Accounting for Development Through Investment Prices$^1$

Roc Armenter  
Federal Reserve Bank of Philadelphia

Amartya Lahiri  
University of British Columbia

October 2008

$^1$We would like to thank the referees and seminar participants at various universities for helpful comments. Lahiri would like to thank SSHRC for research support. The views expressed here do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.
Abstract

In this paper we study the explanatory power of relative prices of investment goods for cross-country relative incomes using an open economy version of the standard neoclassical growth model with intermediate goods. Crucially, we incorporate an extensive margin by allowing for entry into the intermediate goods sector. We formalize an environment in which entry into the intermediate goods sector requires investment of capital. We show that this extensive margin provides an amplification mechanism from relative investment prices to output. The model maps the measured differences in the relative price of investment goods into 16-fold differences in per capita income even with a conventional capital share of 0.40. In contrast to the standard model, which implies 7-fold total factor productivity differences between the richest and poorest countries, our model only requires TFP differences around 1.5 in order to explain the 35-fold income gaps observed in the data. The model also reproduces the cross-section distribution of relative per-capita incomes as well as the dispersion in capital output ratios observed in the data.
1 Introduction

The per capita income gap between the richest and poorest countries of the world are often measured to be 35-fold or more. Concurrently, a systematic feature of the data is that the price of investment goods relative to consumption goods is systematically higher in poorer countries relative to the richer countries. These two facts have induced a number of researchers to investigate the ability of relative investment prices to explain the observed income disparities across countries (see Chari, Kehoe and McGrattan (1997), Jones (1995) and Restuccia and Urrutia (2001)). However, using conventional capital share measures the standard neoclassical growth model typically does not translate these differences in relative investment prices into 35-fold gaps in per capita income even though it does generate quantitatively accurate differences in measured investment rates across countries.\(^1\) A corollary of this is that the observed variation in investment and capital is clearly not enough to explain the measured income disparities. In terms of the neoclassical growth model this shows up as huge disparities in measured productivity across countries. Clearly, there appears to be a missing link in the mapping from investment prices to relative incomes. In this paper we propose a mechanism that can potentially fill this gap.

We formalize a multi-sector model in which intermediate goods are combined to produce the final good. Our environment introduces an extensive margin to the intermediate goods sector. Setting up an intermediate good firm is costly as it involves a set-up cost, which we label as investment in structures. Thus, intermediate goods production have two cost components – a set-up cost of installing the initial structure, and the variable cost of hiring labor and capital to vary output given the structure. In this environment, higher relative investment prices not only reduce investment in variable capital but also reduce entry into the intermediate goods sector. This reduces the varieties of intermediate goods that are produced in economies where relative investment prices are

\(^1\)Using a broader measure of capital including both physical and human capital one could potentially work with larger capital shares than the typical range of 0.30-0.40. Mankiw, Romer and Weil (1992), Chari, Kehoe and McGrattan (1997), Parente and Prescott (1994), amongst others, use capital shares upwards of 2/3 to explain observed income disparities using investment rates.
high. The negative effect on the extensive margin, in turn, reduces output and per capita income. Put differently, the extensive margin on intermediate goods amplifies the effects of higher relative investment prices.

A key feature of the model is that we allow free trade in variable capital goods. The free trade assumption implies that all countries face the same world prices of variable capital goods. Hence, the domestic price of variable capital in any country, when expressed in terms of its consumption good, depends on the real exchange rate of the country, i.e., on the price of the world numeraire good in terms of the domestic final good. Thus the domestic price of variable capital goods will be higher in countries with relatively cheaper consumption goods. This provides the model with the standard intensive margin of relative investment prices; countries with higher investment prices will employ less variable capital and thereby also produce less. Structures are, however, non-traded. Hence, their domestic price depends both on the price of consumption goods as well as the local cost of producing structures. All else equal, countries where structures are more expensive will invest less in them. This implies less entry into the intermediate goods sector, which is the extensive margin in the model.

The model’s predictions for relative incomes boil down to expressions involving the relative prices of structures and variable capital (amongst other factors). We identify these components individually using the model and the available cross-country data on the relative price of investment goods. When these data are put through a calibrated version of the model with a capital share of 0.40, we generate three principal results. First, the predicted relative income series from the model track the data very closely with a correlation coefficient of 0.72. Second, the model generates 16-fold income differences between the richest and poorest countries of the world. This represents an 8-fold increase relative to the standard neoclassical model. Third, the model does well in matching the cross-country dispersion in capital-output ratios as well with the correlation between the model-generated series and the data being 0.62. Fourth, in contrast to the standard model, which implies 7-fold total factor productivity (TFP) differences between the richest and poorest countries, our model only requires TFP differences between 2 and 3 in order to explain the 35-fold
income gaps observed in the data.

While there is a large literature on development accounting, the paper that is closest to our work is Jones (2008). He studies environments where there are linkages and complementarities between intermediate goods and final goods. The key point in Jones’s work is that small distortions in some sectors can have a multiplier effect on final output because of linkages between sectors. In the model employed by Jones final goods are produced using capital, labor and a composite intermediate good. Final goods can be used to produce consumption goods, capital goods as well as intermediate goods. Individual intermediate goods, in turn, combine together to produce the composite intermediate good. This linkage effect along with complementarities between different types of intermediate goods generates multiplier effects of any distortions in intermediate goods sectors. Jones shows that his model can easily generate 50-fold income differences between low distortion and high distortion economies even with a capital share which is within the empirically plausible range.

Our model differs from Jones in three substantive ways. First, our mechanism works through the extensive margin; distortions affect the number of intermediate goods. Jones's model, on the other hand, works through the intensive margin; distortions affect the intensity with which a given list of intermediate goods are produced. Second, Jones works in a closed economy environment whereas we allow for trade in a set of capital goods. Third, we quantify our model using relative investment prices and characterize the whole distribution of relative incomes based on a model-based identification. Due to lack of available cross-country data on distortions, Jones is constrained to highlighting the relative income implications of different levels of distortions.

Our work is also related to literature on development accounting which is focussed on decomposing cross-country income differences into different components. A key finding of this body of work is that measured differences in inputs are insufficient to account for the large relative income disparities in the data. A key conclusion of this literature is that productivity differences across countries must be very large.² The contribution of our work is to demonstrate that the productiv-

²A non-exhaustive list of papers in this context is Klenow and Rodrigues (1997), Hall and Jones (1999), Parente
ity differences that are required to explain large income differences can be much smaller than those implied by the standard model.

The next section develops the model while section 3 presents the key analytical implications for the cross-country facts of interest. Section 4 presents the quantitative results while the last section concludes.

2 Model

In this section we outline the model. We start with the economic environment that we study and then provide details of the competitive equilibrium of the model.

2.1 Environment

We consider a world economy with $J$ countries. We now describe the economy of a given country $j \in J$. We use a superscript to denote country-specific parameters: all non-indexed parameters are taken to be identical across countries.

In each country $j \in J$ there is a representative household who supplies one unit of labor inelastically and consumes the final good $c^{j}_t$. The household values a stream of consumption according to

$$U^j = \sum_{t=0}^{\infty} \beta^t u(c^j_t)$$

where $\beta \in (0, 1)$ is the intertemporal discount rate.

The final good $y^j_t$ is produced by combining together a set $\Omega^j_t = [0, m^j_t]$ of intermediate goods according to the technology:

$$y^j_t = \left[ \int_{\omega \in \Omega^j_t} \left( x^j_t(\omega) \right)^\rho d\omega \right]^{1/\rho}$$

where the parameter $\rho \in (0, 1)$ determines the elasticity of substitution across intermediate goods and $x^j_t(\omega)$ is the amount of intermediate good $\omega$ used in production in country $j$.

and Prescott (2000), etc..
The set of intermediate goods is endogenous as we allow for entry and exit in the sector. Let us start, though, with the production of existing intermediate goods \( \omega \in \Omega_j \). The production technology for intermediate goods is given by

\[
x_j^t(\omega) = A_j^t \left( k_j^t(\omega) \right)^\alpha \left( l_j^t(\omega) \right)^{1-\alpha}
\]

with \( k_j^t(\omega) \) and \( l_j^t(\omega) \) being the composite capital and labor inputs respectively. The parameter \( \alpha \in (0, 1) \) equals the share of working capital expenses in the variable cost. The term \( A_j^t \) determines the total factor productivity in the production of intermediate goods: we allow this term to vary between countries.

To start production of a new intermediate good requires an initial investment in one unit of structure: we can thus equate the measure of active intermediate goods \( m_j^t \) to the stock of structures in the economy. Exit of firms from the intermediate goods sector is exogenous and occurs with probability \( \chi \). We assume that the initial investment in structures depreciates fully on exit.

We now turn to the production of new composite capital. Let \( i_{jt}^j \) denote the investment in the composite capital good in country \( j \). The new composite capital goods are produced by combining together specific capital goods from all countries according to the CES technology

\[
i_{jt}^j = \left[ \sum_{c \in J} \left( n_{jt}^c \right)^\eta \right]^{1/\eta}
\]

where \( n_{jt}^c \) denotes specific capital produced in country \( c \in J \) and used in country \( j \) at date \( t \). Parameter \( \eta \in (0, 1) \) governs the elasticity of substitution between specific capital goods from different countries. The total stock of composite capital \( k_{jt}^j \) in country \( j \) follows the law of motion

\[
k_{jt}^j = (1 - \delta) k_{j,t-1}^j + i_{jt}^j
\]

where \( \delta \in [0, 1] \) is the depreciation rate.

To produce one unit of the specific capital good of country \( j \) requires \( v_j^i \) units of the final good. We assume that countries differ in their ability to produce specific capital goods. Specific capital goods are the only traded goods in the world economy. The production of new structures
is also linear: one unit of new structures costs $f^j_t$ units of the final good. This too can vary across countries.

We are now ready to summarize all the uses of the final good $y^j_t$ in the aggregate resource constraint

$$y^j_t = c^j_t + f^j_t \left[ m^j_{t+1} - m^j_t (1 - \chi) \right] + v^j_t \sum_{c \in J} n^j_{ct}. \quad (6)$$

The resource constraint includes the cost of investment in new structures,

$$f^j_t \left[ m^j_{t+1} - m^j_t (1 - \chi) \right]$$

as well as the total cost of producing the country $c$ specific capital demanded by all countries,

$$v^j_t \sum_{c \in J} n^j_{ct}.$$

### 2.2 Competitive Equilibrium

We now describe a competitive equilibrium. We focus on the determination of prices and the exchange rate. In each country we use the final good as numeraire.

**Households.** The representative household owns the stock of domestic composite capital as well as all claims to domestic firms. Each period the household rents out labor and the composite capital to domestic firms, and trades one-period domestic bonds $b$ in order to maximize utility as given by (1) subject to the flow budget constraint

$$c^j_t + z^j_t \left[ k^j_{t+1} - (1 - \delta) k^j_t \right] + q^j_t b^j_{t+1} \leq w^j_t + r^j_t k^j_t + d^j_t + b^j_t$$

where $z^j_t$ is the price of new composite capital goods, $q^j_t$ is the price of the domestic bond, $w^j_t$ and $r^j_t$ are the rental rates of labor and the composite capital good respectively, and $d^j_t$ are total dividends received.

The necessary first order conditions give the price of the bond,

$$q^j_t = \beta \frac{u' \left( c^j_{t+1} \right)}{u' \left( c^j_t \right)}.$$
We assume bonds are in zero net supply (that is, they are not internationally traded) and we drop them from the competitive equilibrium definition.

From the household optimization problem we also derive

$$u'(c^j_t) = \beta u'(c^j_{t+1}) \left[ \frac{z^j_{t+1}(1-\delta) + r^j_{t+1}}{z^j_t} \right]$$

which is the basic Euler equation governing the optimal consumption saving decision by households.

*Final good producers.* The final goods sector is competitive. Profit maximization by final goods firms implies

$$p^j_t(\omega) = \left( \frac{y^j_t}{x^j_t(\omega)} \right)^{1-\rho}$$

which is the standard inverse demand function for intermediate good $\omega$.

*Structures and capital specific firms.* We assume both sectors are perfectly competitive so the price of structure and capital specific goods equal their marginal costs $f^j_t$ and $r^j_t$ respectively.

*Intermediate good producers.* The intermediate goods sector is monopolistically competitive and subject to entry. We will focus on a symmetric equilibrium: we drop the index $\omega$ for the intermediate goods.

Cost-minimization given technology (3) implies that total costs are

$$C^j_t(x^j_t, w_t, r_t) = \frac{r^\alpha_t w_t^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} x^j_t A^j_{A_t}.$$

We can substitute the cost function and the inverse demand function facing the intermediate goods firm into its profit function to derive the optimal price that the firm charges for its product:

$$p^j_t = \frac{1}{\rho} \frac{r^\alpha_t w_t^{1-\alpha}}{A^j_t \alpha^\alpha (1-\alpha)^{1-\alpha}}$$

This is the familiar mark-up pricing rule.

*Entry in the intermediate sector.* There is free entry into the intermediate sector. However, entrants need one unit of structures to start up production. The present discounted value of flow profits of a firm that chooses to enter at date $t$ is

$$V^j_t = \sum_{s=t}^{\infty} (1-\chi)^{s-t} q^j_s \pi^j_s.$$
where $\pi^j_t = p^j_t x^j_t - C^j_t \left( x^j_t, w^j_t, r^j_t \right)$ are revenues net of variable costs and we have used the bond price for $s$ periods ahead, $q^j_{ts}$, to discount future profits. Substituting (9) and (10) we get that

$$\pi^j_t = \frac{1}{\sigma - 1} A^j_t \alpha^\alpha (1 - \alpha)^{1 - \alpha}.$$

where $\sigma = 1/(1 - \rho)$ is the elasticity of demand of each intermediate good.

Hence there will be entry as long as the present value of profits is higher than the cost of a new structure, given by $f^j_t$. The free entry condition can be written then as

$$V^j_t \leq f^j_t$$

with strict equality if there is positive entry.

*Composite capital sector.* Perfectly competitive firms produce new units of the composite capital good using (4). The profit function is given by

$$z^j_t i^j_t - \epsilon^j_t \sum_{c \in J} \left( \frac{u^c_t n^c_{jt}}{\epsilon^c_t} \right).$$

where $\epsilon^c_t$ is the real exchange rate between country $c$ and some (unindexed) country whose final good acts as numeraire in the international markets. Hence the ratio $\epsilon^c_t / \epsilon^j_t$ is the price of one unit of final good in country $c$ in terms of the final good of country $j$. All prices in the world market are taken as given.

Profit maximization implies that the following inverse demand for specific capital,

$$\left( \frac{\epsilon^j_t}{n^c_{jt}} \right)^{1 - \eta} = \frac{\epsilon^j_t \epsilon^c_t}{\epsilon^c_t \epsilon^j_t.}$$

We assume that trade is balanced each period. Since specific capital goods are the only traded goods in this economy we have that

$$u^j_t \sum_{c \in J} n^j_{ct} = \epsilon^j_t \left( \frac{u^c_t n^c_{jt}}{\epsilon^c_t} \right).$$

*Competitive equilibrium definition.* A competitive equilibrium is a price system $\left\{ q^j_t, z^j_t, f^j_t, u^j_t, r^j_t, w^j_t, p^j_t \right\}$ and an allocation $\left\{ c^j_t, y^j_t, x^j_t, k^j_t, i^j_t, n^j_t, m^j_t \right\}$ such that for all countries $j \in J$ and periods $t \geq 0$:
1. Households maximize (1) subject to the flow budget constraints,

2. Firms maximize profits,

3. All markets clear,

4. and the resource constraint holds (6).

2.3 The Steady State

We now solve for the key equilibrium relationships in the model along a steady state balanced growth path. We shall focus on the symmetric equilibrium wherein \( x^i = x, p^i = p, l^i = l \) and \( k^i = k \) for all \( i \), i.e., allocations across intermediate goods firms are symmetric.\(^3\) We start by using the inverse demand function for specific capital from country \( c \) (see equation (12)) to get

\[
n^j_c = \left( \frac{z^c}{\varepsilon^c v^j} \right)^{\frac{1}{1-\eta}} I^c.
\]

Substituting this expression into the production function for the composite capital good in country \( c \) and suitably manipulating the resulting expression gives

\[
\varepsilon^c = z^c \left[ \int_J (v^j)^{\frac{n}{\eta}} dj \right]^{\frac{1-\eta}{\eta}}.
\]

Clearly, \( \left[ \int_J (v^j)^{\frac{n}{\eta}} dj \right]^{\frac{1-\eta}{\eta}} \) is a world price index for a basket of specific capital goods which is common to all countries. Hence, the relative price of the composite capital good across any two countries is given by the ratio of their real exchange rates:

\[
\frac{z^j}{z^i} = \frac{\varepsilon^j}{\varepsilon^i}.
\]

Intuitively, since the basket of goods that are used to produce the composite capital good is identical across countries and freely available to all countries at the same world price, differences in the domestic basket price of the composite capital good across countries can only arise due to differences

\(^3\) Here and below we suppress time subscripts for notational convenience since we are describing the steady state.
in the real exchange rate across countries, i.e., differences in the rate at which the world numeraire good can be converted into domestic final goods. Recall that all world prices are denominated in terms of the world numeraire good.

Along any symmetric equilibrium path the final output of any economy is given by

$$Y = M^{\frac{1}{\rho}} AK^\alpha L^{1-\alpha},$$

where $\frac{1-\rho}{\rho} = \frac{1}{\sigma-1}$. This expression follows by setting $x^i = x$ for all $i$ and substituting it into the production function (2). Note that by market clearing we must have $kM = K$ and $lM = L$ in a symmetric equilibrium. With no loss of generality we shall set aggregate labor $L = 1$ from here on. This assumption also implies that $Y$ gives per capita output of the final good.

In the appendix we show that the steady state composite capital to structure ratio is given by

$$\frac{K}{M} = \frac{f}{z} \left( \frac{1}{\alpha (\sigma - 1)} \right) \beta \left( \frac{1 - \beta (1 - q)}{1 - \beta (1 - \delta)} \right).$$

This gives the optimal mix of composite and structural capital as a function of the ratio of their relative prices.

The appendix also shows that

$$M = \left[ \frac{z^\alpha f^{1-\alpha} \sigma [1 - \beta (1 - q)]^{1-\alpha} [1 - \beta (1 - \delta)]^\alpha}{A^\alpha \beta^\alpha (\sigma - 1)^\alpha} \right]^{\frac{\sigma - 1}{2 - \alpha (\sigma - 1)}}.$$

where we have used the fact that $L = 1$.

We can substitute this expression for $M$ along with equation (16) into the expression for output (equation 15) to derive output as a function of productivity and capital goods prices:

$$\log Y = \left[ \frac{\sigma - 1}{\sigma - 2 - \alpha (\sigma - 1)} \right] (\log A - \alpha \log z)$$

$$- \left[ \frac{1}{\sigma - 2 - \alpha (\sigma - 1)} \right] \log f + \text{constant}.$$ 

Throughout the paper we shall maintain the restriction $\sigma > 2$. It is easy to see that per capita output will be decreasing in the capital prices $z$ and $f$ as long as $\sigma - 2 > \alpha (\sigma - 1)$. This condition will be satisfied by our parameterization of the model (see Section 4 below).
3 Quantitative Implications

We now turn to an evaluation of the quantitative implications of this model. Our primary focus is on the model’s implications for per capita output. From the previous section the output of country \( c \) relative to any reference country, say country \( r \), is given by

\[
\frac{y_i}{y_r} = \left( \frac{A_i}{A_r} \right)^D (\xi_r \xi_i) \left( \frac{f_r}{f_i} \right)^D,
\]

where \( D \equiv (\sigma - 2 - \alpha (\sigma - 1))^{-1} \). Hence, in order to quantify the relative income predicted by the model we need to identify the price of composite capital \( f \) and the price of structures \( z \) across countries.

From the previous section we know that

\[
\frac{z_r}{z_i} = \frac{\xi_r}{\xi_i}.
\]

Since \( \xi \) denotes the real exchange rate, one can determine the relative price of composite capital by using the ratio of the price of consumption in the two countries (expressed in terms of the same basket). This data is readily available from the Penn World Tables.

Determining the price of structures across countries is more complicated. Structures are one component of capital while the relative price of investment that is available from the Penn World Tables gives the price of a basket of capital goods, not just structures. We get around this problem by using the structure of the model to identify the prices of structures and composite capital individually from the relative price of investment series.

To proceed we consider the problem facing an artificial firm which produces aggregate capital \( X \) using as inputs structures \( M \) and composite capital \( K \). Define aggregate capital as

\[
X = \left[ M^{\frac{1}{1+\alpha}} K^{\alpha} \right]^\frac{\sigma-1}{1+\alpha(\sigma-1)}.
\]

The definition is necessary in order to have constant returns to scale for the artificial firm.\(^4\) Final output is now given by

\[
Y = AX^{\frac{1+\alpha(\sigma-1)}{\sigma-1}}.
\]

\(^4\)Without constant returns to scale, the price of aggregate capital would depend on the quantities.
The artificial firm’s problem is given by

$$\max P^x X - fM - zK$$

where $P^x$ is the price of aggregate capital. Combining the first order conditions for this problem, solving out for $K$ and $M$ in terms of $f$ and $z$ and substituting them back into the expression for $X$ gives

$$X = \left[ \left( \frac{f}{z} \right) \alpha (\sigma - 1) \right]^{\frac{\alpha (\sigma - 1)}{1 + \alpha (\sigma - 1)}} M.$$ 

Finally, we can use the first order condition for $M$ in the above to get

$$\log (P^x) = \frac{\alpha (\sigma - 1)}{1 + \alpha (\sigma - 1)} \log (z) + \frac{1}{1 + \alpha (\sigma - 1)} \log (f) + \text{cte.}$$

Thus, in our case the price index is a geometric average of $z$ and $f$ with constant weights.

To see the implications of this for the model’s cross-country predictions, note first that the expression for the investment goods price index can be used to derive the ratio of structures prices between the reference country $c$ and country $i$:

$$\frac{f_r}{f_i} = \left( \frac{P^x_r}{P^x_i} \right) ^{1+\alpha(\sigma-1)} \left( \frac{z_i}{z_r} \right) ^{\alpha(\sigma-1)}.$$ 

Substituting this expression into the relative income term above gives

$$\frac{y_i}{y_r} = \left( \frac{A_i}{A_r} \right) ^{(\sigma-1)D} \left( \frac{P^x_r}{P^x_i} \right) ^{[1+\alpha(\sigma-1)]D}.$$  \hspace{1cm} (19)

Hence, the relative incomes across countries becomes solely a function of the ratio of relative price of investment and the productivity ratios across countries. This is very similar to the one-sector neoclassical model except for the exponent on the arguments.

Equation (19) can be used to illustrate two key differences in the quantitative implications of our model relative to the standard neoclassical model. Consider a simplified version of the neoclassical model where $P^x$ units of the final good produce one unit of capital. Further assume that the final good is produced using a production function $Y = AK^\kappa L^{1-\kappa}$. It is easy to check that the steady state relative income between countries $r$ and $i$ in the standard model is given by

$$\frac{y_i}{y_r} = \left( \frac{A_i}{A_r} \right) ^{\frac{1}{1-\kappa}} \left( \frac{P^x_r}{P^x_i} \right) ^{\frac{\kappa}{1-\kappa}}.$$  \hspace{1cm} (20)
The terms for relative income in equations (19) and (20) are very similar. The key difference is in the value of the exponent on the two terms on the right hand side. The key parameter in equation (20) is \( \kappa \) which is the share of capital in total output. Work by Gollin (2002) and others indicates that the capital share is somewhere between 0.3 and 0.4 in most countries. Moreover, cross-country data on the relative price of investment suggests that \( \frac{P^x_{poor}}{P^x_{rich}} \approx 5 \), i.e., relative investment prices in the poorest countries are, on average, around 5 times higher than in the richest countries. Plugging a capital share of 0.4 along with \( \frac{P^x_{poor}}{P^x_{rich}} = 5 \) into (20) while keeping the \( A \)'s constant across countries gives \( \frac{y_{rich}}{y_{poor}} = 2.9 \). A corollary of this is that for the model to generate 35-fold income differences one needs a productivity gap \( \frac{A_{rich}}{A_{poor}} = 8.4 \).

What are the corresponding numbers from our model? To quantify the model we need estimates for the parameters in equation (19). To do so we start by noting that profits of intermediate goods firms are used to pay for structures. The profit share of revenues in the intermediates goods sector is \( \frac{1}{\sigma} \). Hence, in terms of the share of final output, a fraction \( \frac{1}{\sigma} \) accrues to structures while a share \( \alpha \) of the remaining \( 1 - \frac{1}{\sigma} \) accrues to composite capital. Hence, the share of output going to capital in our model is

\[
\kappa = \frac{1}{\sigma} + \alpha \left( 1 - \frac{1}{\sigma} \right) = 1 + \alpha \left( \frac{\sigma - 1}{\sigma} \right).
\]

Using this expression one can rewrite equation (19) as

\[
\frac{y_{rich}}{y_{poor}} = \left( \frac{A_{rich}}{A_{poor}} \right)^{\frac{\sigma - 1}{\sigma(1-\kappa)}} \left( \frac{P^x_{poor}}{P^x_{rich}} \right)^{\frac{\alpha \kappa}{\sigma(1-\kappa)}}.
\]

To make the comparison between the standard model and our's meaningful we shall keep the capital share fixed at 0.4. \( \sigma \) represents the elasticity of substitution between intermediate goods. Acemoglu and Ventura (2002) estimated \( \sigma = 2.6 \) which is the number that we shall use in our cross-country quantification below. Here we shall illustrate the predicted income disparities and the required TFP gaps in the model for a range of different values of \( \sigma \). We keep the capital share fixed at \( \kappa = 0.40 \) by varying \( \alpha \) appropriately. Table 1 illustrates the contrast between the models.
Table 1. Contrasting Income and TFP predictions

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Standard model</th>
<th>Armenter-Lahiri</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_{rich}/y_{poor}$</td>
<td>$y_{rich}/y_{poor}$</td>
<td>$y_{rich}/y_{poor}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.6 35</td>
<td>2.9 8.4</td>
<td>19.9 1.2</td>
</tr>
<tr>
<td></td>
<td>2.8 35</td>
<td></td>
<td>14.2 1.4</td>
</tr>
<tr>
<td></td>
<td>3 35</td>
<td></td>
<td>11.2 1.6</td>
</tr>
</tbody>
</table>

Two features stand out from Table 1. First, the model generates between 11 and 20-fold relative income gaps between the richest and poorest countries. This is 4 to 6 times larger than income gaps that are generated by the standard model even when we keep the capital share at conventional levels. Second, and perhaps more interestingly, the implied total factor productivity differences between the richest and poorest countries that are needed to explain 35-fold income differences are fairly small ranging between 1.2 and 1.6. These numbers are an order of magnitude smaller than the 8.4 required in the standard model. Moreover, the required TFP differences also appear to be remarkably robust to changing values of $\sigma$.

We next subject the model to a second test. In particular we evaluate the cross-country fit of the model relative to the data. We express each country’s income relative to the U.S.A.. We use income and relative investment price data on 186 countries for the year 1996 from the Penn World Tables 6.1. We quantify equation (21) for each country $i$ by substituting in $\frac{P_i x_i}{P_{US} x_{US}}$. For our baseline calibration we set $\sigma = 2.6$ and the capital share $\kappa = 0.4$.

Figure 1 plots the result. On the horizontal axis are the relative income numbers in the data while the vertical axis gives the corresponding numbers predicted by the model. We have drawn in a 45-degree line to permit an easy visual examination of the fit of the model. The scatter of points are mostly clustered around the 45-degree line indicating that the fit is reasonably good. The correlation between the predicted and data relative income series is 0.72.

Lastly, we compare the statistics of the model with the data in terms of the income dispersion
between the richest and poorest countries. In particular, we compare the income of the 95th percentile relative to the 5th percentile of the income distribution in the model with that in the data. As an ancillary test, we also compute the predicted capital-output ratio of the 95th percentile relative to the 5th percentile and compare it with the data. We take the data on capital-output ratios from Hall and Jones (1999). To check for robustness we compute these statistics for three alternative values of $\sigma$ while keeping the capital share $\kappa = 0.40$.

Table 2. Predicted Dispersion and Robustness

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\frac{y_{95}}{y_{5}}$</th>
<th>$\frac{K-Y_{95}}{K-Y_{5}}$</th>
<th>$\frac{y_{95}}{y_{5}}$</th>
<th>$\frac{K-Y_{95}}{K-Y_{5}}$</th>
<th>$\frac{A_{95}}{A_{5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>32.7</td>
<td>8.5</td>
<td>16.7</td>
<td>5.2</td>
<td>1.3</td>
</tr>
<tr>
<td>2.8</td>
<td>32.7</td>
<td>8.5</td>
<td>12.1</td>
<td>5.2</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>32.7</td>
<td>8.5</td>
<td>9.7</td>
<td>5.2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 2 reports the results. The Table shows that raising $\sigma$ from 2.6 to 3 lowers the relative income gap predicted by the model from 50 percent of the data to just under a third of the data. It bears repeating however that even at $\sigma = 3$ the model predicts a relative income gap of 9.7 which is more than three times the corresponding prediction of the standard model. Our predicted capital-output ratio for the 95th to 5th percentile is slightly lower than that corresponding number in the Hall-Jones data. To get a better sense of the fit of the capital-output ratio series, Figure 2 plots the predicted capital-output ratios for all countries in our sample against the reported numbers in Hall-Jones for capital-output ratios.\(^5\) The correlation between the data and the model is 0.6 and most of the scatter points are clustered slightly below the 45-degree line indicating that, if anything, we appear to be under-predicting the capital-output ratio. The results show that the magnification of the income disparities achieved in the model is not being generated through

\(^5\)Since the Hall-Jones numbers for capital output ratios are not available for all countries in our sample, we generate Figure 2 by using a shorter sample of 119 countries.
counterfactually high capital accumulation. It is important to note that this fit is independent of \( \sigma \) since the relative capital-output ratios across countries depends only on the ratio of the relative price of investment goods across countries. The last column of Table 2 shows that the TFP differences that are required to fit the relative income dispersion exactly are relatively small (all under 2) and quite robust to changes in \( \sigma \).

We interpret these results as suggestive that the model does a decent job of fitting the data. Moreover, we view the relatively small TFP gaps implied by the model as being a significant improvement over the standard model which typically requires TFP differences around 9 to fit the relative income data. This has been viewed by a number of authors as being unreasonably large.

4 Conclusion

The standard neoclassical growth model attributes a large fraction of the observed income disparity between the richest and poorest countries of the world to productivity differences. Differences in measured inputs, while substantial, are not large enough to account for the income disparity given measured factor shares. This has proved to be a stumbling block for using the dispersion in relative investment prices across countries to account for the observed cross-country income differences. In this paper we have augmented the standard model to allow for an extensive margin in the production process. In particular, we have formalized an environment where higher investment prices not only reduce the total amount of investment in variable capital (the standard intensive margin on capital) but also reduce the number of firms and products in the economy. We have shown that the extensive margin effect magnifies the effect of higher relative investment prices on per capita output. At conventional levels for the capital share, our model can account for up to 16-fold income differences between the richest and poorest countries of the world. Moreover, our model reduces the TFP gaps required to explain the observed income differences to less than 2-fold. We view both these results as significant improvements over the standard model.

A key component of our environment is that entry into the intermediate goods sector involves
a set-up cost. In the quantitative section of the model we take a narrow view of this set-up cost as being the cost acquiring and installing a business fixed structure. More generally, one could view this set-up cost as including a number of other costs of setting up businesses including things like acquiring licenses, paying business agents, costs of conducting business surveys and plans, political contributions as well as outright bribes. Naturally, if one were to use this broader concept of the entry-cost then the accounting exercise would also have to be adjusted to reflect the fact that only a part of the entry costs are for business structures while the rest are for indirect business costs. In as much as the entry cost reflects the latter component, the model would then translate into measured productivity differences across countries instead. We intend to address the issue of endogeneizing productivity differences across countries in future work.
Appendix

A Some key derivations

Here we derive the steady state expressions for $K/M$ and $M$ as functions of the relative prices $f$ and $z$. First, consider the condition that govern the entry decision in the intermediate goods sector in steady state. The present discounted value of flow profits of a firm that chooses to enter at date $t$ and produce intermediate good of variety $i$ is $V_t^i = \sum_{s=t}^{\infty} \beta^{s-t} \pi^i_s$. Recall that to produce in the intermediate goods sector a firm needs one unit of the structure. Hence, the no-entry condition in any sector $i$ is $V_t^i \leq f_t$. Along a balanced growth path (BGP) this condition reduces to

\[
\left(\frac{1}{\sigma - 1}\right) \frac{r^\alpha w^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{x^i}{A} \left(\frac{1}{1 - \beta (1-q)}\right) \leq f
\]

Integrating the optimal labor condition (equation (??)) over all intermediate goods firms $i$ gives

\[
\frac{x}{A} = \left(\frac{\alpha}{1 - \alpha} \frac{w}{r}\right) \frac{L}{M}
\]

where we have used the market clearing condition for labor $\int_i l^i M(i) \, di = L$ and the fact that all intermediate goods firms are symmetric. Substituting this into the free entry condition (which holds with equality in equilibrium) gives

\[
\left(\frac{1}{\sigma - 1}\right) \left(\frac{1}{1 - \beta (1-q)}\right) \frac{wL}{(1-\alpha)M} = f.
\]

From the optimal factor use conditions of intermediate goods firms we also have

\[
rK = \frac{\alpha}{1 - \alpha} wL.
\]

In deriving the above we have used the market clearing condition for capital $\int_i k^i M(i) \, di = K$. Using this condition in the free entry condition and rewriting gives

\[
\frac{K}{M} = \frac{f}{r} \frac{\alpha (\sigma - 1) [1 - \beta (1-q)]}{\sigma - 1}.
\]
Lastly, in steady state the Euler equation for optimal household behavior (equation (7)) reduces to
\[ \frac{1-\beta(1-\delta)}{r} = \frac{\beta}{\bar{f}}. \]
Substituting this into the previous relation gives
\[ \frac{K}{M} = \frac{f}{\bar{z}} \alpha (\sigma - 1) \beta \left[ \frac{1 - \beta (1 - q)}{1 - \beta (1 - \delta)} \right]. \]

Since we are assuming that there is no trade in final goods and there are no international asset markets, all countries must satisfy a balanced trade condition
\[ \int_j n^t v^j d_j = \int_j n^c v^c d_j. \]

Since \( \int_j n^c d_j = n^c \) the above can be rewritten as
\[ n^c v^c = \int_j n^c v^j d_j. \]

By substituting in for \( n^j \) from equation (12) we can rewrite the balanced trade condition as
\[ n^c v^c = I^c \left( \frac{z^c}{x^c} \right)^{\frac{1}{1-\eta}} \int_j (v^j)^{\frac{\eta}{1-\eta}} d_j. \]

Using equation (14) this condition can be rewritten as
\[ \frac{n^c}{I^c} = \frac{z^c}{x^c v^c} = \frac{z^c}{x^c}, \]

where the second equality follows from the first order condition for specific capital firms. This condition gives the optimal specific capital to composite capital ratio as a function of their relative price.

The market clearing condition for composite capital implies that
\[ I_t = K_{t+1} - (1 - \delta) K_t \]
where the left hand side is the supply of composite capital while the right hand side gives the demand. In steady state we must have \( M_t = M_{t-1} \) and \( K_{t+1} = K_t \) for all \( t \). Hence, we must have \( I = \delta K \) in steady state. Using these steady state relationships, the resource constraint reduces to
\[ c + 2\delta zK + qfM = Y, \]
where we have used the fact that $zI = an$ from equation (23).

The last step in solving for the steady state of the model is to derive the steady state solution for $M$. In a symmetric equilibrium the inverse demand function for intermediate goods (equation (8)) can be written as

$$p = M^{\frac{1}{\sigma-1}},$$

where we have used the equilibrium relation $Y = xM^{\frac{1}{\sigma-1}}$. Equating this solution for $p$ with the optimal price of intermediate goods (equation (10)) yields

$$M^{\frac{1}{\sigma-1}} = \frac{r^\alpha w^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha} \rho A} = \frac{[1 - \beta (1 - q)] f}{(1 - \rho) x}.$$

The second equality follows from the free-entry condition given by equation (11).

Using the production function for intermediate goods and imposing symmetry on it gives

$$Mx = AK^\alpha L^{1-\alpha}.$$

This can be used to substitute out for $x$ in the previous expression for $M^{\frac{1}{\sigma-1}}$ which gives

$$M^{\frac{2-\sigma}{\sigma-1}} K^\alpha = \frac{\sigma [1 - \beta (1 - q)] f}{AL^{1-\alpha}}.$$

We can now use equation (16) in the above expression to get

$$M = \left[ \frac{z^\alpha f^{1-\alpha} \sigma [1 - \beta (1 - q)]^{1-\alpha} [1 - \beta (1 - \delta)]^\alpha}{A \alpha^\alpha \beta^\alpha (\sigma - 1)^\alpha} \right]^{\frac{\sigma-1}{\sigma-\sigma^\alpha(\sigma-1)}}.$$

References


Figure 1: Relative Income: Model and Data
Figure 2: Capital-output ratios: Model and Data