, I_{T+2}, \ldots)$. We denote the entire sequence of investments by $I \equiv (I_T, I_\infty)$. Any such sequence uniquely determines both the past and future capacity levels $K_t(I)$. The firm’s market value can then be expressed as:

$$MV_T = \max \{ I_\infty \} \{ \sum_{t=1}^{\infty} [R_{T+t}(K_{T+t}(I_T, I_\infty)) - I_{T+t}] \cdot \gamma^t \}.$$  \hspace{1cm} (10)

Let $I^*_\infty(I_T)$ denote a sequence of future investments that maximizes (10), conditional on $I_T$. The combined sequence $I^* \equiv (I_T, I^*_\infty(I_T))$ then achieves the firm’s market value in the sense that:

$$MV_T = \sum_{t=1}^{\infty} [R_{T+t}(K_{T+t}(I^*)) - I^*_{T+t}] \cdot \gamma^t.$$  \hspace{1cm} (11)

Our principal object of study is the Market-to-Book ratio:

$$MB_T = \frac{MV_T}{BV_T}. $$ \hspace{1cm} (12)

One would expect this ratio to be greater than or equal than one provided two conditions are met, provided the accounting for past investments has been conservative and furthermore the firm does not have “stranded assets” in the sense that past capacity investments were excessively high. To formalize the notion of unconditional conservatism, we employ the following partial ordering for depreciation schedules:

**Definition 1** A depreciation schedule $d$ is (weakly) more accelerated than $d'$ if

$$bv_t(d) \leq bv_t(d')$$

for all $0 \leq t \leq T - 1$. 

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The restriction to “tidy” depreciation schedules, that is, $\sum_{t=1}^{T} d_t = 1$, implies that $bv_T(d') = bv_T(d) = 0$. A more accelerated depreciation rule results in a weakly lower book value at any date prior to $T$. In other words, the cumulative amount depreciated by any point of time must be higher under the more accelerated depreciation schedule. In particular, the ordering in Definition 1 requires that $\alpha > \alpha'$. The notion that a firm has not over-invested in the past can be formalized by comparing future revenues to the economic cost of capacity levels to which the firm has already committed itself. Let $(I_T, 0)$ denote the investment trajectory that is obtained if hypothetically the firm were not make any new (value creating) investments but merely put its existing assets to use in future periods.

**Assumption (A1):** $R_{T+t}(K_{T+t}(I_T, 0)) - c \cdot K_{T+t}(I_T, 0) > 0$, for $1 \leq t \leq T$.

The condition in (A1) says that the net-revenues the firm can obtain with the capacity that has been “locked in” will cover, on a per unit basis, at least the economically relevant unit cost $c$ in each of the next $T$ periods. With this condition, we obtain the following result:

**Proposition 1** Given (A1), suppose $d$ is more accelerated than the R.P.C. depreciation rule. The Market-to-Book ratio then satisfies:

$$MB_T \geq 1.$$

It will be instructive to provide the proof of Proposition 1 here in the text rather than in the Appendix. We invoke the well-known relation expressing market value as book value plus future discounted residual incomes (Edwards and Bell, 1961; Feltham and Ohslon, 1996). Since this relation holds irrespective of the accounting rules, it holds in particular for the R.P.C. depreciation rule. Thus:

$$MV_T = BV_T^*(I_T) + FV_T, \quad (13)$$

where

$$FV_T \equiv \sum_{t=1}^{\infty} [R_{T+t}(K_{T+t}(I^0)) - c \cdot K_{T+t}(I^0)] \cdot \gamma^t. \quad (14)$$

The expression in (13) makes use of Lemma 1 in so far as the residual income in period $T + t$ under the R.P.C. rule is given by $R_{T+t}(K_{T+t}) - c \cdot K_{T+t}$. We identify the following two components of the Market-to-Book ratio:

\[Nezlobin (2010)\] also invokes the residual income formula under unbiased accounting in order to characterize a firm’s equity value.

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CC_T = \frac{BV_T(I_T)}{BV_T(I_T)} \quad \text{(15)}

and

FB_T = \frac{FV_T}{BV_T(I_T)} \quad \text{(16)}

A sufficient condition for \( MB_T \geq 1 \) then is that both \( CC_T > 1 \) and \( FV_T > 0 \). By definition, \( CC_T > 1 \) whenever depreciation is accelerated relative to the R.P.C. rule. With regard to \( FV_T \), (A1) yields the following inequalities:

\[
\sum_{t=1}^{\infty} [R_{T+t}(K_{T+t}(I^o)) - c \cdot K_{T+t}(I^o)] \cdot \gamma^t \geq \sum_{t=1}^{T} [R_{T+t}(K_{T+t}(I^o, 0) - c \cdot K_{T+t}(I^o, 0)] \cdot \gamma^t > 0.
\]

This completes the proof of Proposition 1. For the remainder of our analysis, we refer to the equation:

\[
MB_T = CC_T + FB_T, \quad \text{(17)}
\]

as the decomposition of the Market-to-Book ratio into the Conservatism Correction factor, \( CC_T \) and the Future-to-Book ratio, \( FB_T \). For a firm that operates in a competitive environment, and therefore earns zero economic profits, net-revenues will just cover economic costs, that is, \( R_{T+t}(K) = c \cdot K \) for all \( K \). As a consequence, future value, \( FV_T \) is zero and the Market-to-Book ratio reduces to the conservatism correction factor, \( CC_T \). For those firms \( MB_T = 1 \) if and only if operating assets are accounted for at their replacement cost, that is depreciation is calculated according to the R.P.C. rule. On the other hand, conservative accounting will push the Market-to-Book ratio above one even though firms in competitive industries have no future “growth opportunities” in terms of economic profitability.\(^{19}\)

A firm’s future value can be negative at date \( T \), because anticipated future revenues at date \( T \) are lower than they were at the time the investments were undertaken. A longer lag, \( L \), between the time investment expenditures are made and the time the investments

\(^{19}\)For an all equity firm, Tobin’s \( q \) is then given by:

\[
q = 1 + \frac{FV_T}{BV_T} = \frac{MB_T}{CC_T}.
\]

This representation is in agreement with the description in Lindenberg and Ross (1981, page 3): “...for firms engaged in positive investment, in equilibrium we expect \( q \) to exceed one by the capitalized value of the Ricardian and monopoly rents which the firm enjoys.”
become productive tends to increase the “likelihood” for a negative \( FV_T \). Specifically, the first \( L \) terms in \( FV_T \), that is:

\[
\sum_{t=1}^{L} [R_{T+t}(K_{T+t}(I^o)) - c \cdot K_{T+t}(I^o)] \cdot \gamma^t = \sum_{t=1}^{L} [R_{T+t}(K_{T+t}(I_T)) - c \cdot K_{T+t}(I_T)] \cdot \gamma^t
\]

are determined entirely by past investment decisions. “Future growth opportunities” will affect economic profits only beyond date \( T + L \).

To characterize the behavior of the Market-to-Book ratio in terms of its constituent variables, we first note that ceteris paribus \( MB_T \) will increase as the accounting becomes more conservative in accordance with Definition 1.\(^{20}\) A stronger statement can be made if the degree of conservatism is measured by the percentage of investment expenditures that are directly expensed. We note that book value is decreasing linearly in the parameter \( \alpha \). For any depreciation rule \( d = (\alpha, d_1, d_2, \ldots d_T) \), let \( d_{\alpha=0} \) denote the same depreciation schedule, except that the investment is fully capitalized initially. Thus \( d_{\alpha=0} \equiv (0, d_1, d_2, \ldots d_T) \). It then follows immediately that:

\[
MB_T(I_T, d) = \frac{1}{1 - \alpha} MB_T(I_T, d_{\alpha=0}),
\]

and we obtain the following result.

**Observation 2:** The Market-to-Book ratio, \( MB_T \), is increasing and convex in \( \alpha \).

We next turn to the impact of past growth in investments on \( MB_T \). To that end, the *growth rate* in investments in period \( t \) will be denoted by \( \lambda_t \). This rate is defined implicitly by:

\[
(1 + \lambda_t) \cdot I_{t-1} = I_t.
\]

Any investment history \( I_T \) induces a sequence of corresponding growth rates \( \lambda_T = (\lambda_1, \ldots, \lambda_T) \). and conversely, any initial investment \( I_0 \) combined with growth rates \( (\lambda_1, \ldots, \lambda_T) \) defines an investment history \( I_T \). Therefore, the aggregate book value \( BV_T(I_T, d) \) at date \( T \) can be expressed as:

\[
BV_T(\lambda_T, d|I_0) = I_0 \cdot \left[ bv_{T-1}(1 + \lambda_1) + bv_{T-2}(1 + \lambda_1)(1 + \lambda_2) + \ldots + bv_0 \cdot \prod_{i=1}^{T} (1 + \lambda_i) \right].
\]

(18)

The impact of higher growth on the future value, \( FV_T \), is ambiguous since both the numerator and the denominator in (16) will tend to increase with higher growth in past investments.

\(^{20}\)See also Proposition 2 in Lampenius and Staehle (2010).
investments. With regard to \( CC_T \), it is clear that growth has no impact if the accounting were to be unbiased. On the other hand, if the depreciation schedule deviates from the R.P.C. rule, then the impact of growth on \( CC_T \) depends on how the constituent ratios \( \frac{b_{0t}}{b_{vt}} \) change over time. To state a general result, we introduce the following stronger notion of accelerated depreciation.

**Definition 2** A depreciation schedule \( d \) is uniformly more accelerated than the R.P.C. rule if:

\[
z_t(d) \geq 0 \text{ for } 0 \leq t \leq L - 1 \quad \text{and} \quad \frac{z_t(d)}{x_t} \text{ is monotonically decreasing in } t \text{ for } L \leq t \leq T.
\]

For the R.P.C. depreciation schedule, \( d^* \), the inequalities in Definition 2 are met as equalities and the ratio of \( z_t^*(d^*) \) to \( x_t \) is constant in \( t \) (see (8)). It is readily verified that the criterion of uniformly accelerated depreciation is stronger than of accelerated depreciation: if \( d \) satisfies the inequalities in Definition 2, it will also satisfy the “cumulative” inequalities in Definition 1.\(^{21}\)

**Proposition 2:** If \( d \) is uniformly more accelerated than the R.P.C. depreciation rule, \( CC_T(\cdot) \) is monotone decreasing in each \( \lambda_t \).

With uniformly more accelerated accounting, higher levels of growth in past investments thus lowers the conservatism correction factor, due to a smaller divergence between the stated accounting book values and the “fair” replacement cost values. Observation 2 and Proposition 2 together show that if a greater percentage of investments is directly expensed, the impact is to reinforce the downward effect of growth on \( CC_T \).

**Observation 3:**

\[
\frac{\partial^2}{\partial \lambda_t \partial \alpha} CC_T(\cdot) < 0.
\]

What is the impact of a higher cost of capital, \( r \), on the Market-to-Book ratio? A simple ceteris paribus argument suggests a negative association. Accounting book value in the denominator of \( MB_T \) is independent of \( r \) provided firms use straight-line depreciation (or any other schedule that is independent of \( r \)) for the portion of their investments that were capitalized in the first place. At the same time, the expression for \( MV_T \) in (10) is decreasing in \( r \), because future free cash flows are discounted at a higher rate. Yet, such a simplistic ceteris paribus comparison can be misleading since a higher cost of capital is likely to result in

\(^{21}\)The same criterion of accelerated depreciation is used in Rajan and Reichelstein (2009) to show that the average historical cost (the historical cost per unit of capacity) is decreasing in past growth.
other simultaneous changes. To illustrate, suppose again the firm operates under conditions of zero economic profits such that $FV_T = 0$ because $R_{T+i}(K) = c \cdot K$ for all $K$. A higher cost of capital then results in a higher unit cost of capacity $c$ and therefore higher net-revenues that will be obtained under competitive conditions. The impact of $r$ on $MB_T$ then reduces to the effect of $r$ on $CC_T$. Intuitively, one would expect that a higher cost of capital makes the stock of past investments, $BV_T^*$ more valuable. This turns out to be true subject to a regularity condition on the pattern of productivity levels, $(x_L, ..., x_T)$.

**Proposition 3:** Suppose that $d$ is independent of $r$. Then $CC_T$ is increasing in $r$ provided that for all $t \geq L$, the pattern of productivity declines satisfies

$$\frac{x_t}{x_{t+1}} \leq \frac{x_{t+1}}{x_{t+2}}.$$

The condition on the $x_t$'s in the statement of Proposition 3 is sufficient, but not necessary. This condition is also not very restrictive. For instance, it is satisfied by any $x$ vector that decreases over time in either a linear or geometric fashion. The one-hoss shay scenario, where all $x_t = 1$, is one particular admissible case.

In order to obtain sharper insights about the magnitude and behavior of $CC_T$, we now impose additional structure on the model. Since our ultimate interest is in testing the theoretical predictions about $MB_T$ using data from firms’ financial reports, we confine attention to the following accounting rules for new investments: partial expensing (fraction $\alpha \geq 0$), followed by straight-line depreciation over the period of productive use. This is the predominant form of accounting for operating assets employed in practice.\(^{22}\) Since assets are not depreciated prior to being placed in productive use, we have $z_t = r \cdot (1 - \alpha)$ for $1 \leq t \leq L - 1$. For subsequent periods, straight-line depreciation results in the following capital costs:

$$z_t = (1 - \alpha) \cdot \left[\frac{1}{T - L + 1} + r \cdot \frac{T - t + 1}{T - L + 1}\right] \text{ for } L \leq t \leq T. \quad (19)$$

For the one-hoss shay scenario, where $x_t = 1$ for all $L \leq t \leq T$, we note that the accounting rules in (19) entail three distinct sources of conservatism: (i) an $\alpha$ percentage of investments is never capitalized, (ii) asset values are not compounded during the “construction” phase in periods 1 through $L - 1$ and (iii) assets are depreciated according to the straight-line rule rather than the annuity depreciation rule during their productive phase in

\(^{22}\)It appears that the vast majority of firms in the U.S. applies straight-line depreciation for financial reporting purposes.
periods $L$ through $T$. Even for $\alpha = 0$, the resulting book values $bv_t$ are strictly below the unbiased book values $bv_t^*$\textsuperscript{23}. The difference between the two increases in the parameter $L$.

We henceforth restrict attention to settings with constant growth ($\lambda_t = \lambda$). Denoting $\mu \equiv \frac{1}{1+\lambda}$, $CC_T$ can then be reduced to the following simple representation:

$$CC_T = \frac{BV_T^*}{BV_T} = c \cdot \frac{\sum_{i=L}^{T} x_i \cdot (\gamma^i - \mu^i)}{\sum_{i=1}^{T} z_i \cdot (\gamma^i - \mu^i)}.$$  \hfill (20)

Figure 1 illustrates the magnitude of the resulting conservatism correction factor $CC_T$ for different levels of growth. In addition, the graphs in Figure 1 are based on the following parameter specifications: $L = 1$, $r = 10\%$ and $T = 15$.

\textsuperscript{23}Informally, this inequality follows from the following two observations. (i) on the interval $[0, L - 1]$ it is clearly true that $bv_t^* > bv_t$; (ii) on the interval $[L, T - 1]$ it must also be true that $bv_t^* > bv_t$, because $bv_T^* = bv_T = 0$ and $bv_t^*$ is decreasing and concave on $[L, T]$, while $bv_t$ is a linear function of time.
Figure 1 suggests that the impact of growth on CC\(T\) is rather uneven in the sense that the most significant drop in CC\(T\) occurs for moderately negative growth rates between \(-0.5\) and zero. Thereafter CC\(T\) quickly approaches its asymptotic value, which in all three examples is equal to \(\frac{1}{1-\alpha}\). For extremely negative growth rates CC\(T\) appears to flatten out rather than increase asymptotically without bound. Our final result shows that these observations do indeed hold at some level of generality. In deriving this result, we impose the restriction that the productive capacity of assets declines linearly over time:

\[
x_t = 1 - \beta \cdot (t - L),
\]

for \(t \geq L\). Here, \(\beta \geq 0\) captures the periodic decline in productive capacity once assets are in use. The one-hoss shay scenario corresponds to \(\beta = 0\). We assume that the rate of decline is not too great, in particular that \(0 \leq \beta \leq \beta^* \equiv \frac{r}{1+r-(T-L+1)}\). Under these assumptions, it can be verified that the combination of partial expensing and straight line depreciation represents conservative accounting, and in fact is uniformly more accelerated than R.P.C. It thus meets the more stringent requirement in Definition 2.

It will be notationally convenient to introduce the auxiliary function:

\[
h(s) \equiv \frac{s \cdot (1 + s)^T}{(1 + s)^T - 1},
\]

for \(s\) on the domain \([-1, \infty]\). The economic interpretation of \(h(s)\) is that if this amount is paid annually over \(T\) years, then the present value of this annuity is precisely 1, provided future payments are discounted at the rate \(s\). Therefore \(h(\cdot)\) is increasing and convex over its domain, with \(h(-1) = 0\), \(h(0) = 1/T\) and \(h(\infty) = \infty\).

**Proposition 4:** Suppose \(d\) conforms to straight-line depreciation with partial expensing, \(\lambda_t = \lambda\), and \(x_t = 1 - \beta \cdot (t - L)\). Then, if \(L = 1\),

\[
\frac{2}{3} \leq \frac{CC_T(\lambda = -1) - CC_T(\lambda = 0)}{CC_T(\lambda = -1) - CC_T(\lambda = \infty)} \leq \frac{T}{T + 1}.
\]

If in addition the productivity pattern conforms to the one-hoss shay scenario \((\beta = 0)\):

(i) \(\lim_{\lambda \to -1} CC_T = \frac{1}{1-\alpha} \cdot \frac{T \cdot h(r)}{(1+r)}\);

(ii) \(\lim_{\lambda \to 0} CC_T = \frac{1}{1-\alpha} \cdot \frac{2[T \cdot h(r) - 1]}{r \cdot (1+r)}\);

(iii) \(\lim_{\lambda \to \infty} CC_T(\cdot) = \frac{1}{1-\alpha}\).
Consistent with the observations in Figure 1, Proposition 4 demonstrates that a substantial majority of the drop in $CC_T$ as a result of increases in the growth rate occurs in the region where growth rates are negative. At least two-thirds of the reduction, and up to $\frac{2}{3} T + 1$ of it, takes place as the growth in new investments varies between $-100\%$ and $0\%$. The far smaller remainder of the decline occurs when growth varies between $0\%$ and $\infty$.\textsuperscript{24}

Proposition 4 also demonstrates that for extremely negative growth rates, $\lambda \rightarrow -1$, the conservatism correction factor, $CC_T$ flattens out and assumes finite limit values, which can be expressed in terms of the annuity function $h(\cdot)$.\textsuperscript{25} At the other extreme, we find that, again consistent with the observations in Figure 1, $CC_T$ converges to $\frac{1}{1-\alpha}$ for very high growth rates, irrespective of any of the other parameters.

Our final result examines the impact of a faster decay in the assets’ productivity on the conservatism correction factor.

**Proposition 5:** Suppose $d$ conforms to straight-line depreciation with partial expensing, $\lambda_t = \lambda$, and $x_t = 1 - \beta \cdot (t - L)$. Then $CC_T$ is monotone decreasing in the decay parameter $\beta$.

The intuition behind this result is that if $\beta = \beta^* \equiv \frac{r}{1 + r \cdot (T - L + 1)}$, straight-line depreciation does indeed coincide with the R.P.C. rule. Therefore straight-line depreciation becomes less conservative as $\beta$ increases beyond zero without exceeding the critical value $\beta^*$. Consistent with our earlier findings, $CC_T$ declines as the accounting becomes less conservative.

### 3 Hypotheses

Our decomposition of the Market-to-Book ratio and our analytical predictions regarding its two principal components have been obtained under restrictive modeling assumptions. To align the empirical analysis as closely as possible with the above model, our focus will not be on the "raw" Market-to-Book ratio, commonly defined as the ratio of the market value of equity over the book value of equity. Since the thrust of our notion of accounting conservatism is that operating assets are understated relative to their fair replacement value, we shall instead examine the following *adjusted* Market-to-Book ratio:

$$MB_T = \frac{MV_T - FA_T}{BV_T - FA_T},$$

\textsuperscript{24} For general $L > 1$, it can be shown that at least half of the drop in $CC_T$ occurs in the range of negative growth rates, provided productivity conforms to the one-hoss shay scenario.\textsuperscript{25} This finding can be extended to general values of $\beta$ and $L$. The limit values are available from the authors upon request. We note that $\lim_{\lambda \rightarrow -1} CC_T = \frac{bv_T - 1}{bv_T - 1}$ and $\lim_{\lambda \rightarrow \infty} CC_T = \frac{bv_T}{bv_T}$.
where $FA_T$ denotes financial assets at the observation date $T$. Financial assets here includes working capital, such as cash and receivables, net of all liabilities, including both current liabilities and long-term debt. From that perspective, the book value of operating assets is given by $OA_T = BV_T - FA_T$. Similarly, we view the market value of equity as financial assets (carried at fair value) plus the discounted value of future free cash flows. Given R.P.C. accounting, $MV_T$ can be expressed as:

$$MV_T = FA_T + OA_T^*(I_T) + \sum_{t=1}^{\infty} [R_{T+t}(K_{T+t}(I^o)) - c \cdot K_{T+t}(I^o)] \cdot \gamma^t,$$

in the presence of financial assets.\(^\text{26}\) The adjusted Market-to-Book ratio therefore can be decomposed into:

$$MB_T = \frac{MV_T - FA_T}{BV_T - FA_T} = CC_T + FB_T,$$

where

$$CC_T = \frac{OA_T^*(I_T)}{OA_T(I_T)},$$

and

$$FB_T = \frac{FV_T}{OA_T(I_T)}.$$

As before, the firm’s future value, $FV_T$, is given by the last term on the right-hand side of (22). We note in passing that the focus on adjusted rather than raw market-to-book ratios makes little difference if the raw Market-to-Book ratio is close to one.

The conservatism correction factor in (24) can be computed in terms of the firm’s investment history, the percentage of investments expensed, the estimated useful life of its investments and the estimated cost of equity capital. For our calculation of $CC_T$ we assume that the productivity of assets follows the one hoss-shay pattern and that for financial reporting the firm relies on straight-line depreciation for its capitalized investments. As a consequence, the accounting is accelerated relative to the unbiased standard of R.P.C. depreciation. Proposition 1 then implies that $CC_T > 1$. From an empirical perspective, it is of interest to examine the magnitude of the implied Future-to-Book ratio, given as the residual $MB_T - CC_T$.

\(^\text{26}\)This is, of course, consistent with the studies in Feltham and Ohlson (1995) and Penman, Richardson and Tuna (2007), which presume that financial assets are carried at their fair market values on the balance sheet.
Hypothesis 1: The implied Future-to-Book ratio, $MB_T - CC_T$, is positive on average.

As argued in connection with Proposition 1, it is conceivable that a firm’s future value is negative because past investment decisions, which are irreversible at date $T$, were made with more “exuberant” expectations about future sales revenues than investors believe in at the current date $T$. The statement of Hypothesis 1 reflects that such a shift in expectations should not be expected on average. Hypothesis 1 also reflects that the economic profits beyond date $T + L$ reflect investments to be made optimally in future periods and the associated option value is inherently positive.

The model analyzed in Section 2 characterizes Future Value as the stream of expected future discounted economic profits, that is, the stream of residual income numbers that emerge under the R.P.C. rule. As such, it combines the firm’s investment history with future decisions to be made optimally. One way to estimate Future Value therefore is to extrapolate the current economic profit at date $T$. To reflect the “option value” associated with future investment decisions, we adopt an asymmetric specification that takes as the estimated future value a capitalization of the current economic profit, provided that number is positive. In contrast, our measure of estimated Future Value is set equal to zero if current economic profit is negative.\footnote{It goes without saying that our approach to forecasting future value is ad hoc. There appear to be many promising avenues for refining the approach taken here in future studies.}

Formally, we define the estimated Future-to-Book ratio as:

$$\hat{FB}_T = \frac{(1 - \tau_T) \cdot I\{R_T(K_T) - c \cdot K_T\} \cdot \Gamma^5 \lambda_{OA_T}}{OA_T},$$

(26)

where $\tau_T$ is the statutory income tax rate in year $T$ and $I\{x\}$ is the indicator function corresponding to a call option, that is, $I\{x\} = x$ if $x \geq 0$, while $I(x) = 0$ if $x \leq 0$. The “capitalization” factor $\Gamma^5_{\lambda}$ is given by $\sum_{i=1}^{5} (\frac{1 + \lambda^5_{a}}{1 + r})^i$, where $\lambda^5_{a}$ denotes the geometric mean of investment growth over the past 3 years.\footnote{Our capitalization of current economic profit is broadly consistent with the valuation model developed in Nezlobin (2010). We use the average past growth rate as a proxy for anticipated future growth in the firm’s product markets.}

Since the economic profit $R_T(K_T) - c \cdot K_T$ is not observable, we estimate this number by making suitable adjustments to the firm’s accounting income. The details of this adjustment are described in the next section summarizing our empirical findings.\footnote{Firms obviously do not pay income taxes on their economic profits. Our approach of incorporating income taxes avoids the issues of estimating the firm’s actual tax rate as well as taxes to be paid in future periods.} If our construct of the estimated Future-to-Book ratio does indeed provide a reasonable approximation of the implied Future-to-Book ratio, we would expect both $\hat{FB}_T$ and $CC_T$ to have significant explanatory power for the overall Market-to-Book ratio $MB_T$.
Hypothesis 2: Both $CC_T$ and $\hat{FB}_T$ have significant explanatory power for $MB_T$.

We next formulate several hypotheses related to accounting conservatism. As argued in Section 2, our model has two principal sources of unconditional conservatism. The first of these is that the accounting for intangible assets results in a percentage of investments that is never recognized on the balance sheet. In this context, we seek to test the prediction emerging from Observation 2.

Hypothesis 3: The Market-to-Book ratio, $MB_T$, is increasing and convex in $\alpha$.

Another source of conservatism is that the portion of assets that has been capitalized is usually depreciated according to the straight-line rule. This depreciation schedule is accelerated relative to the form of unbiased accounting embodied in the R.P.C. rule assuming that the pattern of asset productivity conforms to the one-hoss shay pattern. One form of accelerated depreciation is that firms choose a shorter horizon for depreciating the asset than the asset’s actual useful life. In our model, this would correspond to some horizon $\hat{T} < T$. For instance, it has been observed that different airlines employ varying assessments of the useful lives of their aircraft for depreciation purposes, even though these airlines are comparable in terms of their operations. Assuming straight-line depreciation, a shorter assessment of the asset’s useful life amounts to more accelerated depreciation.

Hypothesis 4: Ceteris paribus, more accelerated depreciation tends to increase the Market-to-Book ratio.

The above model has viewed the proportion of intangible investments as exogenous. In particular, we have taken the perspective that economic profitability is a function of the firm’s past and future investment decisions which require a given mix of tangible and intangible investments. The parameter $\alpha$ therefore did not enter as a direct factor in the firm’s future value, $FV_T$. In contrast, many studies have asserted that intangibles are generally a source of innovation and competitive advantage with the promise of abnormal economic profits. If true, a higher proportion of intangible investments would then tend to increase the $MB_T$ ratio on two accounts: through conservatism and higher economic profits in the future. To examine this hypothesis, we note that the implied Future-to-Book ratio $FB_T = MB_T - CC_T$ is affected by $\alpha$ in the same mechanical fashion as $MB_T$: the denominator $OA_T$ is linearly decreasing in $\alpha$ and therefore $FB_T$ is a hyperbolic function of $\alpha$. This suggests an examination of the modified Future-to-Book ratio, defined as:

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30 Shivakumar (2004) stresses this point in contrasting the profitability of Lufthansa and British Airways.

31 See, for instance, Lev (2000) and the references provided therein.
\[ \tilde{FB}_T \equiv (1 - \alpha) \cdot (MB_T - CC_T) \]

Consistent with our model formulation, we take the perspective that a higher proportion of investments in intangibles is by itself not a source of higher economic profitability in the future.

**Hypothesis 5:** The modified Future-to-Book ratio, \( \tilde{FB}_T \) is unrelated to \( \alpha \).

The predicted impact of higher growth rates in past investments on the \( MB_T \) ratio is ambiguous in our model. While the predicted impact on \( CC_T \) is unambiguous according to Proposition 2, both the numerator and the denominator in \( FB_T \) are likely to increase with higher growth rates in the past. To isolate the impact on \( CC_T \), we therefore consider the following estimated Conservatism Correction factor:

\[ \hat{CC}_T = MB_T - \hat{FB}_T \]  \hspace{1cm} (27)

To the extent that \( \hat{FB}_T \) provides a suitable proxy for \( FB_T \), we would therefore expect \( \hat{CC}_T \) to be decreasing in past investment growth. Proposition 4 shows that this downward relation is most pronounced for negative growth rates (assuming constant growth). In contrast, Proposition 1 and Figure 1 suggest that \( CC_T \) flattens out and reaches its asymptotic value \( \frac{1}{1-\alpha} \) quickly once the growth rate in investments turns positive.

**Hypothesis 6:** \( \hat{CC}_T \) is decreasing in past investment growth.

Observation 2 shows that the negative impact of past growth on the conservatism correction factor is stronger for firms that expense a larger percentage of their investments. Figure 1 illustrates this cross-partial result.

**Hypothesis 7:** The negative association between \( \hat{CC}_T \) and past investment growth is more pronounced for firms with a higher percentage of intangibles investments.

Finally, Proposition 4 shows that the drop in \( CC_T \) as a function of past investment growth is far more pronounced for firms with negative growth rates compared to those with positive growth rates. Figure 1 also illustrates this pattern. Accordingly, we formulate the following

**Hypothesis 8:** The negative association between \( \hat{CC}_T \) and past past investment growth is more pronounced for firms with negative average growth in past investments than for firms with positive average growth in past investments.
As observed in the text, a firm’s future value, $FV_T$, should *ceteris paribus* be decreasing in the cost of capital $r$, simply because future free cash flows are discounted at a higher rate. Yet the scenario of a firm operating under competitive conditions provides a good illustration of why such a *ceteris paribus* approach is likely to be misleading. A firm operating in a competitive environment will obtain revenues that match its entire economic cost. Therefore, a higher discount rate must lead to both higher capital costs and corresponding higher sales revenues. The impact of changes in $r$ on the Market-to-Book ratio then reduces to the impact of $r$ on the conservatism correction factor. Proposition 3 established that a higher cost of capital will generally result in a higher replacement cost for the firm’s current assets, that is a higher value $OA_T^*$. Accordingly, we formulate the following

**Hypothesis 9:** $\hat{CC}_T$ is increasing in the cost of capital, $r$.

### 4 Empirical Analysis

Our empirical analysis is designed to test the implications of the model, using a cross-section of firms over time. These tests speak directly to the central questions posed in the previous sections: What are the major components of the Market-to-Book ratio, and how does the ratio and its components relate to conservatism, growth and cost of capital. Section 4.1 discusses our empirical proxies for the theoretical constructs, Section 4.2 describes sample formation, and Section 4.3 presents the empirical methodology and the results.

#### 4.1 Empirical Proxies for Key Constructs

The key variables in our analysis of the Market-to-Book ratio, $MB_T$, are the useful life of assets, $T$, growth in investments, $(\lambda_1, ..., \lambda_T)$, the depreciation schedule $d$, the percentage of intangibles investments, $\alpha_T$ and the cost of capital, $r_T$. These variables jointly determine the two principal components of the M-to-B ratio: $CC_T$ and $FB_T$. In this section, we describe our proxies for these constructs and the assumptions underlying their use. The Compustat Xpressfeed variable names used in our measures are presented parenthetically. Additional details on the measurement of these variables and related variables are included in Appendix 1.

The central focus of our paper is the Market-to-Book ratio. As discussed in Section 3, we focus on the adjusted Market-to-Book ratio, which effectively excludes financial assets, as these are not subject to the forms of conservatism we study in this paper. The market value of equity and book value of equity are measured at the end of the fiscal year. The
useful life of tangible and intangible assets, denoted as $T$ throughout the model, is measured by taking the sum of the gross amount of property, plant and equipment and recognized intangibles divided by the annual charge for depreciation $\frac{PPE + INTAN}{dp}$. The depreciation variable on Compustat, $dp$, includes amortization of intangibles. Although our measure is admittedly an approximation, it provides an estimate of the weighted average useful life of the capitalized operating assets in the firm. This measure does not include investments that are immediately expensed such as R&D and advertising expense; effectively this assumes the omitted assets have a comparable useful life to the recognized assets.

Total investments in the observation year, $T$, are denoted by $INV_T$. This value is calculated as research and development expenses (XRD) plus advertising expenses (XAD) plus capital expenditures (CAPXV). Growth in investment in a given period, $\lambda_T$, is calculated as

$$\frac{INV_T}{INV_{T-1}} - 1.0.$$ 

We also compute the average growth rate over the past $T$ periods by the geometric mean of the rates ($\lambda_1, ..., \lambda_T$).

The model allows for two forms of conservatism: partial expensing of assets and conservatism in depreciation. Our measure of partial expensing, $\alpha_T$, is the ratio of research and development expenses and advertising expenses to total investment, that is, $\frac{XRD + XAD}{XRD + XAD + CAPXV}$. Although there are alternative measures of conservatism in the empirical accounting literature, $\alpha_T$ reflects our construct of partial expensing and is therefore consistent with our theory framework. Our second measure of conservatism, $ConDep$, aims to capture conservatism in depreciation. This variable is premised on the notion that firms may exercise discretion by choosing longer or shorter useful lives for their plant and intangibles. We estimate this propensity by comparing a firm’s estimated useful life $T$ to the median useful life reported by other firms in its industry, measured by its four-digit SIC code. $ConDep_1$ is an indicator variable equal to one if the firm’s useful life is less than the median for its industry and zero if equal to or greater than the median. $ConDep_2$ is the firm’s relative useful life, measured as $T$ minus the median for the industry.

The question of how to measure the equity cost of capital, $r_T$ is certainly not without controversy in the accounting and finance literature. Because our focus is on understanding conservatism and its effect on Market-to-Book ratios, we want a cost of capital measure that does not rely on financial statement numbers. We therefore use the Fama and French (1992) two-factor approach, and estimate the cost of capital with the market return and firm size as factors. If the firm’s implied cost of capital is missing or negative, we substitute the median cost of capital for firms in the same two-digit SIC code and year.

As indicated in Section 3, we estimate the Future-to-Book ratio at date $T$, $FB_T$, by
capitalizing the firm’s current economic profit, that is, \( R_T(K_T) - c \cdot K_T \), provided that profit is positive. In turn, we obtain an approximation of the firm’s current economic profit by current residual income, subject to a correction factor, \( \Delta_T \). This correction is intended to correct for the biases that result from the direct expensing of intangibles investments and the use of straight-line depreciation. Specifically, our proxy for \( R_T(K_T) - c \cdot K_T \) is \( \text{Sales}_T - \text{EconCost}_T \) where:

\[
\text{EconCost}_T = \text{Expenses}_T - dp_T + \frac{1}{\Delta_T} \cdot (dp_T + r_w \cdot OA_{T-1}). \tag{28}
\]

Here \( r_w \) denotes the weighted average cost of capital and the correction factor \( \Delta_T \) is given by:

\[
\Delta_T = \Gamma_w^T \cdot \frac{u_0 + u_1(1 + \lambda_1) + \cdots + u_{T-1} \prod_{i=1}^{T-1} (1 + \lambda_i) + \alpha_T \cdot T}{1 + (1 + \lambda_1) + \cdots + \prod_{i=1}^{T-1} (1 + \lambda_i)}, \tag{29}
\]

where

\[
\Gamma_w^T = \frac{1}{1 + r_w} + \left( \frac{1}{1 + r_w} \right)^2 + \cdots + \left( \frac{1}{1 + r_w} \right)^T
\]

and

\[
u_t = (1 - \alpha_t) \left[ \frac{1}{T} + r_w \cdot \left( 1 - \frac{T-1-t}{T} \right) \right],
\]

for \( 0 \leq t \leq T - 1 \). The correction factor \( \Delta_T \) is the ratio of two historical cost figures: the numerator represents the historical cost obtained with direct expensing for investments in intangibles and straight-line depreciation of all capitalized investments; the denominator is given by the historical (economic) cost under R.P.C. accounting. This correction is applied to operating assets and is based on the weighted average cost of capital \( r_w \). As shown in Rajan and Reichelstein (2009), this ratio exceeds (is below) one whenever the past growth rates have consistently been below (above) the cost of capital, that is, \( \lambda_t \leq (\geq) r_w \) for all \( t \).

### 4.2 Sample Selection

Our empirical tests employ financial statement data from Compustat Xpressfeed, and cost of capital data from the CRSP monthly returns file and K. French’s website on return factors. Our sample covers all firm-year observations with available Compustat data, and covers

\[32\text{Throughout our empirical analysis, we set the lag factor } L \text{ equal to 1. It seems plausible that there are significant variations in } L \text{ across industries, an aspect we do not pursue in this paper.}\]
the time period from 1962 to 2007. We exclude firm-year observations with SIC codes in the range 6000-6999 (financial companies) because the magnitude of these firms’ financial assets likely precludes our detecting the effects on Market-to-Book we are interested in. This gives us a starting point of 316,896 firm-year observations, as indicated in Table 1. We impose several additional criteria to insure firms have the relevant data to measure the variables in our analysis. Specifically, we exclude observations for which market value is not available (94,185 firm-years), book value of operating assets is not available (582 firm-years), market value of net operating assets is zero or negative (13,831 firm-years), there is insufficient history for the calculation of $CC_T$ (37,106 firm-years), the ratio of plant to total assets is less than 10% (28,859 firm-years) and total assets are less than $4 million (6,978). These criteria yield a sample size of 135,358 firm-year observations with data on the primary variables we examine. The number of observations in any given regression varies depending on the availability of additional data necessary for the particular test as well as deletion based on outlier diagnostics.

4.3 Empirical Methodology and Results

We report results based on pooled OLS regressions. To minimize the influence of extreme observations in the parametric regressions, we winsorize included variables at the 2nd and 98th percentile, and exclude observations using deletion filters based on the outlier diagnostics of Belsey, Kuh and Welsch (1980). In addition, we estimate a second set of regressions where the continuous value of the independent variable is replaced with its annual percentile rank. To create these ranks, the continuous variables are sorted annually into 100 equal-sized groups. This second set of regressions makes the less restrictive assumption that the relations between the dependent and explanatory variables are monotonic (Iman and Conover, 1979). In the interests of parsimony, we present the parametric analysis in the tables, as none of our primary inferences are affected if they are replaced by nonparametric estimations.

Descriptive Statistics

Table 2 presents the descriptive statistics for our sample. The average Market-to-Book ratio, is 2.443 and the median is 1.597. The median and skewness of the distribution are consistent with the data in Penman (2009, p. 43). For the adjusted Market-to-Book ratio, $MB_T$, we observe an average Market-to-Book ratio for operating assets of 2.995, consistent with our presumption that financial assets have book values closer to their market values. The average cost of capital is 10.5%, which is consistent with estimates of long-term rates of return on equities by Ibbotson and Associates (2006). The average capital intensity, measured as plant to total assets, is 39.5%, confirming that plant assets are material for our
sample. Advertising intensity and R&D intensity are skewed, with zero expense recognized at the 25th and 50th percentiles. The average useful life of plant and capitalized intangibles is 14.821, with a median of 14. The average annual fraction of partial expensing is 23.4%, with a median of 7.8%, and the growth-weighted average measure, $\alpha_T$, is 21.3% with a median of 9.2%, consistent with skewness in advertising and R&D. Average annual growth in investment is 24.4%, just slightly greater than the geometric mean of historical growth at 20.9%.

The mean of $CC_T$ is 1.83, and the median is 1.321. As a result, the mean of $FB_T$, defined as the residual $MB_T - CC_T$ is 1.165. The sizable magnitude of $CC_T$ and $FB_T$ suggests that both conservatism and future value are very substantial components of $MB_T$. The mean of $CC_T^2$ is 1.985 with a median of 1.356. Thus the calculation of the conservatism correction factor based on a measure of the average constant growth over the past $T$ periods results in a conservatism correction of similar magnitude to that based on the full history of investments over the prior $T$ periods. The variable $\hat{FB}_T$ in Table 2 is an estimate of future value based on on estimated future economic profits. We note that $\hat{FB}_T$ has a mean of 1.040 and a median of 0.179, and thus is fairly comparable to the measure of $FB_T$ derived by subtracting $CC_T$ from $MB_T$. Panel B of Table 2 presents a correlation matrix of the variables, with Pearson correlations above the diagonal and Spearman correlations below the diagonal. The correlations provide support for a number of our measures and constructs.

**Tests of Hypotheses**

Our first hypothesis is that $FB_T \equiv MB_T - CC_T$ is positive on average. This is motivated by the argument that economic profits in future periods resulting from past investments should be positive on average. As documented in Table 2, the mean of $FB_T$ is positive. Panel A of Table 3 shows that the t-statistic for the hypothesis that the mean of $FB_T$ is greater than 0 is 118.46, which is highly significant. Our second hypothesis is that $FB_T$ and $CC_T$ have significant explanatory power for $MB_T$.

Our test of Hypothesis 2 is based on the estimation equation:

$$MB_T = \eta_{01} + \eta_{11} \cdot CC_T + \eta_{21} \cdot \hat{FB}_T + \epsilon_1. \quad (E1)$$

We hypothesize positive coefficients on both $CC_T$ and $\hat{FB}_T$. Panel B of Table 3 presents the estimation results. The findings indicate that both $CC_T$ and $\hat{FB}_T$ have significant explanatory power for $MB_T$. The coefficient on $CC_T$ is 0.777 with a t-statistic of 150.09. The coefficient on $\hat{FB}_T$ is 0.655, with a t-statistic of 130.07. Including both variables in the estimation causes the adjusted $R^2$ to climb from 15% and 18.8% for the single variable regressions to 27.7%, consistent with both variables having significant incremental explanatory
power. The findings indicate that both our conservatism correction factor and our estimate of future value explain substantial a substantial part of the variation in $MB_T$.

Our third hypothesis states that the Market-to-Book ratio is increasing and convex in $\alpha$. We provide evidence along three different lines in our test of this hypothesis. First, we examine the relation visually by plotting the Market-to-Book ratio against $\alpha T$. Second, we test whether the logarithm of the Market-to-Book ratio is negatively associated with the logarithm of $1 - \alpha$. Third, we test whether the Market-to-Book ratio is increasing in $(\alpha_T)^2$.

The corresponding two estimation equations are:

$$
\log(MB_T) = \eta_{02} + \eta_{12} \cdot \log(1 - \alpha_T^a) + \epsilon_2. \quad \text{(E2)}
$$

$$
MB_T = \eta'_{02} + \eta'_{12} \cdot \alpha_T^a + \eta'_{22} \cdot (\alpha_T^a)^2 + \epsilon'_2. \quad \text{(E2')}\n$$

Panel A of Table 4 displays the values of $\alpha_T^a$ partitioned by half-deciles, and the corresponding value of $MB_T$ for each partition. Because many firms do not report advertising or research and development expense to Compustat and therefore $\alpha = 0$, a sizable number are pooled in the bottom 6 ranks (0-5). The mean $MB_T$ for these firms is 2.205. For observations with positive values of $\alpha$, $MB_T$ increases monotonically in $\alpha_T^a$, ranging from 1.823 for the partition with mean $\alpha_T^a=0.001$ to 8.095 for observations with $\alpha_T^a=0.806$. 

Figure 2: *Market-to-Book as a function of the percentage of investments directly expensed.*
Figure 2 presents the graph of Market-to-Book values plotted against $\alpha_T^a$. The figure confirms a convex relation, with $MB_T$ increasing at an increasing rate in $\alpha_T^a$. The plot findings are consistent with the estimation results for E2 and E2’ in Panel B of Table 4. The first estimation documents that the relation between the log of $MB_T$ and $\log(1-\alpha_T^a)$ is highly significantly negative, with a coefficient estimate of -0.833 and t-statistic of -187.15. The second estimation also confirms a convex relation between $MB_T$ and $\alpha_T^a$, as the coefficient $\eta$ is positive and significant after controlling for $\alpha_T^a$. These findings provide strong support for the functional form of $CC_T$, which postulates that the form of correction is convex (and hyperbolic) in the degree of partial expensing.

Our fourth hypothesis is that ceteris paribus more accelerated depreciation increases the Market-to-Book ratio. Our proxy for more accelerated depreciation is a shorter useful life relative to that used by other firms in the industry. We test for a relation between $MB_T$ and more conservative depreciation with $ConDep_1$, an indicator variable equal to one if the firm has a shorter life for depreciation than other firms in its SIC code, and $ConDep_2$, the life of the firm’s plant and intangible assets relative to the median of its industry peers. We test for this association after controlling for $CC_T$, and therefore controlling for partial expensing and differences in the form of depreciation relative to relative practical capacity. The estimation equation is:

$$MB_T = \eta_{03} + \eta_{13} \cdot CC_T + \eta_{23} \cdot ConDep_1 + \epsilon_3.$$  \hspace{1cm} (E3)

We also include controls for factors that could influence useful life but would not reflect conservatism: %Land, the ratio of land to gross plant, %Buildings, the ratio of buildings to gross plant, and %Intangibles, the ratio of recognized intangibles to the sum of net plant and intangibles. We expect that a greater proportion of land and intangibles (buildings) to total operating assets will result in a shorter (longer) estimated useful life than the median firm in the industry, all else equal. The descriptive statistics in Panel A of Table 5 show that the mean of the useful life, $T$, for our sample is 14.82 years. The average number of firm-years used to estimate the median useful life in a firm’s industry for a given year is 29.044, which should result in estimates with a reasonable amount of precision. Due to the skewness in the distribution of $T$, 57.4% of firms have a shorter life than the industry median ($ConDep_1=1$). The estimation results in Panel B of Table 5 document that the Market-to-Book ratio is significantly positively associated with $CC_T$. In addition, as predicted, the findings indicate that the Market-to-Book ratio is significantly greater for firms with shorter useful lives than other firms in their industry ($ConDep_1=1$). Similarly, the Market-to-Book ratio decreases significantly in the years of useful life in excess of the median for the firm’s industry ($ConDep_2=1$). The findings are unaffected by the inclusion of controls for the
percent of land, buildings or intangibles in operating assets. Taken together, these findings provide strong support for our hypothesis that more accelerated depreciation causes a higher Market-to-Book ratio.

Our fifth hypothesis states that $\tilde{F}B_T$ is unrelated to $\alpha^a_T$. Our corresponding test is based on the estimation equation:

$$\tilde{F}B_T = \eta_{04} + \eta_{14} \cdot \alpha^a_T + \eta_{24} \cdot \lambda^a_T + \eta_{34} \cdot r_T + \epsilon_4$$  \hspace{1cm} (E4)

The modified Future-to-book controls for the partial expensing of intangibles. Our test therefore relates an estimate of future value to the proportion of overall investments that are made in intangibles such as R&D and advertising. The findings indicate a positive relation between $\tilde{F}B_T$ and average past growth in investments, consistent with the notion that growth in investment is associated with the value of investment opportunities. The modified Future-to-book is negatively related to the cost of capital, $r_T$, consistent with the notion that the discounting of cash flows by a higher cost of capital reduces future value; however, the nonparametric estimation suggests that this relation is insignificant. If investments in intangibles give rise to positive abnormal returns, we expect a positive coefficient on $\alpha^a_T$, controlling for growth and the cost of capital. The findings in Table 6 indicate that modified Future-to-Book, $\tilde{F}B_T$, is in fact negatively related to $\alpha^a_T$. The coefficient on this variable is -0.200 and the t-statistic is -14.48. The nonparametric estimation results reported in column (2) indicate a significant negative association as well. The findings suggest that controlling for the effect of partial expensing on the denominator of the Future-to-Book ratio, the association between future value and investment in intangibles is negative. The findings call into question the notion that investment in intangibles leads *ipso facto* to above-normal economic profits in the future.

Our sixth hypothesis concerns the relation between $CC_T$ and past investment growth. Proposition 2 states that if depreciation is (uniformly) more accelerated than the unbiased depreciation schedule given by R.P.C. accounting, then $CC_T$ will be monotone decreasing in each $\lambda_t$. In addition to our prediction that past growth has a negative impact on $CC_T$, Observation 2 notes that this negative association should be more accentuated for firms that expense a larger percentage of their investments. This relation is the basis for Hypothesis 7. We base our inferences about Hypotheses 6 and 7 on the following estimation equation:

$$\hat{C}C_T = \eta_{05} + \eta_{15} \cdot \alpha^a_T + \eta_{25} \cdot \lambda^a_T + \eta_{35} \cdot r_T + \epsilon_5$$  \hspace{1cm} (E5)

Panel A of Table 7 presents the values of $MB_T$, $CC_T$, $FB_T$ and their estimated counterparts: $\tilde{F}B_T$ and $\hat{C}C_T \equiv MB_T - \tilde{F}B_T$, partitioned by half-deciles of average growth, $\lambda^a_T$. 31
The findings indicate a declining relation between the Market-to-Book ratio and growth for the first 7 half-deciles, and then an increasing relation. \( CC_T \) and \( FB_T \) exhibit a similar though less pronounced U-shaped relation. The estimation results in Panel B of Table 7 indicate a significant negative coefficient on \( \lambda^a_T \), consistent with Hypothesis 6. With regard to Hypothesis 7, we compare the coefficient on \( \lambda^a_T \) for observations with \( \alpha^a_T \) above versus those below the median. The larger negative (absolute value) coefficients for firms with a high proportion of intangibles emerges both in the parametric and nonparametric (ranks) regressions. One peculiarity in the parametric estimation results is that the coefficient on \( \alpha^a_T \) turns negative for the subsample with low values of \( \alpha^a_T \). This may reflect the relatively lower power of the test resulting from the limited variation in \( \alpha^a_T \) inherent in that subsample. Notably, the coefficient on \( \alpha^a_T \) is positive in the nonparametric results.

Hypothesis 7 is based on our analytical finding in Proposition 4: the decline in \( CC_T \) is more pronounced for firms with negative past growth in investments than for those with positive average growth. Put differently, \( CC_T \) is a monotonically decreasing function of \( \lambda^a_T \), but the function flattens out for larger values of \( \lambda^a_T \). In testing this prediction, we employ the same regression equation as before, except that the variable \( \lambda^a_T \) is partitioned into two subsamples depending on whether average past growth was positive or negative. The two corresponding variables are denoted by \( \lambda^a_T^- \) and \( \lambda^a_T^+ \), respectively.

\[
\hat{CC}_T = \eta_{05} + \eta_{15} \cdot \alpha^a_T + \eta_{25} \cdot \lambda^a_T^+ + \eta_{35} \cdot \lambda^a_T^- + \eta_{45} \cdot r_T + \epsilon_T
\] (E5')

Our findings in Panel C of Table 7 indicate a more negative association between past growth in investments and the estimated conservatism correction factor on account of two forces: (i) negative growth, that is \( \lambda^a_T < 0 \) and (ii) a high percentage of intangibles investments \( \alpha^a_T \). The only exception to that pattern occurs in Column (3) as we move from negative to positive past growth.

Our ninth and final hypothesis concerns the relation between \( CC_T \) and the cost of capital, \( r_T \). We test whether the component of \( CC_T \) embedded in \( MB_T \) has a positive relation to the cost of capital. Accordingly, we again consider \( \hat{CC}_T \equiv MB_T - \hat{FB}_T \) as our dependent variable. Our test of Hypothesis 9 is that the coefficient on \( r_T \) in estimation equation E5 is positive. The findings in Panels B and C of Table 7 indicate strong support for our hypothesis. The coefficient on \( r_T \) is positive and significant in all specifications. These findings are not due to induced measurement error in our estimate of future value, as the correlation matrix in Table 2 Panel B indicates a significant positive correlation between \( MB_T \) and \( r_T \) as well.

To summarize, the empirical findings are strongly supportive of the empirical predictions.
of the model. The magnitude of the conservatism correction factor is material, and our estimate of $CC_T$ has significant explanatory power for the Market-to-Book ratio. Furthermore, as hypothesized, the Market-to-Book ratio is increasing and convex in the percentage of intangibles investments, increasing in the conservatism of depreciation, and decreasing in past investment growth. Finally, the negative relation between the conservatism correction factor and past growth is more pronounced for lower growth firms and for firms with a higher proportion of intangibles.

5 Conclusion

This paper proposes a structural decomposition of the Market-to-Book ratio (M-to-B) into two additive component ratios: the Conservatism Correction factor (CC) and the Future-to-Book ratio (FB). By construction, the CC factor exceeds one if the firm’s operating assets are valued below their replacement cost on the balance sheet. A positive FB ratio reflects investors’ expectation of a stream of positive (discounted) future economic profits. Our decomposition relates to the familiar concept of Tobin’s $q$ insofar as for an all equity firm $q$ is given by the ratio between M-to-B and the Conservatism Correction factor.

Our empirical results document that the Conservatism Correction factor is significantly greater than one, with a mean of 1.83, and the Future-to-Book ratio is significantly positive, with a mean of 1.165. Given that the mean Market-to-Book ratio for our sample is 2.995, the findings indicate that each component is significant in explaining why the Market-to-Book ratio exceeds one. We test this directly by regressing the Market-to-Book ratio on estimates of the Conservatism Correction Factor and of Future Value. This test reveals that each component has significant explanatory power for the Market-to-Book ratio, and the combined model has an adjusted $R^2$ of 27.7%. These findings indicate accounting conservatism and the value of future growth opportunities each have significant impact on a firm’s Market-to-Book ratio.

Our model predicts that the Market-to-Book ratio is increasing and convex in the degree of partial expensing, a prediction that is clearly confirmed by the data. Accelerated depreciation of capitalized investment expenditures is a second source of unconditional accounting conservatism. Our empirical results confirm this prediction by documenting that the Market-to-Book ratio is increasing in the extent to which the assessed useful life of a firm’s assets are shorter than the assessments by the firm’s industry peers.

Another prediction emerging from our model is that the Conservatism Correction factor ratio is decreasing in past investment growth. We document a significant negative relation between $CC_T$ and past growth, and, consistent with the theoretical predictions, also find a
steeper negative relation for firms with a greater percentage of investments in intangibles. In addition, we predict and find that the association between M-to-B and past investment growth is more pronounced for firms with negative average growth in past investments than positive past growth. Lastly, we predict and find that the component of the conservatism correction factor embedded in the M-to-B ratio is positively related to the cost of capital. We document this relation both for the implied Conservation Correction factor and for the overall Market-to-book ratio.

Taken as a whole, our findings provide substantial support for the validity of the decomposition performed in this paper. Additional uses of our approach may emerge in a variety of contexts that seek to explore the relation between the M-to-B ratio and other financial ratios. As indicated in the Introduction, earlier studies have largely focused on the aggregate M-to-B ratio, even though the aggregate ratio is generally believed to reflect a confluence of several distinct factors.
6 Appendix 1-Tables

Description of Variables

\[ MV_T = \text{Market value of equity at end of fiscal year } T (CSHO \ast PRCC, F) \]

\[ MB^un_T = \text{Market value of equity at time } T \text{ divided by book value of equity for fiscal year } T \]

\[ FA_T = \text{Net financial assets at end of fiscal year } t, \text{ measured as total assets minus net plant intangibles minus liabilities } (AT - PPENT - INTAN - LT) \]

\[ MB_T = \frac{MV_T - FA_T}{OA_T} = \frac{MV_T - FA_T}{BV_T - FA_T}. \text{ Adjusted Market-to-Book ratio.} \]

\[ OA_T = \text{Net plant + intangibles, at end of fiscal year } T, (PPENT + INTAN) \]

\[ \text{Total investment}_T = \text{Advertising expense plus R & D expense plus Capital expenditures for period } T - 1 \text{ to } T (XAD + XRD + CAPXV) \]

\[ \alpha_t = \text{Conservatism in fiscal year } t, \text{ measured as } (XAD + XRD)/(XAD + XRD + CAPXV) \text{ where } XAD \text{ is advertising expense, } XRD \text{ is Research and Development expense, and } CAPXV \text{ is capital expenditures} \]

\[ \alpha^a_T = \text{Growth weighted average of directly expensed investments} \]

\[ \frac{\alpha_1(1+\lambda_1)+\cdots+\alpha_T\prod_{i=1}^{T}(1+\lambda_i)}{(1+\lambda_1)+\cdots+\prod_{i=1}^{T}(1+\lambda_i)} \]

\[ T = \text{useful life of plant and intangibles,} \]

\[ (\text{gross plant + intangibles})/(\text{depreciation + amortization of intangibles}) \text{ measured as } (PPEGT + INTAN)/\text{dep.} \]

\[ \lambda_t = \frac{XAD_t + XRD_t + CAPXV_t}{(XAD_{t-1} + XRD_{t-1} + CAPXV_{t-1}) - 1} \]

\[ \lambda^a_T = \text{Geometric mean of growth over } T \text{ periods} \]

\[ \lambda^a_3 = \text{Geometric mean of growth rates } (\lambda_T, \lambda_{T-1}, \lambda_{T-2}). \]
OIADP = Operating income after depreciation, amortization.

NOI = OIADP - (+) Net-Interest.

Expenses = SALE - NOI

\( r \) = cost of capital for firm \( i \) and year \( t \), estimated with coefficients from the Fama-French (1992) two-factor model:

\[
R_i - R_f = \delta_0 + \delta_1(R_m - R_f) + \delta_2(SMB) + \epsilon
\]

using CRSP monthly returns from the 5 preceding years \((t-5 \text{ to } t-1)\), and Ken French’s data on market and size factors (SMB).

\( \gamma \) = \[
\frac{1}{1+r}
\]

\( \Gamma^n \) = \[
\gamma + \gamma^2 + \ldots + \gamma^n
\]

\( N_T \) = \[
\Gamma^1 + \Gamma^2(1 + \lambda_2) + \ldots + \Gamma^T \prod_{i=2}^{T}(1 + \lambda_i)
\]

\( D_T \) = \[
(1 - \alpha_1) \left(1 - \frac{T-1}{T}\right) + (1 - \alpha_2) \left(1 - \frac{T-2}{T}\right)(1 + \lambda_2) + \ldots
\]

\[
+ (1 - \alpha_T-1) \prod_{i=2}^{T-1}(1 + \lambda_i) \cdot \left(1 - \frac{1}{T}\right) + (1 - \alpha_T) \prod_{i=2}^{T}(1 + \lambda_i)
\]

\( CC_T \) = \[
\frac{N_T}{D_T} \cdot \frac{1}{\Gamma_T} \]

where \( T, r, \gamma \) and \( \lambda \) are as defined above.

\( \tau_T \) = statutory income tax rate in year \( T \)

\( CC^\lambda_T \) = Same as \( CC_T \) except that \( \lambda_t = \lambda_T^a \) for all \( t \).

\( r_d \) = Cost of Debt: \((1 - \tau_T)\). Interest Expense divided by the average of beginning and ending balance of interest-bearing debt

\( r_w \) = Weighted average cost of capital: \[
\frac{BV_T}{AT_T} \cdot r + \frac{AT_T - BV_T}{AT_T} \cdot r_d (1 - \tau_T)
\]

\( \text{EconCost}_T \) = Expenses\(_T\) - \( dep_T + \frac{1}{\Sigma_T} (dep_T + r_w \cdot OA_T-1) \). As before, \( dep_T \) includes amortization

\( \Delta \) = \[
\Gamma^T_w \cdot \frac{u_0 + u_1(1+\lambda_1) + \ldots + u_{T-1}(1+\lambda_{T-1}) + \alpha_T \cdot \prod_{i=1}^{T}(1+\lambda_i)}{1+(1+\lambda_1) + \ldots + \prod_{i=1}^{T}(1+\lambda_i)}
\]

\( u_t \) = \[
(1 - \alpha_t) \left[\frac{1}{T} + r_w \cdot \left(1 - \frac{T-1-t}{T}\right)\right] \] for \( 0 \leq t \leq T-1 \)

\( \Gamma^w_T \) = \[
\frac{1}{1+r_w} + (\frac{1}{1+r_w})^2 + \ldots + (\frac{1}{1+r_w})^T
\]

\( FB_T \) = Estimated Future-to-Book Value, defined as

\[
\frac{(1-\tau_T)I(R_t(K_T) - c - K_T)}{OA_T} \cdot \Gamma^5_\lambda
\]

\( I\{x\} \) = \[
x \text{ if } x \geq 0 \text{ and } I\{x\} = 0 \text{ if } x \leq 0
\]

\( \Gamma^5_\lambda \) = Capitalization factor, given by \[
\sum_{i=1}^{5} \left(\frac{1+\lambda_3}{1+r}\right)^i
\]
<table>
<thead>
<tr>
<th>Criteria</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data available in Compustat</td>
<td>316,896</td>
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<tr>
<td>Market value not available</td>
<td>(94,185)</td>
</tr>
<tr>
<td></td>
<td>222,711</td>
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<tr>
<td>Book value of operating assets not available</td>
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<td></td>
<td>222,129</td>
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<tr>
<td>Market value of net operating assets less than or equal to zero</td>
<td>(13,831)</td>
</tr>
<tr>
<td></td>
<td>208,298</td>
</tr>
<tr>
<td>Missing data for calculation of conservatism correction, CC</td>
<td>(37,106)</td>
</tr>
<tr>
<td></td>
<td>171,195</td>
</tr>
<tr>
<td>Ratio of Plant/Total assets less than 10%</td>
<td>(28,859)</td>
</tr>
<tr>
<td></td>
<td>142,336</td>
</tr>
<tr>
<td>Total assets less than $4 million</td>
<td>(6,978)</td>
</tr>
<tr>
<td></td>
<td>135,358</td>
</tr>
<tr>
<td>Variable Label</td>
<td>N</td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
</tr>
<tr>
<td>$MB_{T_1}^{un}$</td>
<td>129,772</td>
</tr>
<tr>
<td>$MB_T$</td>
<td>135,358</td>
</tr>
<tr>
<td>Total Assets</td>
<td>135,358</td>
</tr>
<tr>
<td>Capital intensity</td>
<td>135,358</td>
</tr>
<tr>
<td>Advertising intensity</td>
<td>135,233</td>
</tr>
<tr>
<td>R &amp; D intensity</td>
<td>135,233</td>
</tr>
<tr>
<td>$T_1$</td>
<td>135,358</td>
</tr>
<tr>
<td>$\lambda_3^a$</td>
<td>135,094</td>
</tr>
<tr>
<td>$\alpha_T^a$</td>
<td>135,358</td>
</tr>
<tr>
<td>$\lambda_T^a$</td>
<td>135,358</td>
</tr>
<tr>
<td>$r_T$</td>
<td>135,358</td>
</tr>
<tr>
<td>$CC_T$</td>
<td>135,358</td>
</tr>
<tr>
<td>$CC_T^a$</td>
<td>135,029</td>
</tr>
<tr>
<td>$FB_T$</td>
<td>135,358</td>
</tr>
<tr>
<td>$\tau$</td>
<td>135,358</td>
</tr>
<tr>
<td>$\hat{FB}_T$</td>
<td>117,960</td>
</tr>
<tr>
<td>$\hat{CC}_T$</td>
<td>117,960</td>
</tr>
<tr>
<td></td>
<td>MB&lt;sub&gt;T&lt;/sub&gt;</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
</tr>
<tr>
<td>MB&lt;sub&gt;T&lt;/sub&gt;</td>
<td>1.00</td>
</tr>
<tr>
<td>r&lt;sub&gt;T&lt;/sub&gt;</td>
<td>0.09</td>
</tr>
<tr>
<td>T</td>
<td>-0.31</td>
</tr>
<tr>
<td>α&lt;sub&gt;T&lt;/sub&gt;</td>
<td>0.23</td>
</tr>
<tr>
<td>λ&lt;sub&gt;T&lt;/sub&gt;</td>
<td>0.17</td>
</tr>
<tr>
<td>λ&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.18</td>
</tr>
<tr>
<td>α&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.27</td>
</tr>
<tr>
<td>λ&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.19</td>
</tr>
<tr>
<td>CC&lt;sub&gt;T&lt;/sub&gt;</td>
<td>0.21</td>
</tr>
<tr>
<td>CC&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.17</td>
</tr>
<tr>
<td>FB&lt;sub&gt;T&lt;/sub&gt;</td>
<td>0.84</td>
</tr>
<tr>
<td>FB&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.33</td>
</tr>
<tr>
<td>CC&lt;sub&gt;T&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Table 3:

**Panel A:** Fractiles of the distribution of the Market-to-Book ratio, Conservatism Correction and Future-to-Book value.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
<th>25th Pctl</th>
<th>Median</th>
<th>75th Pctl</th>
</tr>
</thead>
<tbody>
<tr>
<td>MB\textsuperscript{un}</td>
<td>129,772</td>
<td>2.443</td>
<td>2.853</td>
<td>0.987</td>
<td>1.597</td>
<td>2.708</td>
</tr>
<tr>
<td>MB\textsuperscript{T}</td>
<td>135,358</td>
<td>2.995</td>
<td>3.883</td>
<td>1.009</td>
<td>1.585</td>
<td>3.094</td>
</tr>
<tr>
<td>CC\textsuperscript{T}</td>
<td>135,358</td>
<td>1.830</td>
<td>1.349</td>
<td>1.125</td>
<td>1.321</td>
<td>1.928</td>
</tr>
<tr>
<td>FB\textsuperscript{T}</td>
<td>135,358</td>
<td>1.165</td>
<td>3.619</td>
<td>-0.471</td>
<td>0.145</td>
<td>1.387</td>
</tr>
<tr>
<td>\hat{FB}\textsuperscript{T}</td>
<td>117,960</td>
<td>1.040</td>
<td>2.650</td>
<td>0.000</td>
<td>0.179</td>
<td>0.888</td>
</tr>
</tbody>
</table>

**Panel B:** Tests of hypothesized values for CC\textsuperscript{T} and FB\textsuperscript{T}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Hypothesized Value</th>
<th>Standard Error of Mean</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC\textsuperscript{T}</td>
<td>1.830</td>
<td>1</td>
<td>226.51*</td>
</tr>
<tr>
<td>FB\textsuperscript{T}</td>
<td>1.165</td>
<td>0</td>
<td>118.46*</td>
</tr>
</tbody>
</table>

**Panel C:** Estimation results from regression of Market-to-Book on the Conservatism Correction factor and estimated Future-to-Book ratio. Coefficients are shown with t-statistics in parentheses.

\[ MB_T = \eta_0 + \eta_{11} \cdot CC_T + \eta_{21} \cdot \hat{FB}_T + \epsilon_1. \]  \hspace{2cm} (E1)

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.908</td>
<td>1.649</td>
<td>0.604</td>
</tr>
<tr>
<td></td>
<td>(87.63)</td>
<td>(313.10)</td>
<td>(61.87)</td>
</tr>
<tr>
<td>CC\textsuperscript{T}</td>
<td>0.777</td>
<td></td>
<td>0.766</td>
</tr>
<tr>
<td></td>
<td>(150.09)</td>
<td></td>
<td>(151.21)</td>
</tr>
<tr>
<td>\hat{FB}_T</td>
<td></td>
<td>0.457</td>
<td>0.454</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(161.06)</td>
<td>(161.70)</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.150</td>
<td>0.188</td>
<td>0.277</td>
</tr>
<tr>
<td>n</td>
<td>127,594</td>
<td>112,024</td>
<td>111,742</td>
</tr>
</tbody>
</table>

*The probability value of the test statistic is less than .0001.
Table 4: The Relation between $\alpha$ and $MB_T$.

Panel A: Mean Values of Selected Variables Ranked by Half-deciles of $\alpha^a_T$

<table>
<thead>
<tr>
<th>Rank</th>
<th>$n$</th>
<th>$\alpha^a_T$</th>
<th>$MB_T$</th>
<th>$CC_T$</th>
<th>$FB_T$</th>
<th>$FB^*_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>45564</td>
<td>0.000</td>
<td>2.203</td>
<td>1.159</td>
<td>1.044</td>
<td>1.044</td>
</tr>
<tr>
<td>6</td>
<td>1811</td>
<td>0.001</td>
<td>1.823</td>
<td>1.175</td>
<td>0.648</td>
<td>0.648</td>
</tr>
<tr>
<td>7</td>
<td>6768</td>
<td>0.011</td>
<td>1.832</td>
<td>1.177</td>
<td>0.655</td>
<td>0.640</td>
</tr>
<tr>
<td>8</td>
<td>6768</td>
<td>0.036</td>
<td>1.953</td>
<td>1.207</td>
<td>0.747</td>
<td>0.708</td>
</tr>
<tr>
<td>9</td>
<td>6768</td>
<td>0.071</td>
<td>2.118</td>
<td>1.252</td>
<td>0.866</td>
<td>0.782</td>
</tr>
<tr>
<td>10</td>
<td>6768</td>
<td>0.116</td>
<td>2.243</td>
<td>1.319</td>
<td>0.924</td>
<td>0.789</td>
</tr>
<tr>
<td>11</td>
<td>6768</td>
<td>0.168</td>
<td>2.334</td>
<td>1.397</td>
<td>0.937</td>
<td>0.754</td>
</tr>
<tr>
<td>12</td>
<td>6768</td>
<td>0.225</td>
<td>2.544</td>
<td>1.502</td>
<td>1.041</td>
<td>0.789</td>
</tr>
<tr>
<td>13</td>
<td>6768</td>
<td>0.288</td>
<td>2.621</td>
<td>1.645</td>
<td>0.976</td>
<td>0.702</td>
</tr>
<tr>
<td>14</td>
<td>6768</td>
<td>0.353</td>
<td>2.901</td>
<td>1.805</td>
<td>1.097</td>
<td>0.713</td>
</tr>
<tr>
<td>15</td>
<td>6768</td>
<td>0.422</td>
<td>3.339</td>
<td>2.021</td>
<td>1.319</td>
<td>0.782</td>
</tr>
<tr>
<td>16</td>
<td>6768</td>
<td>0.496</td>
<td>3.805</td>
<td>2.325</td>
<td>1.480</td>
<td>0.757</td>
</tr>
<tr>
<td>17</td>
<td>6768</td>
<td>0.579</td>
<td>4.860</td>
<td>2.771</td>
<td>2.089</td>
<td>0.888</td>
</tr>
<tr>
<td>18</td>
<td>6768</td>
<td>0.677</td>
<td>5.938</td>
<td>3.636</td>
<td>2.302</td>
<td>0.768</td>
</tr>
<tr>
<td>19</td>
<td>6767</td>
<td>0.806</td>
<td>8.095</td>
<td>6.426</td>
<td>1.669</td>
<td>0.359</td>
</tr>
</tbody>
</table>

Panel B:

\[
\log(MB_T) = \eta_{02} + \eta_{12} \cdot \log(1 - \alpha^a_T) + \epsilon_2. \quad (E2)
\]

\[
MB_T = \eta'_{02} + \eta'_{12} \cdot \alpha^a_T + \eta'_{22} \cdot (\alpha^a_T)^2 + \epsilon'_2. \quad (E2')
\]

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.329</td>
<td>1.841</td>
</tr>
<tr>
<td></td>
<td>(130.63)</td>
<td>(246.21)</td>
</tr>
<tr>
<td>$\log(1 - \alpha^a_T)$</td>
<td>-0.833</td>
<td>(-182.24)</td>
</tr>
<tr>
<td>$\alpha^a_T$</td>
<td>0.087</td>
<td>(1.36)</td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>2.908</td>
<td>(33.89)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.206</td>
<td>0.095</td>
</tr>
<tr>
<td>$n$</td>
<td>127,796</td>
<td>127,465</td>
</tr>
</tbody>
</table>
Table 5: Market-to-Book and Conservatism in Depreciation


<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
<th>25th Pctl</th>
<th>Median</th>
<th>75th Pctl</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>135,358</td>
<td>14.821</td>
<td>7.127</td>
<td>10.000</td>
<td>14.000</td>
<td>18.000</td>
</tr>
<tr>
<td>Number of firm-years</td>
<td>135,358</td>
<td>29.044</td>
<td>41.405</td>
<td>7.000</td>
<td>14.000</td>
<td>31.000</td>
</tr>
<tr>
<td>Median useful life</td>
<td>135,358</td>
<td>14.283</td>
<td>5.082</td>
<td>11.000</td>
<td>14.000</td>
<td>16.500</td>
</tr>
<tr>
<td>ConDep1</td>
<td>135,358</td>
<td>0.574</td>
<td>0.494</td>
<td>0.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>ConDep2</td>
<td>135,358</td>
<td>0.538</td>
<td>5.302</td>
<td>-2.000</td>
<td>0.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

Panel B: Estimation Results of Regression of $MB_T$ on conservatism proxies.

$MB_T = \eta_{03} + \eta_{13} \cdot CC_T + \eta_{23} \cdot ConDep_i + \epsilon_3.$  \hspace{1cm} (E3)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.552</td>
<td>0.925</td>
<td>1.558</td>
</tr>
<tr>
<td></td>
<td>(54.10)</td>
<td>(89.33)</td>
<td>(66.34)</td>
</tr>
<tr>
<td>CC</td>
<td>0.787</td>
<td>0.786</td>
<td>0.880</td>
</tr>
<tr>
<td></td>
<td>(151.79)</td>
<td>(151.91)</td>
<td>(111.87)</td>
</tr>
<tr>
<td>ConDep1</td>
<td>0.433</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(40.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ConDep2</td>
<td></td>
<td>-0.044</td>
<td>-0.049</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-43.21)</td>
<td>(-25.25)</td>
</tr>
<tr>
<td>% Land</td>
<td></td>
<td>-0.806</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.78)</td>
<td></td>
</tr>
<tr>
<td>% Buildings</td>
<td></td>
<td>-1.490</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-26.43)</td>
<td></td>
</tr>
<tr>
<td>% Intangibles</td>
<td></td>
<td>-0.741</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-17.24)</td>
<td></td>
</tr>
<tr>
<td>Adj $R^2$</td>
<td>0.161</td>
<td>0.162</td>
<td>0.216</td>
</tr>
<tr>
<td>n</td>
<td>127,634</td>
<td>127,637</td>
<td>55,021</td>
</tr>
</tbody>
</table>
Table 6: The Relation between the modified Future-to-Book ratio, $FB_T$, and $a_T^q$.

\[
FB_T = \eta_{04} + \eta_{14} \cdot a_T^q + \eta_{24} \cdot \lambda_T^a + \eta_{34} \cdot r_T + \epsilon_4 \quad (E4)
\]

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.289</td>
<td>42.498</td>
</tr>
<tr>
<td></td>
<td>(44.33)</td>
<td>(195.64)</td>
</tr>
<tr>
<td>$a_T^q$</td>
<td>-0.200</td>
<td>-0.143</td>
</tr>
<tr>
<td></td>
<td>(-14.48)</td>
<td>(-52.97)</td>
</tr>
<tr>
<td>$\lambda_T^a$</td>
<td>1.145</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td>(95.08)</td>
<td>(107.47)</td>
</tr>
<tr>
<td>$r_T$</td>
<td>-0.174</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(-3.86)</td>
<td>(1.47)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.067</td>
<td>0.089</td>
</tr>
<tr>
<td>$n$</td>
<td>129,629</td>
<td>135,358</td>
</tr>
</tbody>
</table>

Model (2) is estimated with ranks of the dependent and independent variables, where each variable is ranked into groups of 100 by year.
Table 7: The Relation between Adjusted Market-to-Book and Growth

**Panel A:** Market-to-Book partitioned by $\lambda$.

<table>
<thead>
<tr>
<th>Rank</th>
<th>$n$</th>
<th>$\lambda_T^n$</th>
<th>$MB_T$</th>
<th>$CC_T$</th>
<th>$FB_T$</th>
<th>$\hat{FB}_T$</th>
<th>$\hat{CC}_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6767</td>
<td>-0.307</td>
<td>2.789</td>
<td>1.945</td>
<td>0.845</td>
<td>0.087</td>
<td>2.226</td>
</tr>
<tr>
<td>1</td>
<td>6768</td>
<td>-0.099</td>
<td>2.578</td>
<td>1.903</td>
<td>0.675</td>
<td>0.241</td>
<td>1.968</td>
</tr>
<tr>
<td>2</td>
<td>6768</td>
<td>-0.034</td>
<td>2.514</td>
<td>1.910</td>
<td>0.604</td>
<td>0.329</td>
<td>1.881</td>
</tr>
<tr>
<td>3</td>
<td>6768</td>
<td>0.003</td>
<td>2.454</td>
<td>1.865</td>
<td>0.589</td>
<td>0.383</td>
<td>1.812</td>
</tr>
<tr>
<td>4</td>
<td>6768</td>
<td>0.029</td>
<td>2.294</td>
<td>1.768</td>
<td>0.526</td>
<td>0.381</td>
<td>1.688</td>
</tr>
<tr>
<td>5</td>
<td>6768</td>
<td>0.049</td>
<td>2.328</td>
<td>1.776</td>
<td>0.552</td>
<td>0.450</td>
<td>1.653</td>
</tr>
<tr>
<td>6</td>
<td>6768</td>
<td>0.066</td>
<td>2.283</td>
<td>1.759</td>
<td>0.524</td>
<td>0.473</td>
<td>1.567</td>
</tr>
<tr>
<td>7</td>
<td>6768</td>
<td>0.082</td>
<td>2.281</td>
<td>1.746</td>
<td>0.536</td>
<td>0.499</td>
<td>1.529</td>
</tr>
<tr>
<td>8</td>
<td>6768</td>
<td>0.097</td>
<td>2.334</td>
<td>1.748</td>
<td>0.586</td>
<td>0.580</td>
<td>1.524</td>
</tr>
<tr>
<td>9</td>
<td>6768</td>
<td>0.113</td>
<td>2.475</td>
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Panel B:

\[ \hat{C}C_T = \eta_{05} + \eta_{15} \cdot \alpha^2T + \eta_{25} \cdot \lambda^2T + \eta_{35} \cdot r_T + \epsilon_5 \]  

(E5)

Estimation results for regression of the estimated Conservatism Correction factor, \( \hat{C}C_T \), on average growth.

<table>
<thead>
<tr>
<th>( \alpha^2T )</th>
<th>( \lambda^2T )</th>
<th>( r_T )</th>
<th>( \text{Adjusted } R^2 )</th>
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Panel C:

\[
\hat{CC}_T = \eta_{05}' + \eta_{15}' \cdot \alpha_T^a + \eta_{25}' \cdot \lambda_T^{a+} + \eta_{35}' \cdot \lambda_T^{-} + \eta_{45}' \cdot r_T + \epsilon_5'
\]  

(E5')

Estimation results for regression of the estimated Conservatism Correction factor, \(\hat{CC}_T\), on average growth, partitioned into positive and negative average growth: \(\lambda_T^{a+}\) and \(\lambda_T^{-}\).

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<td>56,572</td>
<td>117,959</td>
<td>57,713</td>
<td>60,245</td>
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</table>
7 Appendix 2-Proofs

Proof of Proposition 2: We can set $\alpha$ to 0 without loss of generality for this proof. From (9), (15), and (18), $CC_T$ equals (after dividing through by the common term $I_0 \cdot (1 + \lambda_1)$):

$$CC_T = \frac{BV^*_T}{BV_T} = \frac{bv^*_T + bv^*_{T-2} \cdot (1 + \lambda_2) + \ldots + bv^*_0 \cdot \prod_{i=2}^{T}(1 + \lambda_i)}{bv_{T-1} + bv_{T-2} \cdot (1 + \lambda_2) + \ldots + bv_0 \cdot \prod_{i=2}^{T}(1 + \lambda_i)}. \quad (30)$$

This ratio is decreasing in $\lambda_t$ if and only if the sequence $\frac{bv^*_t}{bv_t}$ is an increasing function of $t$.\(^{33}\)

Using the conservation property of residual income, the book value of a dollar investment $t$ periods into its existence (see equation (5)) can be expressed as the discounted sum of future historical costs, as follows:

$$bv_t = (1 - \sum_{i=1}^{t} d_i) = \sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i-t}. \quad (31)$$

Under the R.P.C. rule, $d^*$, we know from (7) and (8) that $z^*_t = x_t = 0$ for $1 \leq t \leq L - 1$ and $z^*_t = c \cdot x_t$ for $L \leq t \leq T$. It follows that for all $t$, $z^*_t = c \cdot x_t$, implying in turn that:

$$\frac{bv^*_t}{bv_t} = \frac{\sum_{i=t+1}^{T} z_i(d^*) \cdot \gamma^{i-t}}{\sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i-t}} = \frac{c \cdot \sum_{i=t+1}^{T} x_i \cdot \gamma^{i}}{\sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i}}. \quad (32)$$

Analogously, we have:

$$\frac{bv^*_{t-1}}{bv_{t-1}} = \frac{c \cdot \sum_{i=t}^{T} x_i \cdot \gamma^{i}}{\sum_{i=t}^{T} z_i(d) \cdot \gamma^{i}} = \frac{c \cdot \left[ x_t \cdot \gamma^{t} + \sum_{i=t+1}^{T} x_i \cdot \gamma^{i} \right]}{z_t(d) \cdot \gamma^{t} + \sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i}}. \quad (33)$$

To establish that (32) $\geq$ (33), we note that for $1 \leq t \leq L - 1$, the inequality follows immediately since $x_t = 0$ and $z^*_t(d) \geq 0$ (see Definition 2). For $t \geq L$, the result holds if and only if

$$\frac{z_t(d)}{x_t} \geq \frac{\sum_{i=t+1}^{T} z_i(d) \cdot \gamma^{i}}{\sum_{i=t+1}^{T} x_i \cdot \gamma^{i}} \quad (34)$$

\(^{33}\)A proof of this assertion can be found in Claim 2 in the proof of Proposition 3 in Rajan and Reichelstein (2009).
We demonstrate that (34) is true by a process of induction. For \( t = T - 1 \), (34) requires:

\[
\frac{z_{T-1}}{x_{T-1}} \geq \frac{z_T}{x_T},
\]

which is true as \( \frac{z_t}{x_t} \) decreases in \( t \). Now, suppose that (34) holds for \( t = k \). Then, for \( t = k - 1 \),

\[
\frac{z_{k-1}}{x_{k-1}} = \frac{z_k \cdot \gamma^k}{x_k \cdot \gamma^k} \geq \frac{z_{k+1} \cdot \gamma^{k+1}}{x_{k+1} \cdot \gamma^{k+1}} = \frac{T}{T-1} \sum_{i=k}^{T} \frac{z_i \cdot \gamma^i}{x_i \cdot \gamma^i},
\]

where the second inequality arises from the induction hypothesis. We have thus shown that (34) holds. To conclude, note that we have stated the proof in terms of weak inequalities. However, if either \( z_t(d) > 0 \) for some \( t \leq L - 1 \) or \( \frac{z_t}{x_t} \) strictly decreases in \( t \) for \( t \geq L \), it follows that \( \frac{bv^*_t}{bv^*_t} \) strictly increases for some subset of values of \( t \), and therefore that \( CC_T \) is monotone decreasing in each \( \lambda_t \).

Proof of Proposition 3: Consider \( CC_T \) as represented in equation (30). The denominator, \( BV_T \), is determined by the depreciation scheme under consideration and is independent of the cost of capital, \( r \). So it is sufficient to show that the numerator, \( BV^*_T \), increases in \( r \), or, equivalently, that it decreases in \( \gamma \). We use the following formulation of \( BV^*_T \):

\[
BV^*_T = bv^*_{T-1} \cdot I_1 + bv^*_{T-2} \cdot I_2 + \ldots + bv^*_0 \cdot I_T.
\]

As in the proof of Proposition 2, we set \( bv^*_t = c \cdot \sum_{i=t+1}^{T} x_i \cdot \gamma^{i-t} \). \( BV^*_T \) therefore equals:

\[
c \cdot \left[ I_1 \cdot \sum_{i=T}^{T} x_i \cdot \gamma^{i-(T-1)} + I_2 \cdot \sum_{i=T-1}^{T} x_i \cdot \gamma^{i-(T-2)} + \ldots + I_T \cdot \sum_{i=1}^{T} x_i \cdot \gamma^{i} \right].
\]

As \( c = \frac{1}{\sum_{i=L}^{T} x_i \cdot \gamma^{i}} \), we need to show that the following expression decreases in \( \gamma \):

\[
\frac{I_1 \cdot x_T \cdot \gamma + I_2 \cdot \sum_{i=T-1}^{T} x_i \cdot \gamma^{i-(T-2)} + \ldots + I_T \cdot \sum_{i=1}^{T} x_i \cdot \gamma^{i}}{\sum_{i=L}^{T} x_i \cdot \gamma^{i}}.
\]

We do so one term at a time. Ignoring the positive constant \( I_t \), an arbitrary term in (35) is of the form:

\[
\frac{\sum_{i=k}^{T} x_i \gamma^{i-k+1}}{\sum_{i=L}^{T} x_i \gamma^{i}}, \quad k \in \{1, 2, \ldots, T\}.
\]
Consider \( k \leq L \). As \( x_1 = \ldots = x_{L-1} = 0 \), \(36\) is equivalent to:
\[
\frac{\sum_{i=L}^{T} x_i \gamma^{i-k+1}}{\sum_{i=L}^{T} x_i \gamma^i} = \gamma^{-(k-1)},
\]
which is decreasing in \( \gamma \) as \( k \geq 1 \).

For \( k > L \), \(36\) decreases in \( \gamma \) if and only if
\[
\left( \frac{\sum_{i=L}^{T} x_i \cdot \gamma^i}{\sum_{i=L}^{T} x_i \cdot i \cdot \gamma^i} \right) \cdot \left[ \frac{\sum_{i=k}^{T} x_i \cdot (i - k + 1) \cdot \gamma^{i-k}}{\sum_{i=k}^{T} x_i \cdot \gamma^{i-k+1}} \right] \leq \left( \frac{\sum_{i=L}^{T} x_i \cdot \gamma^{i-k}}{\sum_{i=L}^{T} x_i \cdot i \cdot \gamma^{i-k-1}} \right),
\]
\[
\sum_{i=L}^{T} x_i \cdot \gamma^{i-1} \sum_{i=L}^{T} x_i \cdot i \cdot \gamma^{i-1} \sum_{i=L}^{T} x_i \cdot (i - k + 1) \cdot \gamma^{i-k}.
\]

With regard to the left-hand side of \(38\), note that:
\[
\frac{\sum_{i=L}^{T} x_i \cdot \gamma^{i-1}}{\sum_{i=L}^{T} x_i \cdot i \cdot \gamma^{i-1}} < \frac{\sum_{i=L}^{T} x_i \cdot \gamma^{i-1}}{\sum_{i=L}^{T} x_i \cdot i \cdot \gamma^{i-1}}
\]
since each additional term in the former has a numerator-to-denominator ratio of less than \( 1/(L + T - k) \). So it is sufficient to demonstrate that
\[
\frac{\sum_{i=L}^{L+T-k} x_i \cdot \gamma^{i-1}}{\sum_{i=L}^{L+T-k} x_i \cdot i \cdot \gamma^{i-1}} \leq \frac{\sum_{i=k}^{T} x_i \cdot \gamma^{i-k}}{\sum_{i=k}^{T} x_i \cdot (i - k + 1) \cdot \gamma^{i-k}} \leq \frac{\sum_{i=L}^{L+T-k} x_i \cdot \gamma^{i-L}}{\sum_{i=L}^{L+T-k} x_i \cdot i \cdot \gamma^{i-L}} \leq \frac{\sum_{i=k}^{T} x_i \cdot (i - L + 1) \cdot \gamma^{i-L}}{\sum_{i=k}^{T} x_i \cdot (i - k + 1) \cdot \gamma^{i-k}}.
\]

Since \( x_i/x_{i+1} \) increases in \( i \), we know that \( \frac{x_1}{x_2} \leq \frac{x_2}{x_{k+1}} \leq \frac{x_{T-k+1}}{x_T} \). The left-hand side of \(39\) places equal weight on these ratios, which implies that
\[
\frac{\sum_{i=L}^{L+T-k} x_i \cdot \gamma^{i-L}}{\sum_{i=L}^{L+T-k} x_i \cdot (i - L + 1) \cdot \gamma^{i-L}} < \frac{\sum_{i=k}^{T} x_i \cdot (i - L + 1) \cdot \gamma^{i-L}}{\sum_{i=k}^{T} x_i \cdot (i - k + 1) \cdot \gamma^{i-k}}.
\]
since the expression on the right-hand side of (40) places increasingly higher weights on the higher ratios. Finally, \( L \geq 1 \) implies that the right-hand side of (39) exceeds the right-hand side of (40). Hence, the inequality in (39) holds, and we have shown that \( BV^*_T \) (and hence \( CC_T \)) increases in \( r \).

\[ \text{Proof of Proposition 4:} \] For \( L = 1 \), we have \( x_t = 1 - \beta \cdot (t - 1) \). The capital charges in (19) simplify to:

\[ z_t = \frac{1 - \alpha}{T} \cdot [1 + r \cdot (T - t + 1)] \]

Using these expressions, as well as the definition of \( c \), we can rewrite \( CC_T \) in (20) as:

\[ CC_T = \frac{T}{\sum_{i=1}^{T} [1 - \beta \cdot (i - 1)] \cdot \gamma^i} \cdot \frac{1}{1 - \alpha} \]

Expanding this expression, it can then be shown that the limit values of the \( CC_T \) function are as follows:

\[ \lim_{\lambda \to -1} CC_T(\cdot) = \left( \frac{1}{1 - \alpha} \right) \cdot \frac{T \cdot r^2 \cdot (1 + r)^{T-1} \cdot [1 - \beta \cdot (T - 1)]}{(r - \beta) \cdot [(1 + r)^T - 1] + \beta \cdot r \cdot T} \]

\[ \lim_{\lambda \to 0} CC_T(\cdot) = \left( \frac{1}{1 - \alpha} \right) \cdot \frac{T \cdot [2 + \beta \cdot (1 - T)] \cdot r^2 \cdot (1 + r)^T}{(r - \beta) \cdot [(1 + r)^T - 1] + \beta \cdot r \cdot T - 2} \cdot \frac{1}{r \cdot [T + 1]} \]

\[ \lim_{\lambda \to \infty} CC_T(\cdot) = \frac{1}{1 - \alpha} \]

The limit results for the \( \beta = 0 \) case follow directly from these expressions.

We next prove the claim regarding the bounds on the ratios of the \( CC_T \) variables. Note that the term \( 1/(1 - \alpha) \) enters in a multiplicative fashion in each of the \( CC_T \) expressions in (42) and, as such, can be ignored. Also, when \( T = 2 \), direct computations on (42) reveal that the ratio in question always equals \( \frac{2}{3} \). We therefore restrict attention to values of \( T > 2 \).

We first show the upper bound result that

\[ \frac{CC_T(\lambda = -1) - CC_T(\lambda = 0)}{CC_T(\lambda = -1) - CC_T(\lambda = \infty)} \leq \frac{T}{T + 1}. \]

To do so, we will demonstrate the equivalent result that

\[ \frac{CC_T(\lambda = 0) - CC_T(\lambda = \infty)}{CC_T(\lambda = -1) - CC_T(\lambda = \infty)} > \frac{1}{T + 1}. \]
Using the limits in (42), (43) reduces to the following inequality:

\[
(T + 1) \cdot \left[\frac{1}{r(1 + T)} \cdot \frac{Tr^2(1 + r)^T(2 + \beta - \beta T)}{\beta r T + (r - \beta)[(1 + r)^T - 1]} - \frac{2}{r(1 + T)} - 1\right]
\]

\[
> \frac{Tr^2(1 + r)^T-1[1 + \beta - \beta T]}{\beta r T + (r - \beta)[(1 + r)^T - 1]} - 1.
\]

\[
\Leftrightarrow (T + 1)Tr^2(1 + r)^T(2 + \beta - \beta T) - (T + 1) \left[2\beta r T + (2(r - \beta) + r(1 + T)) \left[(1 + r)^T - 1\right]\right]
\]

\[
> Tr^2(1 + r)^T-1[1 + \beta - \beta T]r(1 + T) - r(1 + T) \left[\beta r T + (r - \beta)[(1 + r)^T - 1]\right]
\]

\[
\Leftrightarrow Tr^2(1 + r)^T-1(1 + \beta - \beta T) + Tr^2(1 + r)^T - \left[\beta r T + (r - \beta)[(1 + r)^T - 1]\right] (2 + rT) > 0
\]

\[
\Leftrightarrow Tr^2(1 + r)^T-1(2 + \beta - \beta T + r) - (2 + rT) \left[\beta r T + (r - \beta)[(1 + r)^T - 1]\right] > 0. \quad (44)
\]

But (44) is a linear function of \( \beta \). At \( \beta = \frac{r}{1 + rT} \), (44) equals

\[
\frac{Tr^2(1 + r)^T(2 + rT)}{(1 + rT)} - \frac{Tr^2(1 + r)^T(2 + rT)}{(1 + rT)} = 0.
\]

At \( \beta = 0 \), the expression in (44) reduces to (after dividing through by \( r \)):

\[
T \cdot r \cdot (1 + r)^T-1(2 + r) - (2 + rT)[-1 + (1 + r)^T] > 0.
\]

Letting \( s = (1 + r) \geq 1 \), this inequality holds if and only if

\[
T \cdot (s - 1) \cdot s^{T-1} \cdot (s + 1) - 2(s^T - 1) - (s^T - 1) \cdot T \cdot (s - 1) > 0,
\]

or

\[
T \cdot (s - 1)(s^{T-1} + 1) - 2(s^T - 1) > 0.
\]

But this function and its first derivative equal 0 at \( s = 1 \), while the second derivative is

\[
(s - 1) \cdot T \cdot (T - 1)s^{T-3} \cdot (T - 2) > 0,
\]

for all \( s > 1 \). So the function is convex and positive everywhere. Thus (44) > 0 for all \( \beta \in [0, \frac{r}{1 + rT}] \). We have therefore shown that (43) holds.

We next demonstrate the lower bound inequality:

\[
\frac{CC_T(\lambda = -1) - CC_T(\lambda = 0)}{CC_T(\lambda = -1) - CC_T(\lambda = \infty)} \geq \frac{2}{3}.
\]

Expanding these expressions using (42), we are required to show that

\[
\frac{Tr^2(1 + r)^T-1(1 + \beta - \beta T)}{(r - \beta)[(1 + r)^T - 1] + \beta r T} - \frac{6Tr^2(1 + r)^T(2 + \beta - \beta T)}{2r(1 + T)[\beta r T + (r - \beta)[(1 + r)^T - 1]]} + \frac{6}{r(1 + T)} + 2 \geq 0.
\]
\[ \Leftrightarrow Tr^3(1+r)^{T-1}(1+T)(1+\beta - \beta T) - 3Tr^2(1+r)^T (2+\beta - \beta T) + 6[\beta rT + (r - \beta)[(1+r)^T - 1]] + 2r(1+T)[\beta rT + (r - \beta)[(1+r)^T - 1]] \geq 0, \]

\[ \Leftrightarrow Tr^2(1+r)^{T-1}(1+\beta - \beta T)[r+rT - 3(1+r)] - 3Tr^2(1+r)^T + 2[3 + r + rT][\beta rT + (r - \beta)[(1+r)^T - 1]] \geq 0. \]

(45)

Again, this is a linear function of \( \beta \), so it suffices to show that (45) holds at its end points.

At \( \beta = \frac{r}{1+rT} \), the expression reduces to

\[ \frac{Tr^2(1+r)^{T-1}(1+r)}{(1+rT)}[r(1+T) - 3(1+r)] - 3Tr^2(1+r)^T + 2[3 + r + rT] \cdot \frac{r^2(1+r)^TT}{(1+rT)} \]

\[ = \frac{Tr^2(1+r)^T}{(1+rT)}[r + rT - 3 - 3r + 6 + 2r + 2rT] - 3Tr^2(1+r)^T \]

\[ = \frac{Tr^2(1+r)^T}{(1+rT)}[3(1+rT)] - 3Tr^2(1+r)^T = 0. \]

At \( \beta = 0 \), we need to show that

\[ Tr^3(1+r)^{T-1}(1+T) - 6Tr^2(1+r)^T + 6r(1+r)^T - 1) + 2r^2(1+T)((1+r)^T - 1) \geq 0 \]

\[ \Leftrightarrow T(1+T)s^{T-1}(s^2 - 2s + 1) - 6T(s - 1)s^T + 6(s^T - 1) + 2(1+T)(s - 1)(s^T - 1) \geq 0 \]

\[ \Leftrightarrow T(1+T)s^{T+1} - 2T(1+T)s^T + T(1+T)s^{T-1} - 6T s^{T+1} + 6T s^T + 6s^T - 6 \]

\[ + 2(1+T) s^{T+1} - 2(1+T) s^T - 2s(1+T) + 2(1+T) \geq 0 \]

\[ \Leftrightarrow (T - 2)(T - 1)s^{T+1} - 2(T - 2)(T + 1)s^T + T(1+T)s^{T-1} - 2(1+T)s + 2T - 4 \geq 0. \]

Again, this equals 0 at \( s = 1 \). In addition, its derivative is

\[ (T - 2)(T - 1)(T + 1)s^T - 2(T - 2)(T + 1)Ts^{T-1} + T(1+T)(T - 1)s^{T-2} - 2(1+T) \]

\[ \propto (T - 2)(T - 1)s^T - 2(T - 2)Ts^{T-1} + T(T - 1)s^{T-2} - 2. \]

This equals 0 at \( s = 1 \). Its derivative in turn is

\[ T(T - 2)(T - 1)s^{T-1} - 2T(T - 1)(T - 2)s^{T-2} + T(T - 1)(T - 2)s^{T-3} \]

\[ = T(T - 1)(T - 2)s^{T-3}(s - 1)^2 > 0, \]

for all \( T > 2 \) and all \( s > 1 \). We have thus established that (45) is strictly positive for values of \( \beta \) between 0 and \( \frac{r}{1+rT} \). We conclude that for any level of decay in that range, the ratio bounds of \( \frac{2}{3} \) and \( \frac{T}{T+1} \) hold.
Proof of Proposition 5: Consider the representation of $CC_t$ in (20) and note that $\beta$ has no impact on the denominator. Expanding the numerator using (7), we obtain:

$$1 - \frac{\sum_{i=L}^{T} x_i \cdot \mu_i}{\sum_{i=L}^{T} x_i \cdot \gamma_i}.$$  \hspace{1cm} (46)

Suppose that $\mu > \gamma$ (i.e., $\lambda < r$). The denominator of (20) is then negative, so we have to show that the following expression decreases in $\beta$:

$$F(\beta) \equiv \frac{\sum_{i=L}^{T} x_i \cdot k_i}{\sum_{i=L}^{T} x_i \cdot \gamma_i}$$

Since $x_i = 1 - \beta \cdot (i - L)$, for $i \geq L$, it follows that $\frac{\partial x_i}{\partial \beta} = (L - i) \leq 0$. Differentiating $F(\cdot)$ with respect to $\beta$, we obtain $F'(\beta) < 0$ if and only if:

$$\left[ (1 + L\beta) \sum_i \gamma_i^2 - \beta \sum_i i \gamma_i \right] \cdot \left[ L \sum_i \mu_i^2 - \sum_i i \mu_i \right] < \left[ (1 + L\beta) \sum_i \mu_i^2 - \beta \sum_i i \mu_i \right] \cdot \left[ L \sum_i \gamma_i - \sum_i i \gamma_i \right]$$

After suitable simplification, this reduces to the following inequality:

$$\frac{\sum_i i \gamma_i}{\sum_i \gamma_i} < \frac{\sum_i \mu_i^2}{\sum_i \mu_i} \text{ for } 0 < \gamma < \mu.$$  \hspace{1cm} (47)

Finally, consider the function $\psi(s) \equiv \frac{\sum_i i s^i}{\sum s^i}$, $s > 0$. Differentiating $\psi(\cdot)$ with respect to $s$, it is evident that this function is monotone increasing in $s$ provided

$$(\sum s^i) \cdot \sum (i^2 s^i) > (\sum i s^i)^2.$$

This inequality follows as a direct consequence of the Cauchy-Schwarz inequality (see Marsden [1974, p.20]). Therefore, (47) is true, and $F'(\beta) < 0$ as desired. We conclude by noting that if $\mu < \gamma$, the denominator of (20) is positive, and the same proof technique as above then establishes that $F'(\beta) > 0$. In either case, we have shown that $CC_T$ is monotone decreasing in $\beta$. \hfill $\blacksquare$
References


