Position Competition in Sponsored Search Advertising

Tat Y. Chan and Young-Hoon Park*

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* Tat Y. Chan is Associate Professor of Marketing at the Olin School of Business, Washington University, St. Louis, MO, 63130-4899; phone: (314) 935-6096; fax: (314) 935-6359; email: chan@wustl.edu. Young-Hoon Park is Associate Professor of Marketing at the Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853-6201; phone: (607) 255-3217; fax: (607) 254-4590; email: yp34@cornell.edu. The authors would like to thank Chang Hee Park for his excellent research assistance and participants at the 2009 Marketing Science Conference for their comments. The authors also thank a company which wishes to remain anonymous for providing the data used in this study. This research was done while the second author was visiting New York University. He is grateful for the generous and cooperative support that he received from the Department of Marketing at New York University Stern School of Business.
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Abstract

We model how advertisers compete for advertising positions in sponsored search advertising. Similar to location competition in economics and marketing, one of the biggest challenges in estimating these models is the existence of multiple equilibria. Instead of imposing assumptions to fully describe and estimate the position competition among advertisers, we allow the existence of multiple equilibria without requiring researchers to take stance on how an equilibrium is selected. The basic idea of this approach is to infer model parameters from incomplete econometric models, that is, competitive interactions of players in a game are not fully specified in models. Instead, we adopt inequality conditions to avoid imposing restrictive assumptions of equilibrium-generating processes in the advertiser competition. The proposed model is general, and our estimation results are quite robust to various behavioral specifications and assumptions about equilibrium-generating processes in the advertiser competition.

We use the database with bids and clicks of the sponsored search advertising in a particular brand-name keyword provided by a leading search engine site in Korea. The dataset contains individual-IP-level impressions and clicks on the complete set of competing advertisers in the sponsored search list. The search engine offers multiple advertising positions to potential advertisers; each advertising position is sold independently through daily auctions. Similar to online consumer auctions (e.g., eBay), each position auction is designed with a buy-it-now (BIN) price. Our results show that the existence of multiple equilibria is very common; indeed unique equilibrium rarely exists in any time of our data period. Based on the mean profit per click for each advertiser, we quantify the impact of BIN prices on the expected profit of the search engine. We also conduct some “what-if” experiments to investigate the optimality of the pricing policy of the search engine.

Keywords: Sponsored search advertising, Position competition, Click-through; Buy-it-now, Moment inequality, Location competition
1. Introduction

Sponsored search advertising is now one of the largest and fastest growing sources of revenue for Web search engines. Search engines use their proprietary pricing mechanisms for placement of an advertisement in sponsored search list, and pricing formats have been evolved considerably over the past several years. Two widely used payment mechanisms are pay-per-impression, where all advertisers are charged whenever a consumer searches the keyword and their links are displayed, and pay-per-click, where an advertiser is charged only when a consumer clicks on its link. The most pertinent auction formats to sponsored search auctions are multi-item first-price and second-price auctions with single demand (Yao and Mela 2009a). That is, there are multiple spots for an auction; each bidder needs only one of the multiple positions. Payment is calculated based on the bidder’s own bid (first-price) or the highest losing competitor’s bid (second-price).

Advertisers are often engaged in intense bidding competition to obtain the topmost positions in the sponsored search list. The rationale behind these bidding wars is that the higher the bid, the higher the advertiser’s message appears in the sponsored search list, which should typically lead to more sales-leads (click-throughs), and consequently greater sales. Based on this conventional wisdom, most advertisers aggressively seek the topmost positions in their bidding. Numerous empirical studies have found that, when aggregated across keywords, the number of clicks on a sponsored link decreases approximately exponentially as one proceeds down a list of sponsored links (e.g., Feng et al. 2007, Ghose and Yang 2009). In contrast, Agarwal et al. (2008) find that the conversion profitability is often highest at the second or third position and not always the topmost position.

Several analytical studies have focused on the optimal bidding strategies of advertisers for sponsored links and the related optimal design of the auction mechanisms (e.g., Edelman et al.
Katona and Sarvary (2008) focus on consumers’ clicking behavior for both organic and sponsored links, and show that there are multiple equilibrium. Chen and He (2006) assume that consumers engage in costly search and an advertiser’s valuation to sponsored positions depends on the advertiser’s product quality. They show that, in equilibrium, bidders will submit bids equal to their true values. Hence advertisers will be ranked according to their qualities. Athey and Ellison (2008) develop a model that integrates both consumers and advertisers as in Chen and He (2006), and show that high-quality advertisers will bid more aggressively than low-quality advertisers. Jerath et al. (2009) find that a high-quality advertiser may have the incentive to be placed at a lower position. While the theoretical literature in sponsored search advertising offers some important insights regarding optimal bidding strategies of advertisers for sponsored links, perhaps due to data limitation there is little empirical research on understanding how advertisers compete for advertising positions.

This research proposes a framework to structurally model the advertiser competition for sponsored advertising positions. To this end, we employ a unique dataset with bids and clicks of sponsored search advertising provided by a leading search engine site in Korea. This search engine employs a pay-per-impression pricing mechanism. The dataset includes precise data on the price for each advertising position. It also contains individual-IP-level impressions and clicks on the complete set of competing advertisers in the sponsored search list. This search engine offers multiple advertising positions to potential advertisers; each advertising position is sold independently through daily auctions. One unique feature of the dataset is that, similar to online consumer auctions (e.g., eBay), each position auction is designed with a buy-it-now (BIN) price, which allows advertisers to prematurely end an auction to obtain the position. Thus, the auction format can be described as a multi-item first-price auction, designed with BIN price, for single
demand. All advertisers in our data acquired advertising positions through the exercise of the BIN option. Based on this empirical context, we investigate the average profit per click for each advertiser, which is inferred from the observed advertiser’s purchase decisions for different advertising positions. We further examine the characteristics of the multiple equilibria in our results, and the impacts on the profit of search engine when it changes BIN prices and the number of available advertising positions in the sponsored search list. All these results have strong managerial implications for advertisers as well as search engines.

Modeling the position competition in the sponsored search advertising requires full consideration of various complications as follows. As shown in a number of analytical studies, one of the biggest challenges in estimating these models is the existence of multiple equilibria. Similar to the location competition in economics and marketing where a limited number of valuable locations are competed by multiple potential entrants (e.g., Chan, Padmanabhan and Seetharaman 2007, Duan and Mela 2009, Seim 2006), advertisers compete for advertising positions that are associated with BIN prices in our empirical context.1 The BIN exercise decisions depend on an advertiser’s own profit per click, as well as the advertiser’s expectation of competing advertisers’ profits and bidding strategies. In general unique equilibrium does not exist in our empirical context. Furthermore, as positions in sponsored search advertising have directional characteristics, the actual position of an advertiser depending on the auction outcomes can be higher than the nominal position of the advertiser.2 Hence, the choice of advertising

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1 While there are some previous analytical studies on BIN (e.g., Budish and Takeyama 2001, Hidvégi et al. 2006, Matthews 2004, Wang et al. 2008), little empirical studies exist because of the difficulty to model the equilibrium conditions to estimate the willingness to pay of bidders. Park and Bradlow (2005) use a reduced-form model to understand driving factors for the BIN exercise. An exception is Ackerberg et al. (2006) who estimate a structural model for bidding in auctions with eBay’s BIN feature.

2 For example, an advertiser who purchases the second position may end up at the top position, if the top position has not been sold. This is similar to many natural resources (e.g., water, wind) and business markets (e.g., TV broadcasting time, a one-way street) which have directional characteristics (e.g., Lai 2001).
position of an advertiser depends on its expectation of the choices of advertising positions from other advertisers, and different expectations may lead to different equilibrium outcomes.

Our model differs from the traditional location competition literature (e.g., Chan, Padmanabhan and Seetharaman 2007, Thomadsen 2005) where competition among firms comes from the substitutability in demand between neighboring firms. Instead, competition in our model comes from the uniqueness of a good position; the position will be gone if other advertisers exercise the BIN option first. This is consistent with the type of bidder competition modeled in the auction literature where multiple bidders with heterogeneous willingness-to-pay compete for a unique object. However, our empirical context is even more complicated since there are multiple advertising positions available for auctioning. In addition, the actual advertising position of an advertiser may be different from the nominal position of the advertiser. Thus, an advertiser is less willing to purchase a higher position if it expects that other positions will not be sold. All of these imply that a unique equilibrium is unlikely to exist and even if it exists the equilibrium conditions may be difficult to specify.

We use a new concept proposed in recent economic literature, modeling the “necessary” equilibrium conditions in “incomplete” models when multiple equilibria exist (e.g., Andrews et al. 2004, Chernozhukov et al. 2007, Pakes et al. 2007). The basic idea is to allow for the existence of multiple equilibria without taking stance on equilibrium selection. Instead of modeling the sufficient conditions for equilibrium outcomes, we examine the necessary conditions to develop a set of moment inequalities. The biggest advantage of this approach is that a model can be estimated without imposing any restrictive assumptions on how equilibrium is selected. Thus, our estimation results are quite robust to various behavioral specifications and data-generating processes.
Summarizing, we model the position competition among advertisers using necessary conditions of equilibrium outcomes in the sponsored search advertising. As an intermediary for our estimation of the advertiser competition, we also estimate consumers’ clicking behavior when facing multiple search advertisements. We estimate our model on search advertising data for a particular brand-name keyword provided by a leading search engine site in Korea. We find that, without imposing restrictive model assumptions, the existence of multiple equilibria is very common in our results; indeed unique equilibrium rarely exists in any time of our data period. Other important findings are that advertising position, advertiser identity and selling propositions (e.g., assortment-specific information, price-specific information) are significantly associated with the click-choice propensity. While the top (bottom) position attracts most (least) click-throughs for advertisers, the positions in the middle (from second to fourth position) are not significantly different from each others. Based on the mean profit-per-click estimate for each advertiser, we quantify the impact of BIN prices on the expected profit of the search engine. We also conduct some “what-if” experiments to investigate the optimality of the pricing policy of the search engine and the impact of the number of available advertising positions.

Our work considerably differs from the existing empirical studies on search advertising in marketing which have focused on the effect of advertising position, keyword length, presence or absence of brand name, etc. on the click-through rate of the advertisement and purchase conversion (e.g., Ghose and Yang 2009, Rutz and Bucklin 2007, 2008). This stream of research, using the aggregate-level databases obtained mostly from single retailer, has lead to some important insights regarding how keyword-level characteristics influence clicks and purchase conversions. But, little empirical research exists to understand how advertisers compete for advertising positions in sponsored search advertising. One notable exception is Yao and Mela
They model the advertiser competition for advertising positions under the first-price auction mechanism, and investigate the dynamics of such strategic behavior of searchers, advertisers and the search engine firm. In terms of methodology, they use the two-step estimators algorithm developed by Bajari et al. (2007) assuming the existence of a Markov Perfect equilibrium. Unlike their approach, we do not model the dynamic interactions between firms; instead, our main focus is on how to model and estimate a complicated (static) game without imposing restrictive assumptions on how firms compete against each other, and how equilibrium outcomes are generated when multiple equilibria may exist. We only rely on moment inequalities as necessary conditions for equilibrium; as a result, our estimation approach vastly differs from theirs.

The rest of the paper is organized as follows. Section 2 describes our data and section 3 presents our model and discuss our computational approach using inequalities. In section 4, we discuss results, and demonstrate a way to understand the impact of advertising positions on expected profits. We discuss other managerial implications of this research and conclude with directions for future research in section 5.

2. Data Overview

In this section, we describe the data used for this research and propose several sets of descriptive statistics derived from the database to motivate our modeling approach of advertisers’ competition for positions in sponsored search advertising.

2.1 Data Description

We obtained a database of sponsored search advertisement from a leading search engine site in Korea. This search engine employs a pay-per-impression pricing mechanism. For a given
keyword, the search engine offers up to five different advertising positions in the sponsored search list to potential advertisers. Each advertising position is sold independently through daily auctions in which advertisers bid while auctions are in progress. From the perspective of advertisers, multiple position auctions are concurrently in progress. The search engine uses an ascending first-price auction; the highest bidder wins and pays her bid. One unique feature of the dataset is that, similar to online consumer auctions (e.g., eBay), BIN price is set for each position auction. Thus, this auction can be described as a multi-item first-price auction, designed with BIN price, for single demand.

Once an advertiser obtains a particular advertising position through daily auctions, the position of an advertiser does not vary across impressions during a day. We thus have the exact information of where an advertiser’s advertisement is exposed. This is different from almost all existing studies that use aggregate-level advertising position data (e.g., weekly information). They assume that the mean position during the week is the actual position of an advertiser (e.g., Ghose and Yang 2009, Rutz and Bucklin 2007, 2008). Abhishek et al. (2009) demonstrate that applying standard models on aggregate data can lead to biased estimation of the parameters of a random utility model in understanding the factors that drive consumer click and conversion propensities.

The database consists of both cost per impression and clicks (as well as impressions) related to each advertising position in one particular brand-name keyword over the period of 254 days (from January 1, 2008 to September 10, 2008). That is, in each day during the data period, we observe the following: (1) Which advertising position was occupied by which advertiser and the price paid by each advertiser; (2) For each individual consumer, which advertisements were

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3 After an advertiser obtains a position for a given keyword, the advertiser displays its advertisement in the position during the day. It is possible for the advertiser to stop its advertisement at any time during the day which never happened in our database.
exposed in response to the keyword search, and what advertisements were clicked. However, we do not observe on the order of decision making by advertisers in daily position auctions. We also do not observe on the sequence of clicks by a consumer during the day. The keyword is related to a brand of sports goods including footwear, apparel, bags, and accessories.

The data also includes selling propositions that each advertiser displays in its advertisement. They typically consist of a limited number of phrases/words describing the product or service and a hyperlink that refers consumers to the advertiser’s website. In our empirical applications, we include two variables that may impact consumers on their click propensity. The first one is the breadth of assortments which is constructed based on the number of product categories, ranging from a minimum of 0 to a maximum of 4, described in the selling propositions. Another variable is the price discount, ranging from a minimum of 0% to a maximum of 55%. We transform this variable to $\ln(\text{discount}+1)$ to account for advertisers that never offered price discounts. We decided to exclude other variables in the selling propositions (e.g., new arrivals of products, service provision) because we do not observe variations during the data period and hence their impact on the consumer click choice cannot be separately identified from the advertiser fixed effects.

2.2 Empirical Findings

Another unique feature of the database is that the data contains individual-IP-level clicks (as well as impressions) on the complete set of competing advertisers in the sponsored search list. At the daily level, on average, there are 211 exposures and only 6.7 clicks. About 90.8% of searches are resulted with no clicks for any of the sponsored links. Among those who click, 6.5% of searches have just one click, and the rest 2.7% of searches have multiple clicks in the sponsored search list, i.e., about 30% of unique searches in our data involve more than a single click in the
competitive set of sponsored links. We also observe that the higher the (actual) advertising position, the more clicks to the advertisement. Conditional on the position being occupied, the top (second) position generates 5.08% (3.96%) of clicks per exposure, while the bottom two positions generate less than 2% click-through rates. As these summary statistics have not accounted for advertisers’ fixed effects and selling propositions, we caution not to conclude that top advertising positions are more effective than bottom positions in attracting clicks to websites.

There are six advertisers participated in bidding various position auctions of the sponsored links throughout the data period. Interestingly and surprisingly, we observe that all advertisers chose to exercise the BIN option to obtain their positions. A possible explanation could be that advertisers are risk-averse as sponsored positions in the search engine are scarce (e.g., Budish and Takeyama 2001). However, advertisers still choose to exercise the BIN option even when many advertising positions are available in some days. It could be that as managers at advertising firms have to decide many different keywords for sponsored search advertising and their corresponding advertising positions at a daily level, they may prefer to exercise the BIN option in order to minimize the hassle cost (e.g., Wang et al. 2008), instead of participating in bidding various position auctions while auctions are in progress. This is an important empirical observation; based on that we model the advertisers’ competition for advertising positions via the exercise of the BIN option.

Figure 1 illustrates advertising positions obtained by the six advertisers during the data period. In general, advertisers do not change their positions very often. In particular, Advertiser 4 (thin-solid line) started at the second position at day 92, and except for a few days it remained at the same position for the rest of the data period. Similarly, Advertiser 6 (thin-dotted line)

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4 Due to the nature of the data sharing agreement between the search engine and us, we are unable to reveal the names of the advertisers observed in the database.
obtained the fifth position at day 162 and continued to purchase the same position for the remaining data period. At the same time, we also observe some abrupt changes in advertisers’ positions. For instances, Advertiser 1 (thick-solid line) started from the top position, stopped advertising at all for about one and half months, and switched to the fourth position in the second half of the data period. Advertiser 5 (thin-dashed line) did not advertise for a long time, and then obtained the fifth position for some time and finally switched to the top position. Overall, these observations show a lot of “stickiness” in advertisers’ decisions together with occasionally “jumps” in advertising positions. One potential cause for the “jumps” is the changes in BIN prices which might have an impact on advertisers’ expected profitability from each click-in, or that product costs of advertisers have changed during the data period.

**Insert Figure 1 about here**

On average, 2.98 advertising positions were occupied at the daily level over the data period (std. dev. = 1.45). As shown in Figure 1, fewer positions were purchased in early days while most of the positions were sold at the second half of the data period. Two points are worth noting. First, the actual position of an advertiser may differ from the nominal position acquired by the advertiser. At day 50, for example, Advertiser 1 (thick-solid line) buys the top position and Advertiser 3 (dotted line) buys the third position, while the other positions are not sold on that day. The actual position of Advertiser 1 becomes 1, and that of Advertiser 3 becomes 2. This directional characteristics in the sponsored search list substantially increases the complexity to understand the position competition among advertisers since it implies that an advertiser’s bidding strategy will depend on its expectations of other competing advertisers’ bidding strategies, and vice versa. Second, changes in advertising positions that we observe in Figure 1 could be partially explained by the changes in the BIN prices across advertising positions in
Figure 2. As expected, the top position is the most expensive location (mean = 2.8 cents) and the bottom position is the least expensive (mean = 1.5 cents) among the sponsored links. Prices have changed considerably over the data period. In particular, the top position was designed with the BIN price of 3.2 cents per exposure for the first 109 days. On Apr. 20, 2008, the search engine lowered the BIN price of the top position to 2.7 cents per exposure, and lowered it again to 2.3 cents on Jun. 29, 2008. The bottom position also experienced changes in the BIN prices. It started at the BIN price of 1.6 cents per exposure at the beginning of the data period and 1.3 cents at the end of the data period. The BIN prices of all positions declined in general. We were told by the management of the search engine that the price changes were done on purpose in order to attract more demand from advertisers. Furthermore, the cost difference across advertising positions has been shrinking. For example, the difference in the BIN prices from the second to fourth position was very close in the later period; the search engine even offered the same BIN price for the second and third position.

Insert Figure 2 about here

The descriptive statistics discussed in this section are useful as the first step to understand the nature of position competition among advertisers and strategies employed by the search engine. We next explicitly model these aspects to study the optimal pricing policy of the search engine.

3. The Model

In this section, we model the position competition among advertisers in sponsored search advertising. In section 3.1, we discuss how advertisers decide to exercise the BIN option for each

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5 All bids are in Korean currency (won). At the time of the data period, foreign exchange rates were highly volatile, ranging from a minimum of 935 won to a maximum of 1,508 won which corresponded approximately to $1. In this paper, we used 1,000 won to the exchange of $1.
advertising position, and describe related issues which complicate the properties of equilibrium outcomes. In section 3.2, we characterize the necessary conditions for the equilibrium outcomes under the position competition, which are the main components in our structural model estimation. Section 3.3 presents a reduced-form model of consumer click choice in sponsored links. A description of our estimation approach and computational details are provided in sections 3.4 and 3.5, respectively.

3.1 Modeling Advertisers’ Position Competition and Related Issues

Suppose that there are altogether $J$ advertisers as potential bidders who want to advertise in response to a searched keyword. A total of $K$ different advertising positions (1 to 5 in our empirical context, where “1” is for the first, “2” for the second, and so on for various positions) are available to potential advertisers. Given that all advertisers acquired advertising positions through the exercise of the BIN option in our data, this research focuses on how advertisers compete against each other in exercising the BIN option to obtain different positions in the sponsored search list.

In each period (day) $t$, we denote $R_{jt}$ as the nominal position of advertiser $j$, where $R_{jt} = 1$ represents that advertiser $j$ buys the top position at time $t$, and $R_{jt} = 2$ represents that advertiser $j$ obtains the second position, and so on. If advertiser $j$ does not buy any advertising position at time $t$, we denote $R_{jt} = 0$, i.e., the outside options. We observe in every period $t$ $J_t$ ($< J$) firms advertise their advertisements in response to the keyword. An advertiser at position $R_{jt}$ pays $c(R_{jt})$ per exposure, whether or not his advertisement is clicked. If the advertiser chooses not to advertise, its cost is $c(R_{jt}) = 0$.

As advertising positions in the sponsored search list have directional characteristics, the actual position of advertiser $j$, denoted as $r_{jt}$, may be different from the nominal position $R_{jt}$.
which advertiser $j$ obtains at time $t$. If $R_{jt} \neq 0$, $r_{jt}$ could be higher than $R_{jt}$ if other higher positions were not sold. For example, suppose advertiser $j$ buys the second position and advertiser $k$ buys the fourth position, while the other positions are not sold at time $t$. In this case, the actual position of advertiser $j$ ($r_{jt}$) will be 1 and that of advertiser $k$ ($r_{kt}$) becomes 2. The other advertisers $l$ are not displayed in the sponsored list as they did not obtain positions at time $t$, thus $r_{lt} = 0$.

Given that advertiser $j$’s nominal position is $R_{jt}$, let $p_j(R_{jt}, R_{jt})$ be the probability of consumer clicking into its link as a function of $R_{jt}$ and positions of other competing advertisers, i.e., $R_{jt}$. Also let $y_{jt}$ be advertiser $j$’s expected profit per click at time $t$. Thus, the expected net profit of advertiser $j$ per exposure of advertisement is:

$$E\pi_j(R_{jt}, R_{jt}) = y_{jt} \cdot p_j(R_{jt}, R_{jt}) - c(R_{jt}).$$  

Let $\overline{y}_j$ be the mean expected profit of advertiser $j$ per click. We denote

$$y_{jt} = \overline{y}_j + \xi_{jt},$$  

where $\xi_{jt}$ is a stochastic profit shock and by definition $E(\xi_{jt}) = 0$ for each advertiser.

The problem for advertiser $j$ at time $t$, conditional on the choices of advertising positions of other advertisers, is to choose the optimal position $R_{jt}^*$ such that

$$R_{jt}^* = \arg \max_{R \in K / R_{jt}} E\pi_j(R, R_{jt}).$$  

where $R \in K / R_{jt}$ denotes the set of advertising positions that are unoccupied and hence are still available for purchase.

Advertisers compete for advertising positions through exercising the BIN option; their decisions are interrelated. In general unique equilibrium does not exist in our empirical context.

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6 If $R_{jt} = 0$, $p_j(R_{jt}, R_{jt})$ will be equal to 0 for all $R_{jt}$.
For instance, advertisers $j$ and $k$ may both want to advertise at the fifth position (suppose the expected net profits for both advertisers are positive and the highest at that position). If this position has been taken, advertiser $j$ may choose to buy the fourth position (suppose the expected net profit at that position is still positive) while advertiser $k$ may decide not to advertise at all (suppose the expected net profits at other positions are all negative). In this case two equilibria can exist: (a) $R_{jt} = 4$ and $R_{kt} = 5$ or (b) $R_{jt} = 5$ and $R_{kt} = 0$. In our context, which equilibrium will be chosen depends on which advertiser gets to exercise the BIN option first. In the above example advertiser $j$ will buy the fifth position if it makes the decision before advertiser $k$ hence equilibrium (b) is the equilibrium outcome; otherwise we will observe equilibrium (a). Probably, the existence of multiple equilibria creates the biggest challenge for our study since this model cannot be estimated using standard approach such as the maximum likelihood.\(^7\)

What we have discussed above is a simultaneous-move game. It is well-known in the literature that the multiple-equilibria problem may be solved if researchers instead focus on sequential moves. In the above example, equilibrium (a) will be the outcome if advertiser $k$ is the first-mover, otherwise equilibrium (b) will be the outcome. However, we do not observe the order of the BIN exercises by advertisers in our data. Even if one is willing to impose an assumption on the order, there are still other complexities in our empirical context. As noted earlier, the actual position of an advertisement can be higher than the nominal position had other higher positions not been occupied. Therefore, choice of an advertiser depends on its expected responses of other advertisers following its decision, and different expectations may lead to different equilibrium outcomes. For example, if advertiser $j$ expects that no other advertisers are interested in the fourth position, it may choose the fifth position at a lower cost. In this case its

\(^7\) With an assumption of having unique equilibrium researchers can “invert” the error terms that are consistent with the equilibrium to construct the likelihood function. For examples, see Bresnahan and Reiss (1989) and Berry (1992).
actual position is guaranteed to be not lower than the fourth. However, if it expects that the fourth position will be acquired, it may purchase that position instead of the fifth position.

To solve this problem, the standard approach in the literature is to impose rational beliefs. That is, the expectations of the probability that each position will be taken by other advertisers have to be consistent with the equilibrium outcomes. In the cases that this rational beliefs equilibrium is too complicated to be computed even numerically, empirical IO researchers have typically relied on two-step estimators (for examples see Bajari et al. 2007, Hotz and Miller 1993). 8 That is, researchers will first recover the probabilities of firm decisions conditional on observed state variables and assume that is the equilibrium outcome consistent with firms’ (rational) beliefs. In the second step, researchers will investigate the structural parameters from firms’ decisions conditional on these beliefs. Rational beliefs may be an assumption too restrictive in our application and even under such assumption there is no proof that unique equilibrium will exist in a general set-up. Furthermore, data requirement for the first-step estimator is very high, i.e., there should be sufficient observations across combinations of state variables. As discussed in the data section, advertising positions only changed occasionally in our data. Because of this lack of data variation, we believe that it is infeasible to implement the two-step estimators approach in our study.

3.2 Necessary Conditions of Equilibrium Outcomes

Instead of imposing further simplifying assumptions to fully describe and estimate the position competition among advertisers, our estimation strategy is to employ such approach that the existence of multiple equilibria is allowed without requiring researchers to take stance on how an equilibrium is selected (e.g., Andrews et al. 2004, Chernozhukov et al. 2007, Pakes et al. 2007). The basic idea of this approach is to infer model parameters from “incomplete” econometric

8 An empirical study of the sponsored search auctions adopting a similar approach is Yao and Mela (2009b).
models, that is, behaviors of players in a game are not fully specified in models. Instead of modeling the sufficient conditions for equilibrium outcomes, researchers examine the necessary conditions to develop a set of inequality conditions. Estimating from such inequalities would generate a set of model parameters (which may be a singleton) that are consistent with the observed data. Any value within the set is an acceptable candidate for the estimated parameters. Similar strategy has been employed to estimate willingness-to-pay of bidders in auctions (e.g., Chan, Kadiyali and Park 2007, Haile and Tamer 2003).9

Though less precise than traditional econometric methods which generate point estimates, the biggest advantage of this approach is that a model can be estimated without imposing restrictive assumptions on how the equilibrium is selected. For instance, in our application there is no need to assume a specific order of decision making among advertisers. As we will discuss below, it also does not require rational beliefs assumption regarding the strategic responses from other advertisers. In order to estimate the structural model, we adopt the method of moment inequalities proposed in Pakes et al. (2007).10 The key benefit of this method is its lower computational burden in model estimation compared with other methods (e.g., Andrews et al. 2004, Chernozhukov et al. 2007); hence it is easy to implement. Another advantage is analogous to the advantage of using the method of moments over the likelihood approach in estimation: this approach does not rely on the specification of the data-generation process of the error terms in the model. For example, we do not need to specify the distribution of the profit shock $\xi_{jt}$ in equation (2). Indeed, $\xi_{jt}$ can be correlated across advertisers and time periods in a very flexible way, which is very useful for our application. Because of these advantages, our estimation results

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9 Ackerberg et al. (2006) also use the “incomplete” specification for bidder behavior to estimate a structural model for bidding in auctions with eBay’s BIN feature.

10 Previous empirical studies applying this approach can be found in Ho (2009) and Ishii (2008).
are quite robust to various behavioral specifications and assumptions about equilibrium-generating processes in the advertiser competition.

Two necessary conditions should be satisfied in our model so that advertisers’ choices we observe in data are at equilibrium. Let \( \{R_{jt}, ..., R_{jt}\} \) be the observed advertising positions of \( J \) advertisers at time \( t \). The first necessary condition for \( \{R_{jt}, ..., R_{jt}\} \) to be an equilibrium is the following:

1. Suppose advertiser \( j \) can move to a position \( R_{jt} \in K / R_{-jt} \), lower than \( R_{jt} \). Let \( R'_{jt}(R_{-jt}) \) represent advertiser \( j \)’s expectation of other advertisers’ responses (i.e., moving to other positions) if it switches to \( R_{jt} \).\(^{11} \) The fact that advertiser \( j \) chooses not to switch from \( R_{jt} \) to \( R_{jt} \) implies that
\[
E\pi_j(R_{jt}, R_{-jt}) \geq E\pi_j(R_{jt}, R'_{jt}(R_{jt})). \tag{4}
\]

The above inequality means that the expected net profit of position \( R_{jt} \) is higher than that of \( R_{jt} \).

\( R'_{jt}(R_{-jt}) \) may be a distribution depending on advertiser \( j \)’s uncertainty of the response of other advertisers had advertiser \( j \) switched to \( R_{jt} \).\(^{12} \) Suppose there are \( S \) sets of possible responses \( \{ R_{-jt}^1, ..., R_{-jt}^S \} \), and based on advertiser \( j \)’s beliefs each set \( s \) is associated with a probability \( \rho_j^s(R_{jt}) \). The right-hand side of inequality (4) can be written as:
\[
E\pi_j(R_{jt}, R'_{jt}(R_{jt})) = \sum_{s=1}^{S} \rho_j^s(R_{jt}) \cdot E\pi_j(R_{jt}, R_{-jt}^s). \tag{5}
\]

Following the specification in equation (1), we can rewrite inequality (4) as the following:

\(^{11} \) The concept of expecting responses from other advertisers induced by advertiser \( j \)’s action is important to rationalize some of our empirical observations. For example, suppose advertiser \( j \) is currently at the fourth position and the fifth position is unoccupied. If advertiser \( j \) instead chooses the fifth position, its actual advertising position will still be the same assuming that other advertisers do not change their positions. By choosing the fifth position, advertiser \( j \) will pay a lower cost. The only way to justify its unwillingness to switch to the fifth position is that advertiser \( j \) believes other advertisers are likely to take over the fourth position if it switches to the fifth position, which as a result will lower advertiser \( j \)’s actual advertising position and generate fewer clicks.

\(^{12} \) Following the Bayesian Nash equilibrium concept, a game with asymmetric information, i.e., advertiser \( j \) knows its own \( \xi_j \), but is uncertain of other advertisers’ profit shocks, will be consistent with such uncertainty. In a more general set-up, there may be other uncertainties such as how an equilibrium will be selected when multiple equilibria exist.
\[ y_{jt} \cdot p_j (R_{jt}, R_{j-t}) - c(R_{jt}) \geq y_{jt} \cdot \sum_{s=1}^{s} \rho_j^s (R_{jt}) \cdot p_j (R_{jt}, R_{j-t}^s) - c(R_{jt}). \]  

(6)

We will discuss later how the above inequality will help to define a lower bound for the parameter \( \tilde{y}_j \) in equation (2). The second necessary equilibrium condition is similar to the first condition as the following:

2. Suppose advertiser \( j \) can move to a position \( \tilde{R}_{jt} \in K / R_{j-t} \), higher than \( R_{jt} \). The fact that advertiser \( j \) chooses not to switch from \( R_{jt} \) to \( \tilde{R}_{jt} \) implies that

\[ E_{\pi_j} (R_{jt}, R_{j-t}) \geq E_{\pi_j} (\tilde{R}_{jt}, R_{j-t}^j (\tilde{R}_{jt})). \]  

(7)

Analogous to the discussion above, inequality (7) of the second necessary condition can be rewritten as the following:

\[ y_{jt} \cdot p_j (R_{jt}, R_{j-t}) - c(R_{jt}) \geq y_{jt} \cdot \sum_{s=1}^{s} \rho_j^s (\tilde{R}_{jt}) \cdot p_j (\tilde{R}_{jt}, R_{j-t}^s) - c(\tilde{R}_{jt}). \]  

(8)

This inequality condition will help define an upper bound for the parameter \( \tilde{y}_j \) in equation (2).

The two necessary conditions play an important role to develop a set of moment inequalities which will be described in section 3.4.

3.3 Modeling Consumer’s Click Choice

Before proceeding to model estimation using the inequalities (6) and (8), we have to specify, given a set of nominal advertising positions \((R_{1t}, \ldots, R_{jt})\), the probability of consumer clicking on advertiser \( j \)’s link, \( p_j (R_{jt}, R_{j-t}) \) noted in equation (1). Since our main objective is to study the position competition among advertisers in the sponsored search list, we prefer a model that can capture the consumer intrinsic preference for each advertiser and the impact of each advertising position on click-choice behavior. For consumer \( i \) who searches for the keyword at time \( t \), she will be exposed to a list of \( J_t \) sponsored links. She may click into any of the advertisers for more information, but she may also choose not to click into any of them. As noted in section 2,
conditional on clicks, about 30% of unique searches in our data involve more than a single click in the set of sponsored links. A discrete choice model of consumer clicking among sponsored links (e.g., multinomial logit or probit) cannot be applied. Instead we choose a reduced-form binary logit probability model.\textsuperscript{13} Let \( r(R_{jt}, R_{jt}) \) be the actual position of advertiser \( j \)'s link as a function of \( R_{jt} \) and \( R_{jt} \). We specify the probability that during a keyword search in period \( t \) the probability advertiser \( j \)'s link will be clicked in as

\[
P_{jt} = \frac{\exp(\alpha_j + \sum_{k=1}^{K} 1\{r(R_{jt}, R_{jt}) = k\} \cdot \gamma_k + X_j \beta)}{1 + \exp(\alpha_j + \sum_{k=1}^{K} 1\{r(R_{jt}, R_{jt}) = k\} \cdot \gamma_k + X_j \beta)},
\]

where \( 1\{\cdot\} \) is an indicator function, \( \alpha_j \) measures the attractiveness of advertiser \( j \), and \( \gamma_k \) measures the impact of advertising position \( k \) on the click-choice probability.

Previous studies show mixed results on how online consumers process ordered lists. Granka et al. (2004) find through eye-tracking experiments that consumers do tend to scan the search list from top to bottom. This kind of primacy effect is consistent with other empirical studies of the online world. For instance, Hoque and Lohse (1999) found the position effect for electronic yellow pages even when there is no obvious reason to expect any difference due to the serial position in a list. Ansari and Mela (2003) suggested a positive relationship between the serial position of a link in an email and recipients' clicks on that link. As consumer clicks on search results are largely affected by her search pattern, non-sequential search strategy could be used by consumers. For instance, the more complicated the product attribute space, the better non-sequential search models should mirror the reality of consumer search (Yao and Mela 2009a). Recently, Jerath et al. (2009) study the optimal bidding strategies of advertisers based on

\textsuperscript{13} Yao and Mela (2009b) use a similar binary choice model to specify the conditional probabilities of keyword search and consumer downloads.
a model that while some consumers start their search from the top position and go down sequentially, others click the high-quality firm regardless of its position if they already know the identity of the high-quality firm. In this research we treat this as an empirical question to be estimated from data. If non-sequential strategy is common among consumers, we should expect advertisers’ fixed effects $\alpha_j$ to be significant but $\gamma_k$ in equation (9) to be insignificant. Finally, $X_{jt}$ is a vector of covariates which are based on advertiser $j$’s selling propositions (i.e., number of categories and price discounts) displayed in the sponsored link.

Suppose advertisers $j$ and $k$ both advertise in the sponsored search list. Based on equation (9), the probability that a consumer will click into both advertisers are $p_{jt}p_{kt}$, and the probability that only advertiser $j$ will be clicked into is $p_{jt}(1-p_{kt})$. This choice model is simple to estimate and results can be directly incorporated into the model of advertisers’ position competition for sponsored search advertising. However, there are several well-known limitations implied by the binary logit model. First, the model does not take into account that consumer preferences over multiple advertisers may be correlated. Also, depending on the sequence of website search (which we do not observe from data as the data is available at the daily level) consumer’s utility of clicking into subsequent websites may vary depending on the information she has acquired from previous websites she has clicked into. To allow for these components it requires the development of a full dynamic search model. While such a complicated click-choice model could provide valuable insights into consumer search behavior, it is not clear if this will improve our prediction of clicking probability for each advertiser, which is an intermediary for our estimation of the advertiser competition. Hence, we chose to abstract away from potential associations across advertisers through consumer search behavior within a keyword search.
Another limitation of the model is that the identities of the advertisers who occupy other advertising positions do not impact the clicking probability in equation (9). This means that $p_{jt}$ is independent from whether another advertising position is sold to advertiser $k$ or $k'$. To test whether or not this assumption is reasonable, we estimate another binary logit model including an indicator for each of the competing advertisers where they advertise, as covariates in $X_{jt}$. We find that the coefficients for these indicators are all insignificant, and the impacts on clicking probability are mixed. This seems to provide evidence that the identities of other advertisers in the sponsored search list do not affect the consumer clicking choice.

To summarize, though a full-blown structural dynamic search model accounting for multiple-click choices and rich associations across advertisers and positions (both complementarity and substitutability) can be developed, doing so will increase the complexity in model estimation and may require more restrictive assumptions on the consumer clicking behaviors. It is important to note that the click-choice model is only an intermediary for us to estimate the position competition in a given keyword. We thus choose to use a simple binary logit choice model for the purpose of our study.

3.4 Moment Inequalities

With the estimated clicking probabilities, we now turn to the moment inequalities that are based on the two necessary equilibrium conditions. Expectations $E \pi_j (R_{jt}, R_{jt}^j (R_{jt}^j))$ in inequality (5) and $E \pi_j (\tilde{R}_{jt}, R_{jt}^j (\tilde{R}_{jt}))$ in inequality (7) are difficult to define because they depend on the advertiser’s expectations of the responses from other advertisers had it chosen different advertising positions, which we do not observe from the data. However, further inequalities can be utilized without specifying these expectations. Equation (9) implies that the click-choice probability is independent from which advertisers occupy what positions. Let $r_{jt} = r(R_{jt}, R_{jt})$ be
the observed actual advertising position of advertiser $j$ in period $t$. We have from equation (9) that $p_j(R_{jt}, R_j) = p_j(r_{jt})$. We also know that both $R_{jt}$ and $\tilde{R}_{jt}$ are the lowest and the highest actual positions advertiser $j$ may get, independent from the responses of other advertisers. Suppose the clicking probability $p_j$ is higher when the actual advertising position is higher (which is confirmed from our results). This implies that $p_j(R_{jt}, R_{jt}^*) \geq p_j(R_{jt})$ and $p_j(\tilde{R}_{jt}, R_{jt}^*) \geq p_j(\tilde{R}_{jt})$.

That is, the clicking probabilities are higher when actual advertising positions are taken based on the response of other competitors. Based on that we can construct another inequality from the inequality in (5):

$$y_{jt} \cdot p_j(r_{jt}) - c(R_{jt}) \geq y_{jt} \cdot \sum_{s=1}^{S} \rho_s(R_{jt}) \cdot p_j(R_{jt}, R_{jt}^s) - c(R_{jt}) \geq y_{jt} \cdot p_j(R_{jt}) - c(R_{jt}) \quad \text{(10)}$$

where from equation (9)

$$p_j(R_{jt}) = \frac{\exp(\alpha_j + \sum_{k=1}^{K} 1\{R_{jt} = k\} \cdot \gamma_k + X_{jt} \beta)}{1 + \exp(\alpha_j + \sum_{k=1}^{K} 1\{R_{jt} = k\} \cdot \gamma_k + X_{jt} \beta)}. \quad \text{(11)}$$

Using the same rationale, we can construct another inequality from the inequality in (7):

$$y_{jt} \cdot p_j(r_{jt}) - c(R_{jt}) \geq y_{jt} \cdot \sum_{s=1}^{S} \rho_s(\tilde{R}_{jt}) \cdot p_j(\tilde{R}_{jt}, R_{jt}^s) - c(\tilde{R}_{jt}) \geq y_{jt} \cdot p_j(\tilde{R}_{jt}) - c(\tilde{R}_{jt}) \quad \text{(12)}$$

where

$$p_j(\tilde{R}_{jt}) = \frac{\exp(\alpha_j + \sum_{k=1}^{K} 1\{\tilde{R}_{jt} = k\} \cdot \gamma_k + X_{jt} \beta)}{1 + \exp(\alpha_j + \sum_{k=1}^{K} 1\{\tilde{R}_{jt} = k\} \cdot \gamma_k + X_{jt} \beta)}. \quad \text{(13)}$$

From inequality (10), we can estimate a lower bound for parameter $\bar{y}_j$ as:
\[
\frac{\bar{y}_j - c(R_{j\star}) - c(R_{j\star})}{p_j(r_{j\star}) - p_j(R_{j\star})} \geq -\xi_{jt}
\]

\[
=> \bar{y}_j - \frac{1}{T} \sum_{t=1}^{T} \frac{c(R_{j\star}) - c(R_{j\star})}{p_j(r_{j\star}) - p_j(R_{j\star})} \geq -\frac{1}{T} \sum_{t=1}^{T} \xi_{jt} = 0
\]

The last equality comes from the model assumption that \(\xi_{jt}\) is a deviation from \(\bar{y}_j\) with mean zero. Similarly, as the click-choice probability \(p_j(r_{j\star})\) is lower than \(p_j(R_{j\star})\), from inequality (12), we can estimate an upper bound for \(\bar{y}_j\) using the following condition:

\[
\frac{c(R_{j\star}) - c(R_{j\star})}{p_j(R_{j\star}) - p_j(r_{j\star})} - \bar{y}_j \geq \xi_{jt}
\]

\[
=> \frac{1}{T} \sum_{t=1}^{T} \frac{c(R_{j\star}) - c(R_{j\star})}{p_j(R_{j\star}) - p_j(r_{j\star})} - \bar{y}_j \geq \frac{1}{T} \sum_{t=1}^{T} \xi_{jt} = 0
\]

We note that inequalities (14) and (15) do not require unique equilibrium. There is even no need for a unique equilibrium selection process in the data, which is different from the previous literature. We have not imposed assumptions of rational beliefs or the order of decision making among advertisers in the model. All we rely on is the assumption that the expected profit of the observed advertising position of any advertiser is higher than other advertising positions that are still available. For model estimation, it only requires that the average of the random profit shock \(\xi_{jt}\) is zero. Finally, we are agnostic about the data-generating process of \(\xi_{jt}\) which may be highly correlated across advertisers and/or over time (e.g., due to macro shocks).

The above two inequalities are based on the assumption that a higher and a lower advertising position are available. However, it is possible that advertiser \(j\)’s position is at boundaries. That is, it either cannot move down or cannot move up. An example of the former is that advertiser \(j\) does not purchase a position in the sponsored search list (i.e., \(R_{j\star} = 0\)), and for the latter is that advertiser \(j\) purchases the top position (i.e., \(R_{j\star} = 1\)). If we exclude these
boundary observations in the model estimation, the inequalities may be invalid because of the selection issue (i.e., the excluded $\xi$’s are different from those included). Pakes et al. (2007) suggested that in these cases we can substitute a random variable on the left-hand side that is larger than the right-hand $-\xi_t$ in inequality (14) or $\xi_t$ in inequality (15) with probability one, to make sure the inequalities will be indeed valid. In our application, we choose to substitute by our estimate for $y_j$. This implies that $y_j + \xi_t$ and $y_j - \xi_t$ are always positive, which is equivalent to saying that for every advertiser the expected profit per click in each period is always positive and that the magnitude of the random profit shock is not larger than the mean profit $\overline{y}_j$.

If there are multiple higher or lower positions available, we choose the $R_{jt}$ that maximizes $\frac{c(R_{jt}) - c(\tilde{R}_{jt})}{p_j(r_{jt}) - p_j(\tilde{R}_{jt})}$, and the $\tilde{R}_{jt}$ that minimizes $\frac{c(\tilde{R}_{jt}) - c(R_{jt})}{p_j(\tilde{R}_{jt}) - p_j(r_{jt})}$. This guarantees that we will estimate the tightest bounds for $\overline{y}_j$. Suppose no $\overline{y}_j$ can simultaneously satisfy both inequalities (14) and (15), which is the case when $\frac{1}{T} \sum_{t=1}^{T} \frac{c(R_{jt}) - c(\tilde{R}_{jt})}{p_j(r_{jt}) - p_j(\tilde{R}_{jt})}$ is larger than $\frac{1}{T} \sum_{t=1}^{T} \frac{c(\tilde{R}_{jt}) - c(R_{jt})}{p_j(\tilde{R}_{jt}) - p_j(r_{jt})}$. Pakes et al. (2007) suggested choosing the value that minimizes the difference, which will be the average of the two values. Estimated bounds in this case converge to a point estimate. We find in our empirical applications that the lower and upper bounds for every estimate are always different.

Estimated bounds for $\overline{y}_j$ can be further tightened by using other instrumental variables. Suppose we have an $m$-dimensional vector of instruments $Z_{jt}$ such that the moment condition $E(\xi_t | Z_{jt}) = 0$ is satisfied. We can construct more moment inequalities from inequalities (14) and (15):
\[
\frac{1}{T} \sum_{t=1}^{T} \left( \frac{c(R_{jt}) - c(R_{jt})}{p_j(r_{jt}) - p_j(R_{jt})} - \bar{y}_j \right), Z_{jt} \geq 0
\] (16)

and

\[
\frac{1}{T} \sum_{t=1}^{T} \left( \frac{c(R_{jt}) - c(R_{jt})}{p_j(R_{jt}) - p_j(r_{jt})} - \bar{y}_j \right), Z_{jt} \geq 0
\] (17)

Each inequality above is \( m \)-dimensional. We find that using more instruments (other than advertiser indicators as implied by inequalities (14) and (15)) helps to tighten the estimated lower and upper bounds for \( \bar{y}_j \), since the inequalities conditions have become stricter. In our empirical applications \( Z_{jt} \) includes advertiser indicators, costs-per-exposure for all advertising positions, and the vector of selling propositions.

3.5 Model Estimation and Asymptotic Distribution

In this section, we describe the procedure of our model estimation. We estimate our model in two steps. In the first step we estimate the binary logit model of click choice and obtain model estimates (\( \alpha \)'s, \( \gamma \)'s and \( \beta \)'s) in equation (9). In the second step, we estimate the lower bound for \( \bar{y}_j \), denoted as \( \theta_j \), from inequality (16), and the upper bound for \( \bar{y}_j \), denoted as \( \tilde{\theta}_j \), from inequality (17). To this end, we first plug in the first-step model estimates to compute \( \hat{p}_j(r_{jt}) \), \( \hat{p}_j(R_{jt}) \) and \( \hat{p}_j(R_{jt}) \) and substitute into the two inequalities. The lower- and upper-bound estimates for mean profit per click are obtained using the following procedure:

\[
\hat{\theta}_j = \arg \min_{\theta \in \mathbb{R}} \theta
\]

s. t. \[
\frac{1}{T} \sum_{t=1}^{T} \left( \theta - \frac{c(R_{jt}) - c(R_{jt})}{p_j(r_{jt}) - p_j(R_{jt})} \right), Z_{jt} \geq 0
\] (18)

and
\[ \hat{\theta}_j = \arg \max_\theta \in \mathbb{R} \]
\[ s. t. \quad \frac{1}{T} \sum_{t=1}^T \left( \frac{c(R_{jt}) - c(R_{jt})}{p_j(R_{jt})^2 - p_j(R_{jt})} - \theta \right) \cdot Z_{jt} \geq 0 \]  

(19)

Standard errors of these moment inequality estimators are difficult to compute analytically. Pakes et al. (2007) developed a procedure to approximate the confidence intervals for estimates \( \hat{\theta}_j \) and \( \hat{\theta}_j \). They discussed two simulation methods for this procedure, which will lead to the “outer” confidence intervals that will stochastically dominate the true asymptotic distribution, and the “inner” confidence intervals that will be stochastically dominated. The inner confidence intervals will converge to the true limiting distributions of the boundary estimators if the number of binding moments is the same as the number of estimated parameters.

We will describe how to construct the confidence intervals for the lower-bound estimate \( \hat{\theta}_j \) and refer interested readers to Pakes et al. (2007) for the formal proof. Let \( m(Z, \hat{\theta}_j) \) be a \( T \times 1 \) vector of which the \( t \)-th component is

\[ \left( \hat{\theta}_j - \frac{c(R_{jt}) - c(R_{jt})}{p_j(R_{jt}) - p_j(R_{jt})} \right) \cdot Z_{jt} \],

and let

\[ Pm(Z, \hat{\theta}_j) = \frac{1}{T} \sum_{t=1}^T \left( \hat{\theta}_j - \frac{c(R_{jt}) - c(R_{jt})}{p_j(R_{jt}) - p_j(R_{jt})} \right) \cdot Z_{jt} \].

Let \( \Gamma = \frac{\partial}{\partial \theta} Pm(Z, \hat{\theta}_j) \) and \( \Sigma = \text{Var}(m(Z, \hat{\theta}_j)) \). We draw \( Z^* \sim \text{normal}(0, \Sigma) \) and compute

\[ Z_{1-\alpha/2}^*(Z^*) = \min \{ \tau : 0 \leq \tau + Z^* + \sqrt{T} Pm(Z, \hat{\theta}_j) \} \]  

and also

\[ Z_{\alpha/2}^*(Z^*) = \min \{ \tau : 0 \leq \tau + Z^* \} \]  

(20)

Let \( q_{\alpha/2}^* \) and \( q_{1-\alpha/2}^* \) be the \((\alpha/2)\)-th and \((1-\alpha/2)\)-th percentiles of \( Z_{1-\alpha/2}^*(Z^*) \), respectively, and let \( q_{\alpha/2}^{**} \) and \( q_{1-\alpha/2}^{**} \) be the same percentiles of \( Z_{\alpha/2}^*(Z^*) \), respectively. The inner \((1-\alpha)\) confidence
intervals for \( \hat{\theta}_j \) can be constructed as \((\hat{\theta}_j - q_{a/2}^*/\sqrt{T}, \hat{\theta}_j - q_{a/2}^{**}/\sqrt{T})\), and the outer \((1-\alpha)\) confidence intervals for \( \hat{\theta}_j \) as \((\hat{\theta}_j - q_{a/2}^*/\sqrt{T}, \hat{\theta}_j - q_{a/2}^{**}/\sqrt{T})\).

We can also simulate \( T^*(Z^*) \) and \( T^*_1(Z^*) \) for the upper-bound estimate \( \hat{\theta}_j \) using a similar procedure, and compute the quantiles \( q_{1-a/2}^*, q_{1-a/2}^*, q_{1-a/2}^{**} \) and \( q_{1-a/2}^{**} \). The outer \((1-\alpha)\) confidence intervals for \( \hat{\theta}_j \) are \((\hat{\theta}_j - q_{1-a/2}^*/\sqrt{T}, \hat{\theta}_j - q_{a/2}^{**}/\sqrt{T})\), and the inner \((1-\alpha)\) confidence intervals are \((\hat{\theta}_j - q_{1-a/2}^{**}/\sqrt{T}, \hat{\theta}_j - q_{a/2}^*/\sqrt{T})\). With these the inner and outer \((1-\alpha)\) confidence intervals for the mean profit per click \( \bar{y}_j \) will be

\[
(\hat{\theta}_j - q_{1-a/2}^*/\sqrt{T}, \hat{\theta}_j - q_{a/2}^{**}/\sqrt{T}), \ (\hat{\theta}_j - q_{1-a/2}^{**}/\sqrt{T}, \hat{\theta}_j - q_{a/2}^*/\sqrt{T}),
\]

respectively.

4. Results

In this section, there are three major areas that we report upon in summarizing our results. First, we describe the parameter estimates of our click-choice model and report the mean profit per click for advertisers based on the position-competition model. Second, we quantify the impact of BIN prices on the expected profit of the search engine. Finally, we conduct some “what-if” experiments to investigate the optimality of the pricing policy of the search engine.

4.1 Model Results

Table 1 reports the main results of the click-choice model. Model 1 includes such covariates as advertising positions, advertiser identities, and a set of selling propositions. Models 2 and 3 are nested models to Model 1. Comparing the three models, our estimates are quite robust to various specifications. Adding the information from selling propositions performs best in terms of the
model fit. Given the completeness of Model 1 for having accounted the information of selling propositions, we will focus on results of Model 1 hereon.

**Insert Table 1 about here**

The major findings suggested are as follows. First, the estimates of all advertising position indicators (from the first to the fifth position) are significantly associated with the click-choice probability.\(^{14}\) In particular, we find a strong position effect of the advertisement link on the search engine, i.e., all else equal, higher positions attract more clicks. This result is in line with the conventional wisdom in the industry and academic research that the higher the rank of the advertisement, the higher the click-through rate (e.g., Agarwal et al. 2008, Feng et al. 2007, Ghose and Yang 2009). One point is worth noting. Unlike the existing studies assuming that the mean position during the day or the week is the actual position of an advertiser, this research utilizes the exact measurement of advertising positions and confirms the finding reported in the prior research. A simple numerical exercise also reveals another interesting pattern. Suppose the number of categories and \(\ln(\text{discount}+1)\) are all zero, the click-choice probability for Advertiser 6 is only 0.028 if it is at the fifth position. This probability will increase by 31% to 0.037 if it moves up to the fourth position. There is also a 17% increase in click-choice probability when the advertiser moves from the second to the top position. However, moving between the positions in the middle (from the second to the fourth position) does not significantly impact the click-choice probability. In particular, moving from the third to the second position only increases the click-choice probability by 3%. This nonlinear relationship in click-through rates has useful implications for managers interested in quantifying the impact of advertising positions on click-through rates among others.

\(^{14}\) We are able to estimate the impacts of all advertising positions because the fixed effect of Advertiser 6 is normalized to 0 (see Table 1).
Second, the estimates of all advertisers except Advertiser 3 in Table 1 are significant, indicating that advertiser identity can lead to a significant increase or decrease in click-choice probability compared to the base level, Advertiser 6. The most attractive website is Advertiser 3, which is a well-known online retailer in Korea, while the least attractive website is Advertiser 1, which is a popular online auction website. For average consumers who are searching for product and price information, Advertiser 1 is probably the last choice among the six websites they would like to click into. Finally, the estimates of selling propositions are significant indicating that keyword advertisements that contain assortment-specific information and discount-specific information can lead to a significant increase in the click-choice probability. These results are useful for managers because they imply that keyword advertisements that explicitly contain such selling propositions lead to higher click-through rates.

We now turn to our discussion to structural model estimation of the position competition among advertisers. Table 2 reports the estimates of mean profit per click (both lower- and upper-bound estimates) for each advertiser using moment inequalities. In addition, the table includes the simulated 90th and 95th percentile confidence intervals for the lower- and upper-bound estimates. In all the estimates reported in Table 2, the lower- and upper-bound estimates are quite close with each other. Furthermore, the 90th and 95th percentile confidence intervals are very close to the estimates (for most estimates the differences are smaller than two decimal places.) This is a very encouraging sign since it implies that, using only the necessary equilibrium conditions without imposing restrictive model assumptions, our estimators are reasonably precise.

**Table 2 about here**

Table 2 shows that the mean profit per click of Advertiser 1 ($1.68 to $1.89) is significantly higher than the others. In contrast, the advertiser intercept estimate of Advertiser 1
In the click-choice model is the smallest among all advertisers (see Table 1), that is, its sponsored advertisement generated the lowest click-through rate during the data period. As shown in Figure 1, this website purchased advertising positions for its sponsored advertisement most often (165 days out of 254 days) in the data. In addition, out of the 165 days, Advertiser 1 obtained the top position for 76 days. Such aggressive strategy may imply that this advertiser understands the added value of sponsored advertisement to its business. Interestingly, Advertiser 1 is a popular auction website in Korea. Our results may indicate a self-selection process on the part of consumers; consumers know what this website is known for, so those who click in may already have a strong incentive to purchase specific items at this website. Moreover, unlike other online retailers, Advertiser 1 does not purchase products from suppliers. Its revenue mainly comes from the commission charged to auction sellers, hence its marginal cost is much lower than other websites. All these may explain why the mean profit per click of Advertiser 1 is the highest among the six advertisers. In contrast, as the most attractive website generating the highest click-through rate (see Table 1), Advertiser 3 has the smallest mean profit per click (35 to 42 cents). As shown in Figure 1, this advertiser displayed its advertisement least often (64 days in the data period). Its low profit per click may be due to higher operation costs, less attractive product offerings, or simply that consumers who click in do not have strong incentive to purchase products. Instead, they may just want to check for product-related information at this website.

The estimates of mean profit per click are important measures for managers to understand the value of advertisements in sponsored search advertising.

4.2 Characterization of Multiple Equilibria

With the estimated mean profit per click for advertisers, we now investigate the impact of BIN prices on the expected profit of the search engine via the advertiser position competition. In order
to achieve this objective, we face a couple of problems. First of all, as discussed earlier, we can only estimate the boundaries for the profit per click for each advertiser (both lower- and upper-bound estimates). Second, in our modeling approach we do not take position on, when multiple equilibria exist, how an equilibrium will be selected. Each set of estimated profits per click can be mapped to multiple equilibrium outcomes, all consistent with the necessary equilibrium conditions. Because of these problems, we study a set of equilibrium outcomes associated with the $J \times 1$ vectors of lower-bound profit-per-click estimates $\hat{\theta}$ and the upper-bound estimates $\hat{\theta}$, of which the $j$-th elements are $\hat{\theta}_j$ and $\hat{\theta}_j$, respectively. We thus examine the advertising position of each advertiser and the expected profit level for the search engine for all possible equilibria.

Since the true profits per click always fall between $\hat{\theta}$ and $\hat{\theta}$, the true profit for the search engine should also fall between the expected profit generated from $\hat{\theta}$ and $\hat{\theta}$, based on any equilibrium selection criteria. Such exercise will provide some characterization for the multiple possible equilibria in the search advertising context.

Given that for each advertiser $j$ in each period $t$, the actual profit per click is $y_{jt} = \bar{y}_{jt} + \xi_{jt}$ as noted in equation (2), the decisions of advertisers will depend on not only the mean profit per click $\bar{y}_{jt}$ but also the random profit shock $\xi_{jt}$. For each value of $\bar{y}_{jt}$, the first inequalities in (14) and (15) provide us the range for the possible values of $\xi_{jt}$:

$$\frac{c(R_{jt}) - c(R_{jt})}{p_{jt}(r_{jt}) - p_{jt}(R_{jt})} - \bar{y}_{jt} \leq \xi_{jt} \leq \frac{c(\hat{R}_{jt}) - c(\hat{R}_{jt})}{p_{jt}(\hat{R}_{jt}) - p_{jt}(R_{jt})} - \bar{y}_{jt},$$  \hspace{1cm} (22)

such that the observed advertising position $R_{jt}$ for each advertiser is consistent with the necessary equilibrium conditions. We first set $\bar{y}_{jt} = \hat{\theta}_{jt}$ for all advertisers $j$. We uniformly draw
within the range in equality (22) for NS times\textsuperscript{15} for all advertisers in each period \( t \) within the data period. Each draw generates a \( J \times 1 \) vector of simulated \( \hat{\xi}_t^{s} \), \( s = 1, \ldots, NS \), where its \( j \)-th element is denoted as \( \hat{\xi}_t^{s,j} \). Then we construct all potential equilibria by permuting the sponsored link positions \{0,1, ..., 5\} (including the outside option) for all advertisers, under the condition that the top to the bottom position can be occupied at most by one advertiser. Let \( (R_1, \ldots, R_n) \) be one of these combinations for Advertisers 1 to 6. For advertiser \( j \), let \( (\bar{R}_j, R_{-j}) \) be an alternative combination, where \( \bar{R}_j \neq R_j \) is another available position that advertiser \( j \) can purchase (including the outside option), conditional on other advertisers have occupied positions \( R_{-j} \).

Given simulated \( \xi_t^{s,j} \), we compare the expected profit \( E\pi_{\mu}(R_j, R_{-j}; \hat{\xi}_t^{s,j}) \) with the expected profit \( E\pi_{\mu}(\bar{R}_j, R_{-j}; \hat{\xi}_t^{s,j}) \) (see equation (1)). We repeat this exercise for each available alternative position \( \bar{R}_j \) for each advertiser \( j \). If none of the other available positions can generate a higher expected profit for any of the advertisers, \( (R_1, \ldots, R_n) \) is one of the permissible equilibria given \( \hat{\xi}_t^{s,j} \). We search for all possible equilibria for each \( \hat{\xi}_t^{s,j} \) in each period \( t \). After this we use a similar procedure to search for the possible equilibria for the upper-bound profit-per-click estimates \( \hat{\Theta} \).

Table 3 reports some of the statistics for the simulated equilibria. We choose three sets of BIN prices in data: The prices were the highest in Jan. 2008 (3.2, 2.8, 2.4, 2.0 and 1.6 cents from the first to the fifth position, respectively). In order to increase demand for keyword advertising, the search engine lowered the prices in June (2.7, 2.3, 1.9, 1.8 and 1.6 cents from the first to the fifth position, respectively) and lowered further in Aug. 2008 (2.3, 1.9, 1.9, 1.8 and 1.3 cents from the first to the fifth position, respectively). We assume that these three sets of BIN prices

\textsuperscript{15} In practice we fix NS to be 100.
are constant over the whole data period and use the procedures described above to generate equilibria in every day. As shown in Table 3, our results show that the average number of possible equilibria (over all simulated draws in all periods) ranges from 45 under the BIN prices in Jan. 2008 using the lower-bound profit estimates to 121 under the BIN prices in Aug. 2008 using the upper-bound profit estimates.\(^\text{16}\) This highlights that the existence of multiple equilibria is very common; indeed unique equilibrium rarely exists for any of the simulated $\hat{\xi}_{st}$ in any period. In general, the lower the BIN prices and the higher the profit-per-click estimates, more advertisers will be willing to pay for the limited number of advertising positions and hence the larger will be the number of permissible equilibria. The intense competition among advertisers generates various types of interdependences in obtaining advertising positions that may emerge to equilibria.

Insert Table 3 about here

The six rows under the first “Percentage (%) of equilibria with” in Table 3 summarize how many advertising positions are purchased under different equilibria. Under the BIN prices in Jan. 2008, for example, 14.3% of the equilibria (over all simulated draws $\hat{\xi}_{st}$ in all periods $t$) have just one position occupied using the lower-bound profit estimates, regardless of which position is occupied. Using the upper-bound profit estimates it lowers to 12.1% of the equilibria. If we look at the demand for search advertising in terms of how many positions are purchased, another clear pattern emerges. The higher the BIN prices, the less likely that advertising positions will be purchased. For instance, all five advertising positions are purchased only 6.0% and 9.8% of times under the BIN prices in Jan. 2008, depending on using the lower-bound or

\(^{16}\) The total number of unique combinations of nominal advertising positions (including the outside option) is 4051 based on numerical calculation.
upper-bound profit estimates, compared with 21.3% and 23.0% of times under the BIN prices in Aug. 2008.

The next six rows under the second “Percentage (%) of equilibria with” in Table 3 indicates that the top position will be purchased 24.1% and 32.5% of the equilibria under the BIN prices in Jan. 2008, which are far less than the ones under the BIN prices in Aug. 2008 (69.8% and 73.3%, respectively). We also find a similar pattern for the demand for the second position. These results show that the search engine’s purpose of stimulating demand for sponsored search advertising was mainly achieved through lowering the BIN prices in the later part of the data period.

The last three rows in Table 3 compare the total expected profit for the search engine from Jan. 2008 to Aug. 2008 at the three different BIN prices based on different equilibrium selection criteria. We compare three scenarios: “Minimum profit” is calculated choosing the equilibrium with the lowest profit level to the search engine among equilibria, “Maximum profit” choosing the highest profit level among equilibria, and “Expected profit” of all equilibria assuming that each equilibrium has the equal probability of being chosen, all under different profit-per-click estimates. Profits based on the lower-bound and upper-bound profit-per-click estimates are quite close with each other (as shown by comparing entrants in the first with the second, the third with the fourth, and the fifth with the sixth column in the table). This suggests that our boundary estimators for profit per click predict quite tight profits for the search engine. For researchers and the search engine, however, the real “uncertainty” in predicting profitability comes from which equilibrium will be chosen. For example, the range of profit between the minimum and maximum profit equilibria under the BIN prices in Jan. 2008 was from $1,983 to $3,392 using the lower-bound estimates, and from $2,200 to $3,575 using the upper-bound
estimates. Aside from the minimum profit equilibrium, both the maximum profit and the random equilibrium scenario predict that the BIN prices in Jan. 2008 will generate the highest profit for the search engine. This implies that the policy of lowering BIN prices, though stimulating demand for advertising positions, is likely to reduce the profit for the search engine at least in the time periods studied in this research.

4.3 Some “What-If” Experiments

We further examine in this section the optimality of the pricing policy of the search engine. One of the advantages of structurally modeling the competition among advertisers is that we can also compare profit levels in some “what-if” scenarios even though these have never happened in the data. Since we have found that the BIN prices in Jan. 2008 are better than the BIN prices in later months for most equilibrium selection criteria, we explore here how the search engine may further improve its profit by changing its prices from the price level of \{3.2, 2.8, 2.4, 2.0, 1.6 cents\} in Jan. 2008. We experiment by cutting and raising these prices by 5% and 10%. To compute the expected profit, we assume that all equilibria in any period \(t\) with simulated \(\hat{\xi}_t\) have the same probability of being chosen. We then take the average over all \(\hat{\xi}_t\) as the expected profit in period \(t\), and finally sum up over all periods as the total expected profit for the search engine.

The four rows under “Changes in BIN Prices” in Table 4 report the results. If we use the lower-bound profit-per-click estimates, expected profit for the search engine is the highest when the BIN prices are cut by 5%. However, if the upper-bound profit-per-click estimates are used, the best for the search engine is to raise BIN prices by 5%. As the true profit per click falls between the lower and upper bounds, we cannot determine whether the BIN prices should be

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17 We do not assume that a single equilibrium will be chosen throughout the sample period. However, a deeper examination reveals that the BIN prices in Jan. 2008 dominate BIN prices in other periods in terms of total profit for the search engine in most of the equilibrium selection criteria. Results are available upon request from the authors.
increased or decreased from the profit maximization perspective. Still, the results show that either raising or cutting prices by 5% dominates changing prices by a larger extent of 10%, though the difference is marginal. This implies that the BIN prices in Jan. 2008 may be close to the optimal level. The search engine may further experiment with increasing or decreasing prices for various advertising positions within a small range (say, 5%) to test how the actual profit will change.

**Insert Table 4 about here**

We also experiment a policy change of reducing the available number of advertising positions to four from five. This policy change may be beneficial from the managerial perspective because of two reasons. First, it reduces the chance of irritating potential consumers if they find too many sponsored advertisements in search results. Second, by reducing the available number of advertising positions it may strengthen the competition among advertisers and eventually lead to a profit increase. For example, an advertiser may decide to purchase the fifth position at the lowest price if it expects no other advertisers are willing to pay for higher positions. By eliminating the fifth position the advertiser is forced to pay a higher price for the fourth position. We assume in this exercise that the BIN prices remain the same as the original prices from the top to the fourth position. The last row of Table 4 shows the result. Using either the lower-bound or the upper-bound profit-per-click estimates, we find the expected profit for the search engine will be lowered to $2,641 and $2,813, respectively. These are lower than the profit levels with five advertising positions (see the first row in Table 4). However, the profit decreases are much smaller than the revenue generated from the fifth position based on the previous calculation when five positions are available, implying that some advertisers are paying more for higher positions when fewer advertising positions are available. The search engine can use these
results to decide whether or not it is worthwhile to cut down the available number of advertising positions.\(^{18}\)

5. Conclusions

The rapid rise of sponsored search has fueled recent academic research into it. This research focuses on modeling the position competition of advertisers in the sponsored search list. To achieve this goal, we describe necessary conditions of equilibrium outcome to model how advertisers compete for advertising positions in sponsored search advertising. As an intermediary for our estimation of the advertiser competition, we also model the consumer’s click choice.

Using a unique database of a brand-name keyword provided by a leading search engine site in Korea, we find that unique equilibrium in advertising competition rarely exists. Our policy simulation shows that the lower the BIN prices, the higher the profit-per-click estimates, which then generates the larger number of equilibria. Also, the demand generation from advertisers was achieved through lowering the BIN prices. However, this policy of lowering BIN prices reduced the profit for the search engine. This research sheds new light on the important topic of pricing policy for sponsored search advertising. We showed how the search engine can implement the optimal pricing policy by changing the BIN prices and changing the number of available advertising positions in the sponsored search list.

Another interesting set of results is based on the consumer’s click model. We find a strong position effect of advertisement link on the search engine. While this is consistent with the findings in the existing literature, our database helps us find that while the top (bottom) position is the most (least) desirable for advertisers, the positions in the middle (from second to fourth

\(^{18}\) We also experimented by increasing and decreasing the BIN prices for the four positions by 5% and 10%. All results show a lower expected profit for the search engine compared to the one when five advertising positions are available.
position) are not significantly different from each others. In addition, our results show that advertiser identity and selling propositions (assortment-related information and price-related information) can lead to a significant increase or decrease in click-through rates. These results are useful for managers to make better decisions regarding advertising campaigns in the online domains.

Since this research is among the first attempts to study the position competition of advertisers by allowing the existence of multiple equilibria, we have kept the model as parsimonious as possible to highlight the key phenomena that we have identified from the data. We thus note a number of caveats and limitations in our research that should be acknowledged and perhaps addressed in future research. First, our model of consumer’s click choice is based on the binary logit model as in Yao and Mela (2009b). Park and Park (2009) propose a model of interdependent consumer click decisions in sponsored search advertising which models potential associations across advertisers through consumer search behavior. While this would increase complexity, it is a fruitful area for future research to explicitly account for consumer search behavior to the position competition of advertisers in sponsored search advertising. Second, we restrict our interest in this research to sponsored search advertising. Another area for future work is to investigate how position competition is impacted by the organic search results. For example, an advertiser whose link is placed high in organic search results may have less incentive to purchase the topmost positions in the sponsored search list. Lastly, as competing advertisers, in particular, online retailers purchase the same set of multiple similar keywords to attract demand for their business, another area for future research could be to examine the position competition of advertisers across multiple keywords. We hope our model provide a framework for further empirical exploration.
References


Table 1: Model Results of Click Choice

| Coefficients | Model 1 | | Model 2 | | Model 3 | |  
| Positions | | | | | | |  
| 1st position | -2.99 | 0.12 | -2.83 | 0.10 | -2.67 | 0.04 |  
| 2nd position | -3.15 | 0.10 | -3.06 | 0.10 | -2.91 | 0.05 |  
| 3rd position | -3.18 | 0.09 | -3.09 | 0.08 | -2.98 | 0.06 |  
| 4th position | -3.27 | 0.15 | -3.19 | 0.15 | -3.08 | 0.14 |  
| 5th position | -3.55 | 0.19 | -3.46 | 0.19 | -3.36 | 0.18 |  
| Advertisers | | | | | | |  
| Advertiser 1 | -1.19 | 0.15 | -1.05 | 0.13 | -1.16 | 0.12 |  
| Advertiser 2 | -0.79 | 0.11 | -0.70 | 0.10 | -0.83 | 0.07 |  
| Advertiser 3 | 0.19 | 0.18 | 0.02 | 0.16 | 0.10 | 0.15 |  
| Advertiser 4 | -0.37 | 0.18 | -0.19 | 0.05 | -0.19 | 0.05 |  
| Advertiser 5 | -0.80 | 0.18 | -0.63 | 0.16 | -0.64 | 0.16 |  
| Advertiser 6: Base | 0.00 | -- | 0.00 | -- | 0.00 | -- |  
| No. of categories | 0.06 | 0.03 | -- | -- | -- | -- |  
| ln(discount+1) | 0.10 | 0.04 | 0.05 | 0.03 | -- | -- |  
| Log-likelihood | -11895.8 | | -11898.2 | | -11900.0 | |
Table 2: Estimated Mean Profit per Click

<table>
<thead>
<tr>
<th>Advertiser</th>
<th>Estimates</th>
<th>Simulated 95th Percentiles</th>
<th>Simulated 90th Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>Advertiser 1</td>
<td>1.68</td>
<td>1.89</td>
<td>1.67</td>
</tr>
<tr>
<td>Advertiser 2</td>
<td>0.86</td>
<td>1.09</td>
<td>0.85</td>
</tr>
<tr>
<td>Advertiser 3</td>
<td>0.35</td>
<td>0.42</td>
<td>0.35</td>
</tr>
<tr>
<td>Advertiser 4</td>
<td>0.48</td>
<td>0.69</td>
<td>0.48</td>
</tr>
<tr>
<td>Advertiser 5</td>
<td>0.35</td>
<td>0.57</td>
<td>0.35</td>
</tr>
<tr>
<td>Advertiser 6</td>
<td>0.68</td>
<td>1.11</td>
<td>0.67</td>
</tr>
</tbody>
</table>
Table 3: Profit Estimates of Simulated Equilibria

<table>
<thead>
<tr>
<th></th>
<th>BIN prices in Jan. 08 (3.2,2,8,2,4,2,0,1.6)*</th>
<th>BIN prices in Jun. 08 (2.7,2,3,1.9,1.8,1.6)*</th>
<th>BIN prices in Aug. 08 (2.3,1.9,1.9,1.8,1.3)*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>Average no. of equilibria</td>
<td>44.8</td>
<td>57.0</td>
<td>75.1</td>
</tr>
<tr>
<td>Percentage (%) of equilibria with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 position occupied</td>
<td>1.5</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>1 position occupied</td>
<td>14.3</td>
<td>12.1</td>
<td>10.7</td>
</tr>
<tr>
<td>2 positions occupied</td>
<td>30.9</td>
<td>29.7</td>
<td>27.0</td>
</tr>
<tr>
<td>3 positions occupied</td>
<td>25.5</td>
<td>25.3</td>
<td>25.1</td>
</tr>
<tr>
<td>4 positions occupied</td>
<td>21.9</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>5 positions occupied</td>
<td>6.0</td>
<td>9.8</td>
<td>14.1</td>
</tr>
<tr>
<td>Percentage (%) of equilibria with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st position occupied</td>
<td>24.1</td>
<td>32.5</td>
<td>39.9</td>
</tr>
<tr>
<td>2nd position occupied</td>
<td>40.9</td>
<td>43.0</td>
<td>50.3</td>
</tr>
<tr>
<td>3rd position occupied</td>
<td>59.1</td>
<td>60.8</td>
<td>72.3</td>
</tr>
<tr>
<td>4th position occupied</td>
<td>73.0</td>
<td>73.5</td>
<td>70.7</td>
</tr>
<tr>
<td>5th position occupied</td>
<td>73.0</td>
<td>75.0</td>
<td>65.2</td>
</tr>
<tr>
<td>Profit ($) among equilibria</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum profit</td>
<td>1,983</td>
<td>2,200</td>
<td>2,165</td>
</tr>
<tr>
<td>Maximum profit</td>
<td>3,392</td>
<td>3,575</td>
<td>3,183</td>
</tr>
<tr>
<td>Expected profit</td>
<td>2,749</td>
<td>2,939</td>
<td>2,732</td>
</tr>
</tbody>
</table>

* Numbers in parentheses represent the BIN prices (cents) from the first to the fifth position, respectively.
Table 4: Results of “What-If” Scenarios

<table>
<thead>
<tr>
<th></th>
<th>Estimated Total Profits ($)</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original BIN prices in Jan. 2008: (3.2,2.8,2.4,2.0,1.6)</strong></td>
<td></td>
<td>2,749</td>
<td>2,939</td>
</tr>
<tr>
<td>Changes in BIN prices</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut by 5%</td>
<td></td>
<td>2,758</td>
<td>2,923</td>
</tr>
<tr>
<td>Cut by 10%</td>
<td></td>
<td>2,756</td>
<td>2,907</td>
</tr>
<tr>
<td>Raised by 5%</td>
<td></td>
<td>2,737</td>
<td>2,953</td>
</tr>
<tr>
<td>Raised by 10%</td>
<td></td>
<td>2,712</td>
<td>2,952</td>
</tr>
<tr>
<td>Changes in the number of advertising positions</td>
<td>BIN prices (3.2,2.8,2.4,2.0)</td>
<td>2,641</td>
<td>2,813</td>
</tr>
</tbody>
</table>

* Numbers in parentheses represent the BIN prices (cents) from the first to the fifth position, respectively.

** Numbers in parentheses represent the BIN prices (cents) from the first to the fourth position, respectively.
Figure 1: Advertising Positions

![Advertising Positions Chart]

- Advertiser 1
- Advertiser 2
- Advertiser 3
- Advertiser 4
- Advertiser 5
- Advertiser 6
Figure 2: BIN Prices across Advertising positions