Conflict and Mobility:
Resource Sharing Among Groups

Sourav Bhattacharya∗  Joyee Deb†  Tapas Kundu‡
University of Pittsburgh  New York University  University of Oslo

PRELIMINARY

Abstract

We study a political competition between two groups, where the winner has the decision rights to allocate resources, like political parties deciding on sharing of patronage goods. What factors determine how resources are shared? We highlight an important force that affects distribution of resources, namely the ability to move between groups. In many contexts, group sizes are determined endogenously. For example, allocation of jobs based on party allegiance may influence individuals’ choices of switching party membership. We analyze how the ease of inter-group mobility affects resource allocation. One insight from existing literature is that the threat of conflict can also act as a constraint to how exploitative the elite can be. We investigate the combined effect of both factors. We show how inter-group mobility affects the possibility of conflict and in turn the extent of resource sharing? We find that sharing occurs in equilibrium. There are two reasons why the incumbent wants to shares resources with the opposition. First, if the incumbent retains too much surplus, it may attract switchers, which reduces the per capita share. Second, sharing resources increases the oppositions opportunity cost of engaging in conflict. There are thus two constraints on expropriation - the switching constraint and the conflict constraint. Optimal sharing is dictated by whether the constraint s bind. We also find a non-monotonic relationship between resource sharing and the cost of mobility. Our predictions are consistent with several stylized facts that cannot be explained by earlier models.

JEL Code: D72, D74, D78

∗Email: sourav@pitt.edu
†Email:joyee.deb@nyu.edu
‡Email: tapas.kundu@econ.uio.no
1 Introduction

Examples of competition and conflict between groups over the sharing of society’s resources are ubiquitous. Different groups in society may not have compatible goals and are in competition for scarce resources [2]. Farmers prefer more resources to be allocated to agriculture while industrialists would lobby for the opposite. Different religious, caste-based groups are vying for group-based reservations of limited resources, such as government jobs. In autocratic regimes, parties decide on the division of patronage goods such as high-paying jobs, key positions in the legislature, directed subsidies or even direct monetary transfers. Not surprisingly, the objective of political struggle in the society often turns out to be to gain the decision rights to allocate resources among groups. What factors determine how resources are shared? The broad objective of this paper is to better understand the relationship between conflict and resource sharing between groups.

The relationship between conflict and rent redistribution between groups has been studied before (see for instance, [1]). One of the key insights from this literature is that potential of conflict or revolution can act as a constraint to how exploitative the ruling elite can be. Excessive resource extraction by the elite leaves a low share for the opposition, also reducing the opposition’s opportunity cost of engaging in conflict. If the outcome of conflict is very costly to the ruling group, its rent seeking behavior can be constrained.

In this paper, we highlight a different and important force that affects redistribution of resources between groups. Existing literature studies competition between groups of fixed sizes. However, in many contexts, group sizes are determined endogenously. For example, the sectoral redistribution of resources between the agricultural and industrial sector affects the opportunity costs of individuals and can alter their decision to work in their respective sector. Similarly, redistribution of resources based on geographical regions can affect the incentives for people to migrate. Allocation of jobs based on party allegiance may influence individuals’ choices of switching membership between two political parties. The main question we ask here is how this option of inter-group mobility can affect how groups compete with each other and share resources? In particular, we investigate the combined effect of both factors - inter-group mobility as well as conflict - on how groups compete and study the interconnection between them. We look at situations where groups can collectively engage in conflict and individuals can each choose which group to belong to, and ask how inter-group mobility affects the possibility of conflict and in turn the extent of resource sharing?

Substitutability between political activism and switching groups as alternate response mechanisms is akin to the “exit and voice” mechanisms that have been studied extensively in different socio-political (and business) contexts. A large body of existing work in the collective action literature and in history and sociology suggests that the availability of an exit option typically implies a decrease in collective action or revolution as a means of political protest. For instance, the possi-
bility of emigration from a country would prevent revolution. Conversely, if the costs of revolution decrease, emigration should decrease as the balance of incentives shifts from "exit" to "voice" as a mechanism to improve one’s situation (See for instance, [6] - give more examples here). To the best of our knowledge, while there is much anecdotal evidence documenting this substitutability, there is little formal theoretical work on this.

There is some evidence to support that mobility is also related to resource sharing. North (1981) provides examples of rulers granting more concessions to citizens who have greater opportunities for mobility than to those who do not. Again, there is little theoretical foundation for this.

What effect does endogenous group membership have on resource sharing and conflict? Clearly, keeping a higher share of surplus for one’s own group increases each individual’s share of surplus in the group (keeping the group size constant). So why doesn’t the ruling group exploit the opposition and keep all the resources for itself? The possibility of conflict or changing group membership affects the ruling group’s incentives to share resources. On the one hand, if the ruling group retains a very big share of the resources, this reduces the opposition’s opportunity cost of political action, and thus raises the possibility of costly conflict. Further, retaining a large share induces people from the opposition to switch to the ruling group, thus increasing the size of the group. This implies that the per capita share of surplus decreases for each individual initially in the group (the same pie has to be shared with many more people). On the other hand, an increase in group size has the positive impact of increasing the political strength of the group. The extent of sharing is determined by the balancing these tradeoffs.

We consider a simple two-period game of political competition. Members of society are divided into two groups who compete for political power. In each period, the ruling group gets elected either through a democratic process, or as a result of conflict. The ruling group earns the right to decide how society’s resources should get divided between the two groups. Agents all participate in some economic activity, and the resources are productive inputs that agents can use to enhance their payoffs from economic activity. In each period, once the ruling group announces the split of resources, the losing group (opposition) can choose to either accept its share or can collectively engage in conflict to change the incumbent regime (and improve their payoff in the next period). Conflict lowers the probability of the re-election of the current ruler, but waging conflict is costly. If the opposition engages in conflict, they cannot carry out their economic activity, and so get zero payoff from economic activity. If conflict occurs, the ruling group also loses a fraction of their payoff from economic activity. Each group’s objective is to maximise the expected per capita payoff of the current members of the group. If the opposition decides to accept the share offered by the ruling group and no conflict occurs, individuals (in both groups) can still choose whether they want to stay in their respective group or switch. Individuals can switch groups at a cost. If an agent switches, she gets a share of the new group’s resources. An individual’s objective is to maximise
own expected payoff. We characterize the equilibrium resource allocations in this model.

Notice that the actions that agents can take to improve their payoffs are costly. Conflict is costly for the opposition because it requires time and effort, and members must give up the opportunity to participate in productive economic activity. Conflict is costly for the ruling group, because it implies a higher probability that they do not get re-elected in the future. Switching group membership is also costly. Indeed, the cost of mobility between groups can vary widely. For instance, in the extreme case, one can think of ethnic or racial groups. Changing ethnic identity is essentially impossible (infinitely high cost) except perhaps by marriage in some cases. Changing professions or geographies is less costly. On the other extreme, the cost of changing membership of political parties is relatively low. We analyze how the nature of competition between groups varies based on the cost of mobility.

We find that sharing does occur in equilibrium. Even though the incumbent can decide to expropriate all the resources, it does not do so always. There are two different reasons why the incumbent wants to share resources with the opposition. First, if the incumbent keeps too much surplus for itself, it may attract switchers from the opposition which would reduce the per capita share for the original members of the ruling group: thus the incumbent might want to share in order to prevent switching. Second, the ruling group might want to share resources with the opposition so that economic activity is sufficiently attractive for the opposition, and they do not engage in conflict. Put differently, there are two constraints on expropriation by the incumbent - the switching constraint and the conflict constraint, and the optimal sharing is dictated by whether and which constraint binds.

When the cost of switching is very low, i.e. \( \phi < \phi_1 \), the opposition never engages in conflict. The conflict constraint does not restrain the choices of the incumbent. The incumbent is only concerned about switching and shares just enough resources to prevent switching from the opposition.

As the cost of mobility increases, conflict becomes more costly for the incumbent, and at the same time, the payoff obtainable by just preventing switching increases. There is a threshold cost of switching below which allowing conflict by extracting all surplus is more attractive to the incumbent than offering just enough to prevent switching. In this intermediate range of mobility cost, the incumbent expropriates all resources and there is conflict in equilibrium. Above this threshold, the incumbent prefers to avoid conflict, and also share just enough to prevent switching.

However, if the cost of mobility is too high, the opposition may prefer to engage in conflict. In this case, it is the political action constraint that binds the incumbent. The incumbent needs to share enough resources to prevent conflict. Since the cost of mobility is very high, switching is anyway prevented at this offer.

Our model yields several new and interesting empirical predictions. As described above, we find a non-monotonic relationship between resource sharing and the cost of mobility.
Conflict will not arise in equilibrium if the cost of mobility between groups is extremely low. The incumbent will share enough resources to prevent switching. In an intermediate range of cost of mobility, we can find complete expropriation of resources by the incumbent and conflict by the opposition. At higher costs of mobility, we will again find sharing by the incumbent. Finally if the cost of mobility is extremely high, the incumbent will share enough resources to just prevent conflict. In this range, while conflict does not occur the opposition is exactly indifferent between conflict and economic activity. In this sense, this is a region of peaceful belligerence - the threat of conflict by the opposition forces the incumbent to share.

Our predictions are consistent with some stylized facts that cannot be explained by earlier models. For instance, [5] points out that existing models cannot explain why in some autocratic regimes (like Houpouet-Boigny in Ivory Coast) rulers actually transfer resources to the opposition ethnic groups. In our model this could be explained by the peaceful belligerence region (where cost of mobility is very high, the opposition’s opportunity cost of conflict is low, the ruler wants to avoid conflict.)

Our model further predicts that the political constraint will bind only if the incumbent is a majority group, and the threat of political action by the minority is strong (in the sense that while under no conflict the majority is more likely to retain authority, conflict would make the minority more likely to win power). In other words, in practice some resource sharing will be observed in a situation of conflict only when the incumbent is a majority and the minority poses a strong threat of conflict. On the other hand, in situations of conflict with a minority incumbent we should observe complete expropriation of resources. A systematic empirical analysis of these would be interesting.

Another prediction is that when conflict is very costly ($k$ small enough) for the incumbent, then we should not observe the incumbent expropriating all resources and inducing conflict. Instead, we should observe some resource sharing by the incumbent (enough to just prevent conflict or switching, depending on the cost of mobility).

Closest in spirit to this paper is work by Caselli and Coleman (2006) who develop a model to show that conflict does not occur when switching group identities is easy since it is anticipated that the winning coalition would expand. They suggest that higher switching costs obtain for members of groups that are easily distinguished. However there are many examples where intense conflict arises between groups whose members cant be reliably distinguished. While our model confirms that conflict does not arise when cost of mobility is extremely low, we can explain why conflict can still arise even when cost of mobility is not high.
2 Model

2.1 The Environment

There is a continuum of agents in society of measure 1. Members of society are divided into two groups $A$ and $B$. In each period, groups compete for political power. The winning group gets elected either through a democratic process, or as a result of conflict. The winning or ruling group earns the right to decide how society’s resources should get divided between the two groups. The size of the society’s resources is exogenous. Once a group’s share is determined, the group’s resources are evenly divided among its members\(^1\). Agents in society all participate in some economic activity, and the resources can be thought of as some productive inputs that agents can use to enhance their payoffs from economic activity. If a group of size $\pi$ gets fraction $\alpha$ of society’s total resources $x$, the per capita payoff that its members get from economic activity is given by $\frac{\alpha x}{\pi}$. The winning or ruling group decides the split $\alpha$. The assumption of linear payoff from resources is made here mainly for tractability.

Once the ruling group announces the split of resources, the losing group (opposition) can choose to either accept its share or reject it and engage in conflict instead. We model conflict in a reduced form. We can think of conflict as any action taken by the opposition that is costly (wasteful) in the short-run, but increases the opposition’s chances of becoming the ruler in future. For instance, it could be violent conflict or simply mobilization of voters (making them more politically active) in a pure democratic process. Formally, engaging in conflict lowers the probability of the re-election of the current ruler in the next period. However, waging conflict is costly. If the opposition engages in conflict, they cannot carry out their economic activity, and so get zero payoff from economic activity. Conflict can also be costly for the ruling group. If conflict occurs, the ruling group also loses a fraction $k \in (0, 1)$ of their payoff from economic activity. We can think of this loss as the time that the ruling group must spend in trying to control the conflict situation. Note that waging conflict is a group decision taken by the opposition\(^2\). The group’s objective is to maximize the expected per capita payoff of the current members of the group.

If the opposition decides to accept the share offered by the ruling group and no conflict occurs, now individuals (in both groups) can choose whether they want to stay in their respective group or switch. Individuals can switch groups at a cost $\phi$. If an agent switches, she gets a share of the new group’s resources. An individual’s objective is to maximise own expected payoff.

We study a two-period game of political competition. Below, we formally describe how the game proceeds. Members of society are divided into two groups $A$ and $B$, who compete for political

\(^1\)In many contexts, it would be more reasonable to assume that resources are shared within groups unequally, based on some power structure or hierarchy. In this paper, for simplicity we do not address this issue. However, it would be an interesting extension of our model to study how the effects of inter-group and intra-group competition interact.

\(^2\)We ignore the collective action problem here. Think of a leader being able to coordinate the decision to wage conflict.
power. At the start of the game, the two groups \( A \) and \( B \) are of sizes \( \pi_0^A \) and \( \pi_0^B = 1 - \pi_0^A \) respectively.

### 2.2 Timing of the Game

**Stage 1:** At the start of each period \( t \in \{1, 2\} \) of the game, one of the groups gets chosen as the winning or ruling group, denoted by \( W_t \). The opposition (or losing group) is denoted \( L_t \). The winning group \( W_t \in \{ A, B \} \) is either chosen as a result of a democratic process or as a result of conflict waged by the opposition.

- If there was no conflict, the winning group is chosen as a result of a democratic process, where the probability of election depends on the size of the group. Let \( \Pr(W_t = W_{t-1}) = p_d(\pi_{t-1}^W) \). We make three assumptions on the function \( p_d(\cdot) \). We first assume that the election probabilities are increasing in group size. Second, the probabilities of re-election for the groups are symmetric in the sense that \( 1 - p_d(\pi) = p_d(1 - \pi) \). Further, we assume that \( \frac{d}{d\pi} [\pi(1 - \pi)p_d(\pi)] < \frac{1}{2} \), and call it the "bounded derivative" condition. While this third assumption is made for technical reasons, it turns out that many reasonable political contest success functions do satisfy all these assumptions.

- If there was conflict, the incumbent group has a lower chance of getting re-elected relative to the democratic process. We assume \( \Pr(W_t = W_{t-1}) = p_c(\pi_{t-1}^W) \), where \( p_c(\pi) < p_d(\pi) \) for all \( \pi \in (0, 1) \).

**Stage 2:** The group that is elected, \( W_t \), decides how to share society’s resources among the two groups. The size of available resources is \( x_t \). \( W_t \) announces a fraction \( \alpha_t^W \in [0, 1] \), i.e., the fraction of resources \( x_t \) that the ruling group \( W_t \) gets. Group \( L_t \) gets the remaining share \( \alpha_t^L = 1 - \alpha_t^W \). For ease of exposition, we assume that the total quantity of resources of society in the first period is normalized to \( x_1 = 1 \) and in the second period \( x_2 = x \).

**Stage 3:** After observing the allocation decision of the ruling group, the opposition group (denoted by \( L_t \)) decides collectively whether or not to accept the proposed \( \alpha_t^W \).

- If \( L_t \) accepts \( W_t \)’s announced allocation, we go to stage 4 of period \( t \).

- If \( L_t \) decides to reject the allocation, it engages in political conflict. Conflict results in a re-election (i.e. we go to Stage 1 of period \( t + 1 \)). The benefit of conflict for the opposition is that it decreases the probability of the ruling group \( W_t \) being re-elected as

\(^3\)For instance, the proportionate representation rule with \( p_d(\pi) = \pi \) satisfies the assumptions. Another example that satisfies our assumptions is the family of functions (See for instance Hirshleifer (1989)) \( p_d(\pi) = \frac{e^{\alpha(\pi - \frac{1}{2})}}{1 + e^{\alpha(\pi - \frac{1}{2})}} \) for \( \alpha > 0 \).
the winner in the next period. However, conflict is costly. In particular, since group $L_t$ engages in conflict, it cannot carry out its regular economic activity, and gets 0 payoff from economic activity. Conflict is also costly for the ruling group $W_t$, who gets a fraction of the payoff it would get from economic activity in the case of no conflict. Payoff to group $W_t$ from economic activity is $k \frac{\alpha_t \pi_{wt} x_t}{\pi_t}$, where $k \in [0, 1]$.

**Stage 4:** If no conflict occurred, each individual (in $W_t$ and $L_t$) now decides whether he wants to remain in his own group or switch to the other group. Individuals can change groups at a cost $\phi \in (0, 1)$. Switching activity changes the size of the groups. Let $\pi_t$ and $1 - \pi_t$ denote the new sizes of the groups. is determined. Given $\pi_t$, each member in a group $J$ gets payoff from economic activity given by $\frac{\alpha_t \pi_{jt} x_t}{\pi_t}$.

Figure 1 gives a pictorial representation of the game. The solution concept is the standard sub-game perfect Nash equilibrium.

![Figure 1: Timing: Sequence of play in any period $t$](image-url)
3 Analysis

We solve the two stage game by backwards induction.

3.1 Equilibrium Play in Period 2

Consider play in period 2. The following proposition describes equilibrium play in the second period. In particular, it characterizes the allocation choice of the incumbent, and the resulting conflict or switching decisions.

**Proposition 1 (Equilibrium Play in Period 2).** Suppose the ruling group is of size $\pi_1^W$.

i) The ruling group allocates a fraction $\alpha_2^* = \pi_1^W + \phi \pi_1^W (1 - \pi_1^W)$ to itself and the remainder $(1 - \alpha_2^*)$ to the opposition.

ii) The opposition does not engage in conflict.

iii) No switching occurs across groups. In particular, members of the ruling group strictly prefer to remain in the group and members of the opposition are indifferent between switching and not switching.

iv) The per capita payoff of the winning group in period 2 is given by $x + \phi (1 - \pi_1^W)$ and that of the losing group is $x - \phi \pi_1^W$.

Before we prove the proposition it is worthwhile to make a few observations. The proposition implies that the second period per capita payoff of the ruling group is increasing in the cost of mobility, and that of the other group is decreasing in the cost of mobility. If agents could move freely across groups then per capita payoffs in society would be equalized (and would equal $x$). In a society with positive costs of mobility, the premium from gaining political power in the second period (i.e. the difference between per capita payoffs of the two groups) is exactly equal to the cost of mobility $\phi$. The higher the cost of mobility, higher is the benefit from gaining political power in the second period. Consequently, as the cost of mobility goes up, the opposition group in period 1 has a higher propensity to reject the incumbent’s offer and launch conflict, and the incumbent on the other hand has a stronger incentive to avoid conflict.

Notice that there is no conflict in equilibrium in the second period. This is just an artifact of the two period game. We will see later that conflict does arise in equilibrium in non-terminal periods of the game. In equilibrium there is no switching either. Indeed, the ruling group will share just enough of society’s resources to make the opposition indifferent between switching and not. Since the switching constraint may bind in this sense, if we were to introduce some heterogeneity in switching costs, switching would occur in equilibrium. We make the assumption of uniform mobility costs just for simplicity.
In the rest of this section, we prove that the strategies described in proposition 1 are optimal. The proof proceeds in three steps. We first characterize the switching rule in period 2 (and resulting group sizes) as a function of the announced allocation. Next, we show that conflict never arises in period 2. Finally, we characterize the optimal equilibrium allocation for the ruling group, and show that it indeed induces no switching by either group.

Proof of Proposition 1. Consider the sub-game where players must decide whether or not to switch groups in period 2. The following lemma describes the switching decisions in equilibrium.

Lemma 1 (Switching Decision in Period 2). Suppose the size of the ruling group at the start of Period 2 is $\pi_1^W$, and let $\alpha_2^W$ be the allocation announced by it. Define functions $f_2(\pi) := \pi + \phi \pi (1 - \pi)$ and $g_2(\pi) := \pi - \phi \pi (1 - \pi)$.

The following describes the switching rule and size of the ruling group at the end of Period 2:

i) If $\alpha_2^W \in [g_2(\pi_1^W), f_2(\pi_1^W)]$, then no switching occurs and $\pi_2^W = \pi_1^W$.

ii) If $\alpha_2^W > f_2(\pi_1^W)$, then some switching occurs from the losing group to the winning group and new group size $\pi_2^W = f_2^{-1}(\alpha_2^W)$.

iii) If $\alpha_2^W < g_2(\pi_1^W)$, then some switching occurs from the winning group to the losing group and $\pi_2^W = g_2^{-1}(\alpha_2^W)$.

Proof of Lemma: An individual in the opposition group ($L_2$) will not switch if and only if his payoff from staying in his own group is at least as large as that from switching to the other group.

\[
\frac{(1 - \alpha_2^W) x}{1 - \pi_1^W} \geq \frac{\alpha_2^W x}{\pi_1^W} - \phi \\
\frac{\phi}{x} \geq \frac{\alpha_2^W}{\pi_1^W} - \frac{1 - \alpha_2^W}{1 - \pi_1^W} = \frac{\alpha_2^W - \pi_1^W}{\pi_1^W (1 - \pi_1^W)}
\]

\[
\alpha_2^W \leq \pi_1^W + \frac{\phi}{x} \pi_1^W (1 - \pi_1^W).
\]

Similarly, an individual in the ruling group ($W_2$) will not switch if and only if

\[
\alpha \geq \pi_1^W - \frac{\phi}{x} \pi_1^W (1 - \pi_1^W).
\]

The sizes of each group in the next period will be determined as an outcome of the individual switching decisions. Let $\pi_2^W$ and $(1 - \pi_2^W)$ denote the new sizes of $W_2$ and $L_2$ respectively. It is easy to see from above that if the announced share $\alpha_2^W$ is such that $\alpha_2^W \in [g_2(\pi_1^W), f_2(\pi_1^W)]$, then no one switches and we have $\pi_2^W = \pi_1^W$. If $\alpha_2^W > f_2(\pi_1^W)$, then members of the opposition group have an incentive to switch. Individuals from group $L_2$ start switching to $W_2$, making the
size of group $W_2$ larger. In equilibrium, individuals perfectly anticipate the switching decisions of others, and so switching happens until $W_2$ reaches a threshold size beyond which any further switching from $L_2$ to $W_2$ is not optimal. In particular, since everyone in group $W_2$ has the same incentives to switch or not switch, the new group size $\pi_2$ is such that every member of the group is indifferent between switching or not switching. This implies that $\pi_2 = f_2^{-1}(\alpha_2^W)$. A similar argument applies if $\alpha_2^W < f_2(\pi_1^W)$. In this case, individuals switch from the winning group $W_2$ to the opposition $L_2$, and $\pi_2^W = g_2^{-1}(\alpha_2^W)$. It is easy to show that if $\phi \in (0, 1)$, the functions $f_2(\cdot)$ and $g_2(\cdot)$ are strictly monotonic and therefore their inverses are well-defined.

Next, consider the sub-game starting in period 2 where members of the opposition group $L_2$ must decide whether to accept the announced resources share $(1 - \alpha_2^W)$ or to engage in political activism and conflict. As there is no third period, payoff from political action is zero for the opposition. On the other hand, payoff from playing the switching game is always strictly positive, for any given $\alpha_2^W$. Therefore the opposition will not choose political action regardless of the value of $\alpha_2^W$.

It now remains to characterize the equilibrium allocation and show that it does not induce switching. At the start of period 2, the group that is elected as the winner $W_2$ must decide how to allocate society’s resources. Since conflict does not arise, the winning group $W_2$ must pick $\alpha_2^W$ to maximize its payoff from economic activity, which is given by $\frac{x_1^W}{\pi_1^W}$. This is equivalent to maximizing $y := \frac{\alpha_2^W}{\pi_2^W}$. Three cases can arise.

i) If the winning group $W_2$ chooses $\alpha_2^W$ in the “no switching range”, i.e., $\alpha_2^W \in [g_2(\pi_1^W), f_2(\pi_1^W)]$, then no one will switch, and $\pi_2^W = \pi_1^W$. In this range, $\alpha_2^W$ has no effect on $\pi_2^W$. In other words, we have $\frac{dy}{d\alpha_2^W} = 1 - \frac{\pi_2^W}{\pi_1^W}$.

ii) If $\alpha_2^W$ is chosen such $\alpha_2^W > f_2(\pi_1^W)$, then there is switching from group $B$ to group $A$. The new size of the winning group will be $\pi_2^W$ is given by $\alpha_2^W = \pi_2^W + \phi \pi_2^W (1 - \pi_2^W)$. Now,

$$\alpha_2^W = \pi_2^W + \frac{\phi}{x} \pi_2^W (1 - \pi_2^W) \implies \frac{d\pi_2^W}{d\alpha_2^W} = \frac{1}{1 + \frac{\phi}{x}(1 - 2\pi_2^W)}$$

So, we have $\frac{dy}{d\alpha_2^W} = \frac{1}{\pi_2^W} - \frac{\alpha_2^W}{(\pi_2^W)^2} \left( \frac{1}{1 + \frac{\phi}{x}(1 - 2\pi_2^W)} \right)$

which simplifies to $\frac{dy}{d\alpha_2^W} = -\frac{\phi}{x} \frac{1}{1 + \frac{\phi}{x}(1 - 2\pi_2^W)}$

The above expression is negative if and only if $\pi_2^W < \frac{2 + \phi}{2\phi}$. Indeed since $\phi \in (0, 1)$, this is always true (for any $\pi_2^W$) and so the function $y$ is decreasing in this range of $\alpha_2^W$. \hfill $\Box$
iii) If \( \alpha_2 \) is chosen such \( \alpha_2^W < g_2(\pi_1^W) \), then there is switching from group \( A \) to group \( B \), then the size of the group \( \pi_2^W (\alpha_2^W) \) is given by \( \alpha_2^W = \pi_2^W - \frac{\phi}{x} \pi_2^W (1 - \pi_2^W) \). Now,

\[
\alpha_2^W = \pi_2^W - \frac{\phi}{x} \pi_2^W (1 - \pi_2^W) \implies \frac{d\pi_2^W}{d\alpha_2^W} = \frac{1}{1 - \frac{\phi}{x} (1 - 2\pi_2^W)}
\]

So, we have

\[
\frac{dy}{d\alpha_2^W} = \frac{1}{\pi_2^W} + \left( -\frac{\alpha_2^W}{\pi_2^W} \right) \left( \frac{1}{1 - \frac{\phi}{x} (1 - 2\pi_2^W)} \right)
\]

which simplifies to

\[
\frac{dy}{d\alpha_2^W} = \frac{\phi}{1 - \frac{\phi}{x} (1 - 2\pi_2^W)}.
\]

This expression is always positive for \( \phi \leq 1 \), i.e., the function \( y \) is increasing in this range.

So, we see that the function \( y \) increases and then decreases. It is easy to see that the maximum is attained at \( \alpha_2^W = \pi_1^W + \frac{\phi}{x} (1 - \pi_1^W) \). So, the winning group will choose this split of resources. This in turn implies that there will be no switching in equilibrium in period 2, i.e., \( \pi_2^W = \pi_1^W \). It follows that the equilibrium per-capita stage-game payoff in period 2 to members of group \( W_2 \) will be \( x + \phi (1 - \pi_1^W) \). The per capita payoff to the losing group \( L_2 \) will be \( x - \phi \pi_1^W \) respectively. \( \square \)

### 3.2 Equilibrium Play in the First Period

Without loss of generality, suppose group \( A \) was elected as the winning group at the start of the game, i.e., \( W_1 = A \). Recall that the initial size of group \( A \) was \( \pi_0^A \). Let \( \pi_1^A \) denote the size of group \( A \) that will be realized at the end of period 1 after switching decisions of period 1 have been taken. Group \( A \) must choose an optimal allocation of resources \( \alpha_1^A \). Once the allocation is announced, play will either proceed along the path of conflict, or along the path of economic activity in period 1. Let \( E_A(\alpha_1^A, \pi_1^A) \) and \( E_B(\alpha_1^A, \pi_1^A) \) denote the per capita payoffs to members in group \( A \) and \( B \) respectively, when play proceeds along the path of economic activity (i.e., the opposition accepts the allocation announced by the ruling group), given allocation \( \alpha_1^A \) and new group size \( \pi_1^A \). Similarly, let \( P_A(\alpha_1^A, \pi_0^A) \) and \( P_B(\alpha_1^A, \pi_0^A) \) denote the per capita payoffs to members in group \( A \) and \( B \) respectively, when play proceeds along the path of conflict (i.e., the opposition rejects the allocation announced by the ruling group and engages in conflict), given \( \alpha_1^A \) and \( \pi_0^A \).

#### 3.2.1 Switching Decision in Period 1

Below, we characterize the size of group \( A \) after players have taken switching decisions in period 1 (conditional on choosing economic activity), for any allocation \( \alpha_1^A \) and initial group size \( \pi_0^A \):

**Lemma 2.** Suppose \( A \) is the incumbent group in period 1 with size \( \pi_0^A \). If the announced allocation
Similarly, the per capita payoff of group B's decisions. For any allocation \( \pi \) of others. Let \( f \) where functions \( A \) switch to group \( E \). Consider the decision of an individual to switch groups or not at the end of period 1. An individual will switch if his expected payoff from switching to the other group is higher than that of staying in his own, and in equilibrium players will perfectly anticipate the switching decisions of others. Let \( \pi^A \) denote the new size of group A realized after players have taken their switching decisions. For any allocation \( \alpha^A \) that group A can choose, conditional on group B choosing to undertake economic activity, groups A’s expected per capita payoff will be as follows.

\[
E_A(\alpha^A, \pi^A) = \frac{\alpha^A}{\pi_2^1} + p_A(\pi^A) + (1 - p_A(\pi^A)) (x - \phi(1 - \pi_2^1))
\]

\[
= \frac{\alpha^A}{\pi_2^1} + x + \phi(1 - \pi_2^1)(2p_A(\pi^A) - 1).
\]

Similarly, the per capita payoff of group B if it accepts the allocation is

\[
E_B(\alpha^A, \pi^A) = \frac{1 - \alpha^A}{1 - \pi_2^1} + (1 - p_A(\pi^A)) (x + \phi\pi_2^1) + p_A(\pi^A) (x - \phi\pi_2^1)
\]

\[
= \frac{1 - \alpha^A}{1 - \pi_2^1} + x + \phi\pi_2^1 (1 - 2p_A(\pi^A)).
\]

Now, for any \( \pi^A \) and any announced allocation \( \alpha^A \), it is optimal for a member of group B to switch to group A if and only if

\[
E_A(\alpha^A, \pi^A) - \phi > E_B(\alpha^A, \pi^A)
\]

\[
\iff \frac{\alpha^A}{\pi_2^1} + x + \phi(1 - \pi_2^1)(2p_A(\pi^A) - 1) - \phi > \frac{1 - \alpha^A}{1 - \pi_2^1} + x + \phi\pi_2^1 (1 - 2p_A(\pi^A))
\]

\[
\iff \frac{\alpha^A}{\pi_2^1} - \frac{1 - \alpha^A}{1 - \pi_2^1} > \phi - \phi\pi_2^1 (2p_A(\pi^A) - 1) - \phi (1 - \pi_2^1) (2p_A(\pi^A) - 1)
\]

\[
\iff \frac{\alpha^A}{\pi_2^1} > \frac{1 - \alpha^A}{1 - \pi_2^1} > 2\phi (1 - p_A(\pi^A))
\]

\[
\iff \alpha^A > \pi_2^1 + 2\phi\pi_2^1 (1 - \pi_2^1) (1 - p_A(\pi^A)).
\]
Similarly, for any $\pi^A_1$ and $\alpha^A_1$, it is optimal for a member of group $A$ to switch to $B$ if and only if
\[
E_B(\alpha^A_1, \pi^A_1) - \phi > E_A(\alpha^A_1, \pi^A_1) \iff \alpha^A_1 < \pi^A_1 - 2\phi(\pi^A_1)(1 - \pi^A_1)p_d(\pi^A_1).
\]

It is easy to see that if the announced share $\alpha^A_1$ is such that $\alpha^A_1 \in [g_1(\pi^A_0), f_1(\pi^A_0)]$, then no one switches and we have $\pi^A_1 = \pi^B_0$. If $\alpha^A_1 > f_1(\pi^A_0)$, then members of group $B$ have an incentive to switch. Individuals from group $B$ start switching to $A$, making the size of group $A$ larger. In equilibrium, individuals perfectly anticipate the switching decisions of others, and so switching happens until $A$ reaches a threshold size beyond which any further switching from $B$ to $A$ is not optimal. (Note that if members in any group have an incentive to switch, then in equilibrium it must be the case that the payoff of players who switch is the same as that of those who do not switch.) This implies that $\pi_1 = f^{-1}_1(\alpha^1_1)^4$. A similar argument applies if $\alpha^A_1 < g_1(\pi^A_1)$. In this case, individuals switch from group $A$ to $B$, and $\pi^A_1 = g^{-1}_1(\alpha^1_1)$.

### 3.2.2 Preferences for Conflict in Period 1

Above we characterized switching decisions of agents conditional on the opposition choosing economic activity. Next, we characterize each group’s preferences over conflict and economic activity. In period 1, after observing $\alpha^A_1$, group $B$ has to decide whether to accept this division of resources or to engage in conflict. Group $B$ will choose conflict if and only if its expected payoff from conflict is higher than that from accepting the split. As before, we maintain the assumption that at the start of the game, group $A$ is the winning group of size $\pi^A_0$. Any choice of allocation by group $A$ will either induce play to proceed along the path of conflict or economic activity.

**Definition 1 (Feasibility).**

Let $E := \{\alpha : E_B(\alpha, \pi^A_1(\alpha, \pi^A_0)) \geq P_B(\alpha, \pi^A_0)\}$

Let $P := \{\alpha : E_B(\alpha, \pi^A_1(\alpha, \pi^A_0)) < P_B(\alpha, \pi^A_0)\}$

---

It follows from our assumptions on $p_d(\cdot)$ that $f_1$ and $g_1$ are increasing functions. Note that for $g_1(\pi)$ to be strictly increasing for all $\phi \in (0, 1)$, we need
\[
\frac{d}{d\pi} [\pi(1 - \pi)p_d(\pi)] < \frac{1}{2\phi}.
\]

By assumption, we have $\frac{d}{d\pi} [\pi(1 - \pi)p_d(\pi)] < \frac{1}{\phi}$, and so if $\phi \in (0, 1)$, we have $g_1$ is increasing. Similarly, the condition for $f_1(\pi)$ to be strictly increasing is
\[
\frac{d}{d\pi} [\pi(1 - \pi)p_d(1 - \pi)] > -\frac{1}{2\phi}.
\]

The assumption $\frac{d}{d\pi} [\pi(1 - \pi)p_d(\pi)] < \frac{1}{\phi}$ and the symmetry of $p_d$ imply that this condition is satisfied.
We say an allocation \( \alpha \) is feasible along the path of economic activity if \( \alpha \in E \). We say an allocation \( \alpha \) is feasible along the path of conflict if \( \alpha \in P \).

Notice that \( E \cup P = [0, 1] \) and \( E \cap P = \emptyset \). So feasibility on each path is well-defined. If group \( A \) announces an allocation \( \alpha \in E \), then group \( B \) will accept it. Likewise, if \( A \) announces an allocation \( \alpha \in P \), then group \( B \) will reject it and engage in conflict.

**Proposition 2 (Characterizing Allocations Feasible Along Conflict Path).** For any given \( \pi_0 \), there exists a threshold cost of mobility \( \phi_1 \in [0, 1] \) such that

i) for \( \phi \leq \phi_1 \), the opposition (group \( B \)) accepts any allocation \( \alpha \in [0, 1] \) proposed by the incumbent (group \( A \)).

ii) for \( \phi > \phi_1 \), there exists a threshold allocation \( \bar{\alpha} \in (0, 1] \) such that the opposition accepts any allocation \( \alpha < \bar{\alpha} \), and rejects any allocation \( \alpha > \bar{\alpha} \) (and is indifferent between accepting and rejecting \( \alpha = \bar{\alpha} \)).

Accordingly, we define the following.

**Definition 2 (Opposition’s Conflict Threshold).** The threshold cost of mobility \( \phi_1 \) identified in Proposition 2 above is called the opposition’s political threshold.

The above proposition characterizes the set of allocations feasible along the path of conflict. In particular, it says that if the cost of inter-group mobility is low enough, any offer made by the incumbent will be accepted by the opposition in period 1 (i.e. \( P = \emptyset \)). When the cost of mobility is so low, the premium from gaining power in period 2 is not large enough for the opposition to give up the benefit from economic activity. If on the other hand, the cost of mobility is larger than the threshold \( \phi_1 \), then only “high enough” offers (those that leave more than \( 1 - \bar{\alpha} \) for the opposition) will be accepted, i.e. \( P = (0, \bar{\alpha}) \). The premium from gaining power in the second period is high enough so that the incumbent has to provide higher incentives for economic activity for the opposition to accept.

**Proof.** Please refer to the appendix for a proof of the above proposition.

Next, we turn to the preferences of the incumbent, and find conditions under which the incumbent prefers to induce conflict (or economic activity). We introduce some notation: \( \alpha^e \) is the most preferred offer of the incumbent if it knew that any offer it made would be accepted and \( \alpha^p \) is the most preferred offer if it knew that any offer it made would be rejected.

**Definition 3.** Define \( \alpha^e \) and \( \alpha^p \) as follows:

\[
\alpha^e = \arg\max_{\alpha^A} E_A (\alpha^A, \pi^A(\alpha^A, \pi^A_0))
\]

\[
\alpha^p = \arg\max_{\alpha^A} P_A (\alpha^A, \pi^A_0)
\]
Lemma 3. The incumbent's maximal possible payoffs along the paths of economic activity and conflict are as follows:

\[ \alpha^e = f_1(\pi_0^A). \]

\[ \alpha^p = 1. \]

Lemma 3 characterizes the maximal payoffs possible along the economic path and political path respectively. If the incumbent knew that his announced offer would be accepted (i.e., feasible on the economic path) then it would retain the maximal surplus possible without attracting switchers and consequent dilution of per capita payoffs. On the other hand, if the incumbent knew that its offer would be rejected (and conflict would arise) then it would expropriate all surplus. Note that this lemma does not characterize equilibrium payoffs. In particular, it does not say that an offer of \( \alpha^e \) will be accepted by the opposition.

Proof of Lemma 3. Recall that \( P^A(\alpha^1_A, \pi^A_0) = \frac{k\alpha^A_1}{\pi^A_0} + x + \phi(1 - \pi^A_1)(2p_d(\pi^A_1) - 1) \). Since, \( P_A \) is increasing in \( \alpha^A_1 \) (for \( k > 0 \)), it is maximized at \( \alpha^A_1 = 1 \). We next show that \( \alpha^e = f_1(\pi^A_0) \). We first show that \( E_A \) is increasing over the range \( \{\alpha : \alpha \leq g(\pi^A_0)\} \). Consider \( \alpha^A_1 < g_1(\pi^A_0) \). We know that this would induce switching from group A to B according to the rule we derived in Section 3.2.1, i.e. the new size of group A would be \( \pi^A_1 = g^{-1}(\alpha^A_1) \). So we have,

\[
E_A(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) = \frac{\alpha^A_1}{\pi^A_1} + x + \phi(1 - \pi^A_1)(2p_d(\pi^A_1) - 1)
\]

\[
= \frac{\pi^A_1 - 2\phi\pi^A_1(1 - \pi^A_1)p_d(\pi^A_1)}{\pi^A_1} + x + \phi(1 - \pi^A_1)(2p_d(\pi^A_1) - 1)
\]

\[ = 1 + x - \phi(1 - \pi^A_1), \]

which is increasing in \( \pi^A_1 \). We know that \( g \) is an increasing function, and so \( \pi^A_1 = g^{-1}(\alpha^A_1) \) is an increasing function of \( \alpha^A_1 \). It follows that \( E_A(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) \) is increasing in \( \alpha^A_1 \).

We next show that \( E_A \) also increases over the interval \( \alpha^A_1 \in [g_1(\pi^A_0), f_1(\pi^A_0)] \). We know that for allocations \( \alpha^A_1 \) in this range, no switching occurs and \( \pi^A_1(\alpha^A_1, \pi^A_0) = \pi^A_0 \). Now,

\[ E_A(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) = \frac{\alpha^A_1}{\pi^A_0} + x + \phi(1 - \pi^A_0)(2p_d(\pi^A_0) - 1) \]

which clearly is increasing in \( \alpha \).

Finally, we show that \( E_A \) is decreasing over the range \( \{\alpha : \alpha \geq f_1(\pi^A_0)\} \). Consider \( \alpha^A_1 > f_1(\pi^A_0) \). We know that this would induce switching from group B to A and the new size of group
A would be $\pi_1^A = f^{-1}(\alpha_1^A)$. So we have,

$$E_A(\alpha_1^A, \pi_1^A(\alpha_1^A, \pi_0^A)) = \frac{\alpha_1^A}{\pi_1^A} + x + \phi(1 - \pi_1^A)(2p_d(\pi_1^A) - 1)$$
$$= \frac{\pi_1^A + 2\phi\pi_1^A(1 - \pi_1^A)(1 - p_d(\pi_1^A))}{\pi_1^A} + x + \phi(1 - \pi_1^A)(2\pi_1^A - 1)$$
$$= 1 + x + \phi(1 - \pi_1^A),$$

which is decreasing in $\pi_1^A$. Since $f_1$ is an increasing function, $f^{-1}(\alpha_1^A)$ is an increasing function of $\alpha_1^A$. So, $E_A(\alpha_1^A, \pi_1^A(\alpha_1^A, \pi_0^A))$ is decreasing in $\alpha_1^A$ in this range. It follows immediately, that the function $E_A$ is maximized at $\alpha_1^A = f_1(\pi_0^A)$.

We want to understand the conditions under which the incumbent would always prefer to induce economic activity rather than induce conflict. Notice from the lemma above, that the maximal surplus for the incumbent group is increasing in $\phi$ if the allocation proposed is $\alpha^e$. However, it does not directly depend on $\phi$ when $\alpha^p$ is proposed. This indicates that for large enough $\phi$, we should expect that the incumbent would prefer to avoid conflict. The next lemma below confirms this intuition. We show that if the cost of mobility is above a certain threshold, then the maximal payoff that the incumbent can get by inducing the opposition to choose economic activity is higher than the maximal payoff he can get by inducing the opposition to choose conflict.

**Lemma 4.** There exists a threshold cost of mobility $\phi_2 \in [0, 1]$ such that

$$\phi \geq \phi_2 \iff E_A(\alpha^e, \pi_1^A(\alpha^e, \pi_0^A)) - P_A(\alpha^p, \pi_0^A) \geq 0.$$  

**Proof.** For any allocation of resources $\alpha_1^A$ that group $A$ can choose, if opposition group $B$ rejects the split and engages in conflict, then group $A$’s expected per capita payoff will be as follows.

$$P_A(\alpha_1^A, \pi_0^A) = \frac{k\alpha_1^A}{\pi_0^A} + p_c(\pi_0^A)(x + \phi(1 - \pi_0^A)) + (1 - p_c(\pi_0^A))(x - \phi(1 - \pi_0^A))$$
$$= \frac{k\alpha_1^A}{\pi_0^A} + x + \phi(1 - \pi_0^A)(2p_c(\pi_0^A) - 1).$$

Recall also that

$$E_A(\alpha^e, \pi_1^A(\alpha^e, \pi_0^A)) = \frac{\alpha^e}{\pi_0^A} + x + \phi(1 - \pi_0^A)(2p_d(\pi_1^A) - 1).$$
So, we have

\[ E_A(\alpha_e, \pi_1^A) - P_A(\alpha_e, \pi_0^A) \geq 0 \]
\[ \iff \alpha_e \geq \frac{1}{\pi_0^A} x + \phi(1 - \pi_0^A)(2p_d(\pi_1^A) - 1) - k\phi \]
\[ \iff \phi \geq \frac{(k - \pi_0^A)}{2\pi_0^A(1 - p_c(\pi_0^A))} := \phi_2. \]

Accordingly, we define the following.

**Definition 4 (Incumbent's Conflict Threshold).** The threshold cost of mobility \( \phi_2 \) identified in Lemma 4 above is called the incumbent’s conflict threshold.

The lemma above implies that if \( \phi \geq \phi_2 \), the incumbent will induce economic activity (and get the maximal payoff \( \alpha^{e} \)) if this is feasible. The next result characterizes the conditions under which the allocation \( \alpha^{e} \) is indeed feasible along the path of economic activity.

**Lemma 5.** There exists a threshold cost of mobility \( \phi_3 \) such that

\[ \alpha^{e} \in E \iff \phi \leq \phi_3. \]

Moreover, for any \( \phi \) such that \( \phi_1 < \phi \leq \phi_3 \), we have \( \alpha^{e} \leq \bar{\alpha} \).

**Proof.** Recall that \( \alpha^{e} = f_1(\pi_0^A) \) and at this allocation choice, no switching would take place and so \( \pi_1^A = \pi_0^A \). Hence, we have

\[ \alpha^{e} \in E \iff E_B(\alpha^{e}, \pi_0^A) \geq P_B(\alpha^{e}, \pi_0^A) \]
\[ \iff E_B(\alpha^{e}, \pi_0^A) \geq P_B(\alpha^{p}, \pi_0^A) \]
\[ \iff \frac{E_A(\alpha^{e}, \pi_0^A)}{(1 + x) - \pi_0^A} \geq \frac{P_A(\alpha^{e}, \pi_0^A)}{(1 - \pi_0^A)} \]
\[ \iff E_A(\alpha^{e}, \pi_0^A) - P_A(\alpha^{p}, \pi_0^A) \leq \frac{1 - k}{\pi_0^A} \]
\[ \iff 1 + 2\phi(1 - \pi_0^A)(1 - p_d(\pi_0^A)) + \phi(1 - \pi_0^A)(2p_d(\pi_0^A) - 1) \]
\[ \leq \frac{1 - k}{\pi_0^A} \]
\[ \iff \phi \leq \frac{1}{2\pi_0^A(1 - p_c(\pi_0^A))} := \phi_3. \]

Now note that the threshold \( \phi_3 \) is always (weakly) greater than the threshold \( \phi_1 \) defined in Proposition 2. So, if \( \phi \in (\phi_1, \phi_3] \), then \( P = (\bar{\alpha}, 1] \). Since, \( \alpha^{e} \in E \), it is immediate that \( \alpha^{e} \leq \bar{\alpha} \).

### 3.2.3 Incumbent's Allocation Choice in Period 1

The next proposition is the main result of the paper and describes the equilibrium resource allocations. It turns out that the equilibrium resource allocation is a non-monotonic function of the cost.
of inter-group mobility. Recall the three thresholds $\phi_1$, $\phi_2$ and $\phi_3$.

$$
\phi_1 = \frac{1}{1+\pi_0^A(1-2p_c(\pi_0^A))} \quad \phi_2 = \frac{k-\pi_0^A}{1-\pi_0^A \frac{1}{2\pi_0^A(1-p_c(\pi_0^A))}} \quad \phi_3 = \frac{1}{2\pi_0^A(1-p_c(\pi_0^A))}
$$

It is easy to see that $\phi_1 \leq \phi_3$ and $\phi_2 \leq \phi_3$. Notice that $k$ appears only in $\phi_2$, and $\phi_2$ is increasing in $k$. Recall that $\phi_2$ is the threshold above which the incumbent prefers not to induce conflict. Intuitively, if $k$ is low then conflict is more costly to the incumbent. This means that the incumbent will prefer not to induce conflict over a larger range of $\phi$ or simply $\phi_2$ is low. In fact, $\phi_2 - \phi_1$ can be positive or negative depending on the value of $k$. In particular, $\phi_2$ is greater (less) than $\phi_1$ if and only if $k$, the fraction of economic surplus retained by the incumbent group when the opposition engages in conflict, is greater (less) than some number $K(\pi_0) \in (0,1)$. It is easy to check that

$$
K(\pi_0) = \pi_0 + \frac{2\pi_0(1-\pi_0)(1-p_c(\pi_0))}{1+\pi_0(1-2p_c(\pi_0))}.
$$

**Proposition 3 (Resource Sharing in Equilibrium).** Suppose $A$ is the incumbent group in period 1 with size $\pi_0^A$, and when the opposition engages in conflict, the incumbent retains a share $k$ of the economic payoff. The equilibrium choice of allocation $\alpha_1^*$ is characterized below.

i) Case 1: If $k > K(\pi_0)$, we have $\phi_1 < \phi_2 < \phi_3$.

- If $\phi \leq \phi_1$, then $\alpha_1^* = \alpha^e$: We call this the “no conflict” range. The incumbent shares enough resources to prevent switching from the opposition.
- If $\phi \in (\phi_1, \phi_2)$ then $\alpha_1^* = 1$. We call this the “open conflict” range. In this range, the incumbent prefers to allow conflict and expropriates all resources.
- If $\phi \in [\phi_2, \phi_3]$ then $\alpha_1^* = \alpha_1^e$. This is another “no conflict” range. In this range, the incumbent again shares enough resources to prevent switching from the opposition.
- If $\phi > \phi_3$, then $\alpha_1^* = \alpha$. If cost of mobility is very high, the incumbent prefers not to induce conflict, and therefore shares just enough resources to prevent conflict. We call this the “peaceful belligerence” range.

ii) Case 2: If $k \leq K(\pi_0)$, we have $\phi_2 \leq \phi_1 < \phi_3$.

- If $\phi \leq \phi_3$, then $\alpha_1^* = \alpha^e$.
- If $\phi > \phi_3$, then $\alpha_1^* = \alpha$.

The interested reader may refer to the appendix for the proof of Proposition 3. Below, we describe the main intuition behind the proof.

There are two different reasons why the incumbent may want to share resources with the opposition. First, if the incumbent keeps too much surplus for itself, it may attract switchers from the opposition which would reduce the per capita share for the original members of the ruling group:
thus the incumbent might want to share in order to prevent switching. Second, the ruling group might also want to share resources with the opposition so that economic activity is sufficiently attractive for the opposition, and they do not engage in conflict. Put differently, there are two constraints on expropriation by the incumbent - the switching constraint and the conflict constraint, and the optimal sharing is dictated by whether and which constraint binds.

When the cost of switching is very low, i.e. $\phi < \phi_1$, we know from proposition 2 that the opposition will accept any offer, i.e. the conflict constraint does not restrain the choices of the incumbent. The incumbent is only concerned about switching and offers $\alpha^e$. To follow the optimal choice for $\phi \geq \phi_1$, it would be instructive to first see the role of the other two thresholds $\phi_2$ and $\phi_3$. As the cost of mobility increases, conflict becomes more costly for the incumbent, and at the same time, the payoff obtainable by just preventing switching increases. $\phi_2$ is the precise threshold below which allowing conflict by extracting all surplus is more attractive to the incumbent than offering just enough to prevent switching. In other words, provided $\alpha^e \in E$ and $\{\alpha = 1\} \in P$, the incumbent would prefers $\alpha^e$ for $\phi \geq \phi_2$ and $\alpha^p$ for $\phi < \phi_2$. Therefore, if $\phi \in (\phi_1, \phi_2)$, then the optimal choice is $\alpha^p = 1$, and we observe actual conflict in equilibrium.

For $\phi > \phi_2$ the incumbent wants to get $\alpha^e$, but this may not always be feasible in equilibrium. In particular, if $\phi$ is very high, $\alpha^e$ may be so high that it leaves too little for the opposition, and at the same time, the benefit from conflict is also high for the opposition. Precisely, $\alpha^e$ is feasible on the economic path if and only if $\phi \leq \phi_3$. In other words, if $\phi > \phi_3$, then it is the political action constraint that binds the incumbent. The optimal choice of the incumbent is $\tilde{\alpha}$ which is the amount the incumbent must share to prevent conflict. Switching is anyway prevented at this offer. On the other hand, if $\phi < \phi_3$, then $\alpha^e < \tilde{\alpha}$, and the amount that has to be shared to prevent switching is enough to avoid conflict. Thus, for the range $(\max\{\phi_1, \phi_2\}, \phi_3)$, the optimal choice is $\alpha^e$, where the switching constraint binds.

### 3.2.4 Discussion and Empirical Implications

The proposition characterizes the extent of resource sharing (or exploitation) by the incumbent group in equilibrium for different levels of the cost of mobility across groups. The optimal choice of $\alpha$ takes three values ($\alpha^e$, $\tilde{\alpha}$ and $\alpha^p = 1$) for three different parameter ranges.

**Remark 1. Resource Sharing Non-monotonic in $\phi$:** Notice that the proposition predicts a non-monotonic relationship between the extent of expropriation and the cost of inter-group mobility. When the optimal choice is $\tilde{\alpha}$, i.e. the political constraint determines the choice of $\alpha$, there is less expropriation as the cost of mobility increases. Since the premium from gaining power in the second period is increasing in the cost of mobility, the incumbent has to share more in the current period to keep the opposition indifferent between economic and political activity as $\phi$ increases. On the
other hand, when the optimal choice is $\alpha^e$, i.e. determined by the switching constraint, it is easy to see that the incumbent can expropriate more and still prevent switching as the cost of mobility increases.

**Remark 2. Empirical Predictions:** Our model yields several empirical predictions. Conflict will not arise in equilibrium if the cost of mobility between groups is extremely low (below $\phi_1$). The incumbent will share enough resources to prevent switching ($\alpha^e$). In an intermediate range of cost of mobility (in the range $(\phi_1, \phi_2)$), we can find complete expropriation of resources by the incumbent and conflict by the opposition. At higher costs of mobility, we will again find sharing by the incumbent. Finally if the cost of mobility is extremely high (above $\phi_3$), the incumbent will share enough resources to just prevent conflict. In this range, while conflict does not occur the opposition is exactly indifferent between conflict and economic activity. In this sense, this is a region of peaceful belligerence - the threat of conflict by the opposition forces the incumbent to share.

Another prediction is that when conflict is very costly ($k$ small enough) for the incumbent, then we should not observe the incumbent expropriating all resources and inducing conflict. Instead, we should observe some resource sharing by the incumbent (enough to just prevent conflict or switching, depending on the cost of mobility).

Note that since $\phi \in (0, 1)$, some of the intervals indicated in the proposition may be vacuous. In particular, to have $\phi_3 < 1$, we need $\pi_0^A(1 - p_c(\pi_0^A)) > \frac{1}{2}$. Therefore, necessary conditions for the political constraint to bind are that $\pi_0^A > \frac{1}{2}$, i.e. a majority incumbent, and $p_c(\pi_0^A) < \frac{1}{2}$. By the symmetry assumption, we have $p_d(\pi_0^A) > \frac{1}{2}$. So, our model predicts that the political constraint will be observed to be binding in equilibrium only if the incumbent is a majority group, and the threat of political action by the minority is strong (in the sense that while under no conflict the majority is more likely to retain authority in the next period, a conflict would make the minority more likely to win power in the next period). In other words, in practice some resource sharing (as opposed to complete expropriation) will be observed in a situation of conflict only when the incumbent is a majority and the minority poses a strong threat of conflict. On the other hand, in situations of conflict with a minority incumbent we should observe complete expropriation of resources.

Our predictions are consistent with some stylized facts that cannot be explained by earlier models. For instance, Miquel (2007) points out that existing models cannot explain why in some autocratic regimes (like Houphouet-Boigny in Ivory Coast) rulers actually transfer resources to the opposition ethnic groups. In our model this could be explained by the peaceful belligerence region (where cost of mobility is very high, the opposition’s opportunity cost of conflict is low, the ruler wants to avoid conflict.)

Caselli and Coleman (2006) show that conflict does not occur when switching group identities is easy since it is anticipated that the winning coalition would expand$^5$. They suggest that higher

$^5$However, in [3], the objective of conflict is to have complete control of resources. The question of resource sharing
switching costs obtain for members of groups that are easily distinguished (skin color or other physical features). However there are many counterexamples [4] where intense conflict arises between groups whose members can't be reliably distinguished. While our model confirms that conflict does not arise when cost of mobility is extremely low, we can explain why conflict can still arise even when cost of mobility is not high.

**Remark 3. Intra-group Structure:** Here, we treat all members of a group uniformly in the sense that resources are shared equally by all members. We make this modeling choice in order to focus on inter-group incentives. An interesting extension would be to incorporate some organizational structure within the groups, and then ask whether intra-group hierarchy and competition affects inter-group sharing of resources. For example, each group may have a hierarchical structure with a elite and a non-elite such that the elite gets a higher share of the group's resources. Notice that this would change the opportunity costs of conflict for each sub-group. Another example of group structure could be that new members (switchers) are treated differently from original members. This would change the payoffs from switching for the sub-groups. A systematic investigation is beyond the scope of this paper.

**Remark 4. Switching with Heterogeneous Costs:** While the proposition predicts that there will be no switching in equilibrium, this is an artifact of uniform switching costs. Switching would be observed in equilibrium if there was some noise or heterogeneity in costs.

## 4 An Example

In the previous section, we considered general contest functions $p_d(\pi)$ (under democracy) and $p_c(\pi)$ (under conflict) satisfying certain regularity conditions. As mentioned earlier, these regularity conditions are satisfied in many common contest functions like proportional representation. In this section, we present an example with specific contest functions to illustrate the main results of the paper. Consider the following contest function under democracy.

\[
p_d(\pi) = \frac{e^{b(\pi - \frac{1}{2})}}{1 + e^{b(\pi - \frac{1}{2})}}, \quad b > 0
\]

A version of this function was first introduced in Hirshleifer (1989), and is used in a range of contexts. This function has the property that the difference in group sizes (resources) rather than the ratio of group sizes matters. It is an S-shaped function with the point of inflexion at $\pi = \frac{1}{2}$. The parameter $b$ determines the steepness at the point of inflexion, and thus measures an institutional does not arise in their model. Further, in their model there is no reason for a group to increase its membership. The ruling elite never wants to increase its size. 

21
feature: how much political advantage the majority enjoys in terms of probability of reelection. In particular, when $b$ approaches infinity, $p_d(\cdot)$ approximates a step function that takes value 0 for $\pi < \frac{1}{2}$ and 1 for $\pi > \frac{1}{2}$, which is the case when we have the winner elected deterministically by a majoritarian election. On the other hand, as $b$ approaches 0, the winning group is randomly selected irrespective of its size. In a first-past-the-post set-up like the one in United States, one would expect $b$ to be high, while in a system with proportional representation like in Germany, or in a multi-party democracy like in India, one would expect $b$ to have a lower value.

The function is plotted for different values of $b$ in Figure 2. It is easy to check that this function satisfies all the restrictions on $p_d(\pi)$ for all $b > 0^6$.

![Figure 2: Political Contest Function Under Democracy](image)

Suppose the re-election probability under conflict is given by

$$p_c(\pi) = p_d(\pi) \cdot \pi^c, \ c > 0.$$

Here, the parameter $c$ measures the effectiveness of the conflict waged by the opposition in reducing the reelection probability of the incumbent. Holding group size $\pi$ fixed, $p_c(\pi)$ is decreasing in $c$. In particular, at $c = 0$, $p_c(\pi) = p_d(\pi)$, i.e. conflict has no effect, and on the other hand at $c = \infty$, $p_c(\pi) = 0$, i.e. conflict ensures that the opposition captures political power. Also, holding $c$ fixed, the incumbent’s probability of reelection under conflict increases with its size.

When the incumbent offers the opposition too low a share, the opposition punishes the incumbent by launching political conflict. Such punishment works through two distinct channels. First,

---

6To check the bounded derivatives condition, notice that the function $h(\pi) = \pi(1 - \pi)\frac{e^{(\pi - \frac{1}{2})}}{1 + e^{(\pi - \frac{1}{2})}}$ has a decreasing and well-defined derivative. This implies that the maximum value of the derivative occurs at $\pi = 0$, and the maximum value of the derivative at 0 is strictly less than $\frac{1}{2}$.
the incumbent is denied a share \((1 - k)\) of its economic surplus. The parameter \(k\) thus measures the extent of destructiveness of conflict. Second, the incumbent’s reelection probability goes down, and the extent of reduction is measured by the parameter \(c\). A low value of \(k\) and/or a high value of \(c\) are therefore strong threats to the incumbent, and induce higher sharing.

Below, we show how the equilibrium offer \(\alpha^*\) depends on the parameters \(\phi, \pi_0, k, b\) and \(c\).

4.1 Features of Equilibrium

- **Equilibrium Regimes:** One of three regimes can occur in equilibrium. Figure ?? depicts the three different regimes corresponding to each type in \((\phi, \pi_0)\) space.

![Equilibrium Sharing Regimes](image.png)

Figure 3: Equilibrium Sharing Regimes

First, notice that ”peaceful beligerence” occurs only for high values of both \(\pi\) and \(\phi\). In other words, sharing is driven by threat of conflict only if the incumbent is a majority and mobility is highly restricted. Open conflict, on the other hand, occurs when the incumbent has a smaller group size in general, but mobility is still high. Therefore, our model suggests that in a racially divided society, if the majority group assumes power, then it will share some spoils with the minority to keep it from engaging in conflict, but if the minority is in power, then it will likely extract all surplus and get into open conflict. If, however, the cost of mobility is low enough, then we are in the large no-conflict zone irrespective of the size of the ruling group. Thus, social mobility helps reduce intergroup conflict, which is reminiscent of the main result in Caselli and Coleman (2010).

- **Extent of Sharing:** Next, we look at how the extent of sharing/exploitation changes with the cost of mobility. Note that in each of the three zones, the relationship between \(\alpha^*\) and \(\phi\) is different. In the no-conflict zone, the ruling group keeps for itself just enough to prevent
switching. So, as switching becomes more costly, the incumbent can keep more for itself, i.e. $\alpha^*$ is increasing in $\phi$. In the "peaceful belligerence” zone, $\alpha^*$ is the maximum that the incumbent can keep without provoking conflict. An increase in $\phi$ raises the premium from winning political power, and thus enhances the incentive for conflict. The opposition has to be offered more to be prevented from engaging in conflict, and, $\alpha^*$ is decreasing in $\phi$ in the peaceful belligerence zone. Lastly, since there is full extraction in the "open conflict” zone, $\alpha^*$ is independent of $\phi$.

**FIGURE 3a, 3b, 3c HERE**

Figures 3a, 3b and 3c show how $\alpha^*$ changes with $\phi$ for different values of incumbent group size $\pi_0$. For a small sized incumbent group (e.g. $\pi_0 = ?$), for low values of $\phi$ we have no-conflict and for high values open conflict: thus we expect to have a weakly increasing relationship between the amount the incumbent reserves for itself and the cost of mobility (see figure 3a). For a moderately large sized incumbent (e.g. $\pi_0 = ?$), there is open conflict for moderate values of $\phi$, and no conflict for lower and higher values. Thus, $\alpha^*$ is increasing except for an interval in the middle where it is flat and takes the maximum value. For a moderately large sized incumbent (e.g. $\pi_0 = ?$), there is open conflict for moderate values of $\phi$, and peaceful belligerence for very high values of $\phi$ (see figure 3b). For all other values, there is no conflict. Thus, $\alpha^*$ has an interesting non-monotonic pattern against $\phi$: it is first increasing, then flat at the maximum value of 1 for an interval, then increases following a discontinuous drop, and finally starts decreasing. For a very large sized incumbent (e.g. $\pi_0 = ?$), the open conflict zone vanishes, and the extent of sharing and cost of mobility have a U-shaped relationship (see figure 3c).

- **Incumbent's Cost of Conflict**: The parameter $k$ measures the extent to which the incumbent retains its economic payoff even under conflict. As $k$ decreases, i.e. conflict becomes more destructive, the incumbent’s incentive to allow conflict goes down. In the simulations, we find that with a drop in $k$, the open conflict zone becomes smaller while the peaceful belligerence zone remains the same. This is demonstrated by Figure ???. In fact, it is easy to see that there can be no open conflict if $k < K(\pi_0)$. Since within a regime $\alpha^*$ is independent of $k$, the only effect of a drop in $k$ is the replacement of part or all of the open conflict zone by no conflict. Therefore, as conflict becomes more destructive for the incumbent, there is a reduction in conflict and a weak increase in sharing of resources.

- **Effectiveness of Conflict**: The parameter $c$ measures the extent to which political conflict increases the probability of the opposition to win power.

24
In our simulations (Figure 5), we find that an increase in the effectiveness of conflict reduces both the open conflict and peaceful belligerence zones. In fact, for a large enough $c$, the peaceful belligerence zone vanishes. Also, it is easy to see analytically that while $\alpha^c$ and $\alpha^P$ are independent of $c$, $\bar{\alpha}$ is decreasing in $c$. Thus, an increase in $c$ tends to bring about a reduction in conflict in the society and weakly increase the extent of sharing.

- **Political Advantage of Majority:** The parameter $b$ measures how much political advantage the majority enjoys. An increase in $b$ has opposite effects, depending on whether the incumbent is a majority or a minority.

![Figure 4: Equilibrium Regimes for Different Values of $k$](image)

![Figure 5: Equilibrium Regimes for Different $p_c(\pi)$](image)

25
5 Conclusion

The broad objective of this paper was to better understand the relationship between conflict and resource sharing between groups. One of the key insights from existing literature is that the potential of conflict or revolution can act as a constraint to how exploitative the ruling elite can be. Excessive resource extraction by the elite leaves a low share for the opposition, also reducing the opposition’s opportunity cost of engaging in conflict. If the outcome of conflict is very costly to the ruling group, its rent seeking behavior can be constrained. In this paper, we highlight a different and important force that affects redistribution of resources between groups. In many contexts, group sizes are determined endogenously. We ask how the option of inter-group mobility can affect how groups compete with each other and share resources? In particular, we investigate the combined effect of both factors - inter-group mobility as well as conflict - on how groups compete and study the interconnection between them. We look at situations where groups can collectively engage in conflict and individuals can each choose which group to belong to, and characterize how inter-group mobility affects the possibility of conflict and in turn the extent of resource sharing.

An important substantive question that arises now is what kind of groups would form if leaders could choose the basis for group formation. When would groups choose to form along ethnic lines (with high cost of mobility) and when would they form along ideological lines (relatively low cost of mobility)? This is the subject of future work.

Figure 6: Equilibrium Regimes for Different $b$
6 Appendix

6.1 Proof of Proposition 2

To characterize the set of allocations feasible along the path of conflict, we need to analyze the opposition’s preferences over political vs. economic activity, given any offer made by the incumbent. Formally, this reduces to comparing the functions $E_B$ and $P_B$.

$$E_B(\alpha^A_1, \pi^A_1) = \frac{1 - \alpha^A_1}{1 - \pi^A_1} + x + \phi\pi^A_1(1 - 2p_d(\pi^A_1)).$$

$$P_B(\alpha^A_1, \pi^A_0) = (1 - p_c(\pi^A_0))(x + \phi\pi^A_0) + p_c(\pi^A_0)(x - \phi\pi^A_0) = x + \phi\pi^A_0(1 - 2p_c(\pi^A_0)).$$

We show that $P$ is either empty or an interval. First we show that the function $E_B(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0))$ increases with $\alpha^A_1$ in the range $\alpha^A_1 < g_1(\pi^A_0)$ and then decreases after that.

Consider the range where $\alpha^A_1 < g_1(\pi^A_0)$. In this case, switching would occur along the path of economic activity, and $\pi^A_1 = g_1^{-1}(\alpha^A_1)$. Substituting for $\alpha^A_1 = g_1(\pi^A_1)$ in the above expression, we find $E_B(\alpha^A_1, \pi^A_1(\pi^A_0)) = 1 + x + \phi\pi^A_1$ which increases in $\pi^A_1$ and therefore also in $\alpha^A_1$. Now consider the range, $\alpha^A_1 \in [g_1(\pi^A_0), f_1(\pi^A_0)]$. In this range, no switching occurs ($\pi^A_0 = \pi^A_1$). So clearly, $E_B$ is decreasing in $\alpha^A_1$. Finally, when $\alpha^A_1 > f_1(\pi^A_0)$, switching would occur along the path of economic activity, and $\pi^A_1 = f_1^{-1}(\alpha^A_1)$. Substituting for $\alpha^A_1 = f_1(\pi^A_0)$ in the above expression, we find $E_B(\alpha^A_1, \pi^A_1(\pi^A_0)) = 1 + x - \phi\pi^A_1$ which decreases in $\pi^A_1$ and therefore also in $\alpha^A_1$.

Next, we compare the function $E_B(\alpha^A_1, \pi^A_1(\pi^A_0))$ with $P_B(\alpha^A_1, \pi^A_0)$ at $\alpha^A_1 = 0$. If $\alpha^A_1 = 0$, switching would occur from $B$ to $A$ and $\pi^A_1 = g_1^{-1}(0) = 0$. Consequently, $E_B(0, \pi^A_1(0, \pi^A_0)) = 1 + x$. Now, $P_B(\alpha^A_1, \pi^A_0)$ is a function independent of $\alpha^A_1$ and equals $x + \phi\pi^A_0(1 - 2p_c(\pi^A_0))$, which is clearly less than $1 + x$. At $\alpha^A_1 = 0$, $E_B$ is greater than $P_B$. Moreover we have just shown above that the function $E_B$ first increases and then decreases. This implies that $P_B$ intersects $E_B$ at at most one point. Two cases can arise.

i) First, $P_B$ is lower than $E_B$ in the entire range of $\alpha^A_1 \in [0, 1]$. In this case, $P = \emptyset$ since for any allocation $\alpha^A_1$, the payoff to group $B$ from accepting the split is higher than that from rejecting it.

ii) There exists a unique $\bar{\alpha}$ that solves $E_B(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) = P_B(\alpha^A_1, \pi^A_0)$. In this case, for all allocations $\alpha^A_1 \leq \bar{\alpha}$, group $B$ gets a higher payoff from accepting the split than from conflict, and for all $\alpha^A_1 > \bar{\alpha}$, $B$ prefers the conflict path.

We know that $P = \emptyset$ if and only if $P_B < E_B$, at $\alpha^A_1 = 1$. Since $P_B$ is independent of $\alpha^A_1$ and
is lower than \( E_B \) at \( \alpha_1^A = 0 \), it would follow immediately that for all \( \alpha_1^A, E_B > P_B \), and so \( P = \emptyset \). Suppose \( \alpha_1^A = 1 \).

Recall that for \( \alpha_1^A > f_1(\pi_0^A) \), we have \( \alpha_1^A = f_1(\pi_1^A) \). So as \( \alpha_1^A \to 1 \), \( \pi_1^A \) also goes to 1. Also,

\[
\lim_{\alpha_1^A \to 1} \frac{1 - \alpha_1^A}{1 - \pi_1^A} = \lim_{\alpha_1^A \to 1} \frac{1 - \pi_1^A - 2\phi\pi_1^A(1 - \pi_1^A)(1 - p_d(\pi_1^A))}{1 - \pi_1^A} = 1 - 2\phi(1 - p_d(1)).
\]

Now,

\[
\lim_{\alpha_1^A \to 1} E_B(\alpha_1^A, \pi_1^A(\alpha_1^A, \pi_0^A)) - P_B(\alpha_1^A, \pi_0^A) = \lim_{\alpha_1^A \to 1} \frac{1 - \alpha_1^A}{1 - \pi_1^A} + x + \phi\pi_1^A(1 - 2p_d(\pi_1^A)) - (x + \phi\pi_0^A(1 - 2p_c(\pi_0^A)))
\]

\[
= \lim_{\alpha_1^A \to 1} \frac{1 - \alpha_1^A}{1 - \pi_1^A} + \lim_{\pi_1^A \to 1} [\phi\pi_1^A(1 - 2p_d(\pi_1^A)) - \phi\pi_0^A(1 - 2p_c(\pi_0^A))]
\]

\[
= [1 - 2\phi(1 - p_d(1))] + \phi(1 - 2p_d(1)) - [\phi\pi_0^A(1 - 2p_c(\pi_0^A))]
\]

\[
= 1 - \phi(1 + \pi_0^A(1 - 2p_c(\pi_0^A)))
\]

When is this greater than zero?

\[
\lim_{\alpha_1^A \to 1} E_B(\alpha_1^A, \pi_1^A(\alpha_1^A, \pi_0^A)) - P_B(\alpha_1^A, \pi_0^A) \geq 0
\]

\[
\implies 1 - \phi(1 + \pi_0^A(1 - 2p_c(\pi_0^A))) \geq 0
\]

\[
\implies \phi \leq \frac{1}{1 + \pi_0^A(1 - 2p_c(\pi_0^A))} := \phi_1.
\]

### 6.2 Proof of Proposition 3

First consider the case where \( \phi_1 < \phi_2 < \phi_3 \).

- For \( \phi \leq \phi_1 \), we know from Proposition 2 that \( P = \emptyset \). In other words, the opposition will accept any allocation proposed by the incumbent. Lemma 3 then implies that the optimal allocation choice for the incumbent is \( \alpha^e = f(\pi_0^A) \).

- For \( \phi \in (\phi_1, \phi_2) \), we know from Lemma 4 that the incumbent prefers \( \alpha^p \) to \( \alpha^e \). Lemma 3 shows that \( \alpha^p = 1 \). In this range, we also know from Proposition 2 that \( \alpha^P \) is feasible along the path of conflict.

- For \( \phi \in [\phi_2, \phi_3] \), we know from Lemma 4 that the incumbent prefers \( \alpha^e \) to \( \alpha^p \). Lemma 5 further implies that \( \alpha^e \) is also feasible along the path of economic activity, in this range of \( \phi \).

- Finally, suppose \( \phi > \phi_3 \). Lemma 5 implies that \( \alpha^e \) is not feasible on the economic path. So, \( \alpha^e \in P \). We also know, from Lemma 3, that when \( P \neq \emptyset \), \( \alpha^P = 1 \). So to find the equilibrium allocation choice, we need to compare \( P_A(\alpha^P, \pi_0^A) \) with \( \max_{\alpha \leq \alpha} E_A(\alpha, \pi_1^A(\alpha, \pi_0^A)) \).
Now, \( \alpha^e \in P \) implies (by Proposition 2) that \( \alpha^e > \bar{\alpha} \). So, we have \( \bar{\alpha} < f_1(\pi^A_0) \). The function \( E_A(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) \) is an increasing function in the range \( \alpha^A_1 < f_1(\pi^A_0) \). It follows immediately that \( \max_{\alpha \leq \bar{\alpha}} E_A(\alpha, \pi^A_1(\alpha, \pi^A_0)) = E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) \). So now, it suffices to compare \( E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) \) and \( P_A(\alpha^P, \alpha^A_0) \).

First, we find an explicit expression for \( \bar{\alpha} \). Recall that \( \bar{\alpha} \) is the allocation that makes the opposition indifferent between accepting and rejecting, i.e. \( E_B(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) = P_B(\bar{\alpha}, \pi^A_0) \).

We know that \( \bar{\alpha} < f_1(\pi^A_0) \). From the proof of Proposition 2, it is easy to see that \( \bar{\alpha} > g_1(\pi^A_0)^7 \). Since \( \bar{\alpha} \in (g_1(\pi^A_0, f_1(\pi^A_0))) \), this allocation will not cause any switching. In other words, \( \pi^A_1(\bar{\alpha}, \pi^A_0) = \pi^A_0 \). Now, using the definition of \( \bar{\alpha} \), we have

\[
E_B(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) = P_B(\bar{\alpha}, \pi^A_0) \\
\iff \frac{1 - \bar{\alpha}}{1 - \pi^A_1} + x + \phi \pi^A_0(1 - 2p_d(\pi^A_0)) = x + \phi \pi^A_0(1 - 2p_c(\pi^A_0)) \\
\iff \bar{\alpha} = 1 - 2\phi \pi^A_0(1 - \pi^A_0)(p_d(\pi^A_0) - p_c(\pi^A_0)).
\]

Plugging in the value for \( \bar{\alpha} \), we can now compare \( E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) \) and \( P_A(\alpha^P, \pi^A_0) \).

\[
E_A(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) - P_A(\alpha^P, \pi^A_0) \\
= \frac{\bar{\alpha}}{\pi^A_0} + x + \phi(1 - \pi^A_0)(2p_d(\pi^A_0) - 1) - \frac{k}{\pi^A_0} - x - \phi(1 - \pi^A_0)(2p_c(\pi^A_0) - 1) \\
= \frac{1 - k}{\pi^A_0}.
\]

Clearly, \( \frac{1 - k}{\pi^A_0} \geq 0 \). So, in equilibrium, the ruling group \( A \) would choose allocation \( \bar{\alpha} \), thus inducing the opposition to accept the share.

Next, consider the case where \( \phi_2 \leq \phi_1 < \phi_3 \). The proof is identical to the case above. Here the situation \( \phi \in (\phi_1, \phi_2) \) does not arise.

References


\footnote{To see why, recall that the functions \( E_B(\bar{\alpha}, \pi^A_1(\bar{\alpha}, \pi^A_0)) \) and \( P_B(\bar{\alpha}, \pi^A_0) \) can only possibly intersect in a region where \( E_B \) is decreasing, and we know that \( E_B(\alpha^A_1, \pi^A_1(\alpha^A_1, \pi^A_0)) \) is increasing in the range \( \alpha^A_1 < g_1(\pi^A_0) \).}

