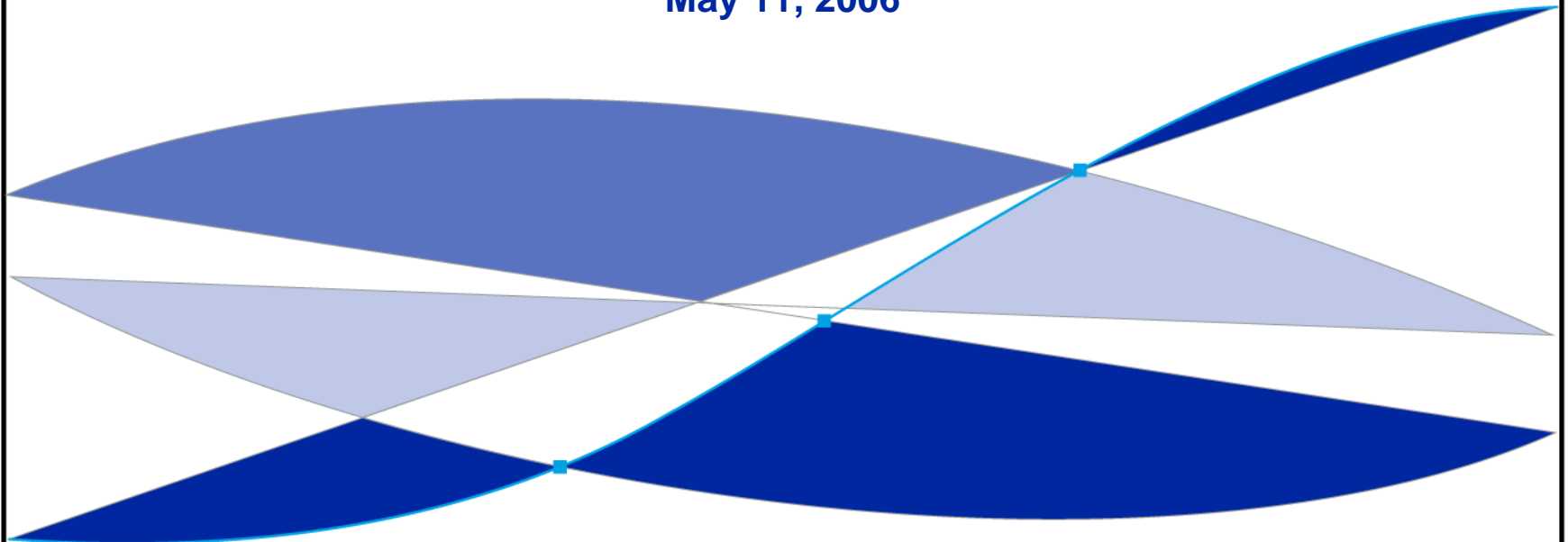


Humpbacks in Credit Spreads

May 11, 2006



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Term structure of credit spreads

Important area of study

- Slope of term structure may contain information about future credit spreads and future default likelihood.
- Focus on low quality credits : Empirical results are not unanimous, suggest upward as well as downward sloping term structures.
- Usual explanation in terms of conditional PD term structure. PD models are often validated by examining the term structures of spread.

This paper

- Show that (a) Recovery model, (b) bond price relative to its par value are important determinants of shape of spread term structure.
- Reconcile divergent empirical results on Speculative grade term structure.
- Evidence on slope of hazard function as credit quality worsens.

Determinants of spread term structure

Focus on low quality credits

Upward sloping hazard rate function (forward default probability term-structure).

	RT	RMV	RF
Zero coupon Spreads	Humped	Upward sloping	Humped
Par Spreads	Humped	Upward sloping	Upward sloping

Downward sloping hazard rate function gives downward sloping or slightly humped curves.

Empirical findings on slope of speculative grade credit curves

Sarig and Warga (1989), Fons (1994), Kealhofer (2003)

- Group bonds by credit rating.
- Speculative grade term structures are mostly downward sloping.

Helwege and Turner (1999)

- Sample selection bias – safer firms issue longer tenor bonds.
- Group bonds issued by the same issuer.
- Majority of speculative grade term structures are upward sloping.

Our findings

- Use HT methodology, a large sample of secondary market prices.
- Majority of speculative grade term structures are downward sloping.

Slope of par curves

Empirical Results:

- HT99: Par spreads are upward sloping even for low quality credits.
- Hazard function is upward sloping as well (assuming RF).
- Our results: Downward sloping zero curves (discount bond curves) can be consistent with upward sloping par curves.

Theory:

- However, many theoretical models suggest a downward sloping par curves for low quality credits.

Resolution:

- Need to go to sufficiently low quality to get downward sloping par curves and hence downward sloping hazard function.

Term structure of zero coupon spreads

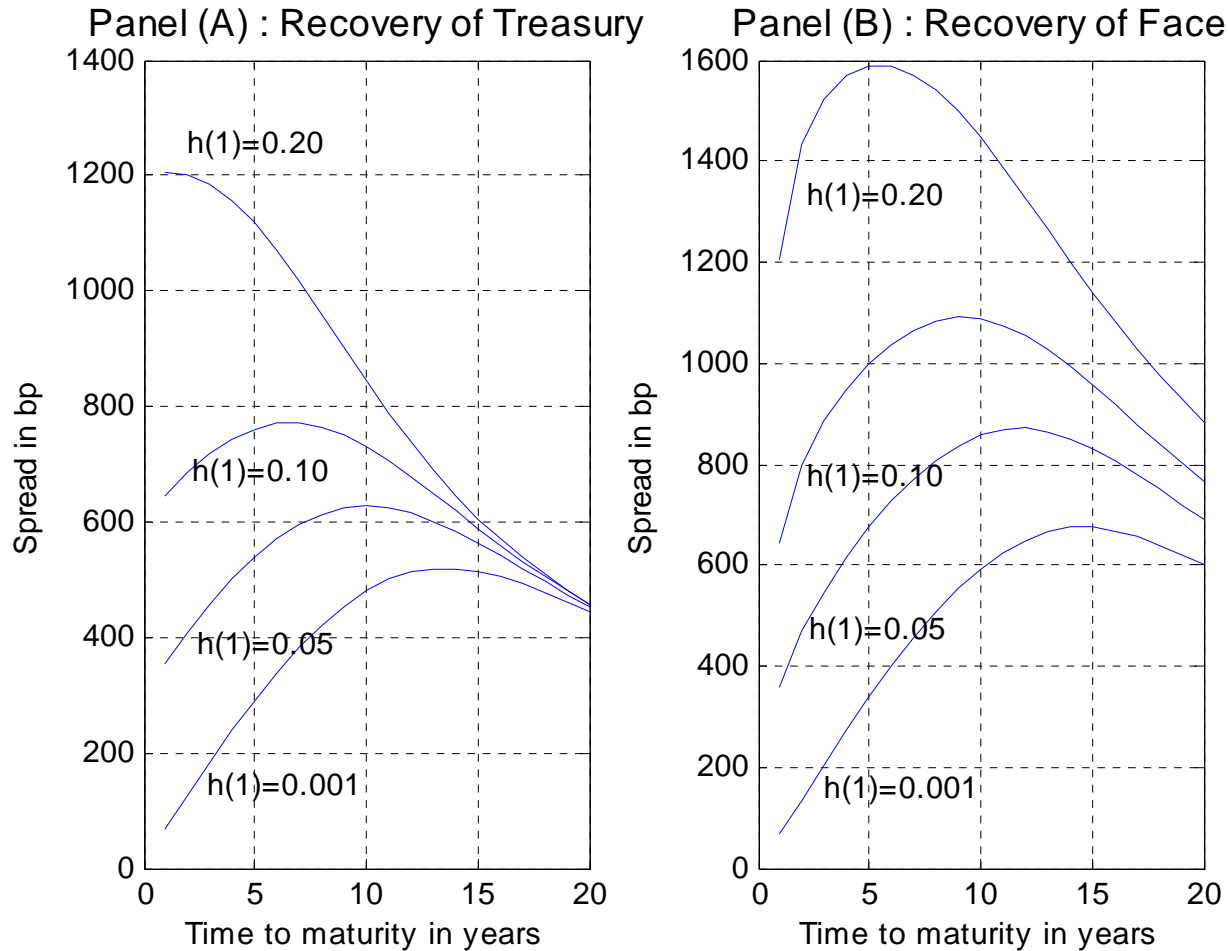
Assume,

Linear hazard rate function $h_t = h_1 + h_2 t$

Fix the slope to be positive $h_2 = 0.02$.

Allow level h_1 to vary from 0.01 to 0.20.

Zero Coupon Spreads, RT and RF models



Term structure of zero coupon spreads

RT:

$$S_{0T} = -\frac{1}{T} \ln(1 - LQ_{0T})$$
$$= \frac{1}{T} \left[LQ_{0T} + \frac{L^2 Q_{0T}^2}{2} + \dots \right]$$

For low quality credits and longer tenors, T increases faster than Q_{0T} , S_{0T} falls as T increases.

Spread reaches an upper bound of $-\frac{1}{T} \ln(1 - L)$, assume $L < 1$.

Spread is expected loss (risk neutral measure), annualized over T .

Term structure of zero coupon spreads

RF:

$$S_{0T} = -\frac{1}{T} \ln \left((1 - Q_{0T}) + \frac{1}{P_{0T}} (1 - L) \int_0^T A_t P_{0t} q_t dt \right)$$

$$= -\frac{1}{T} \ln \left((1 - Q_{0T}) + V_{R,T} \right)$$

Similar to RT, S_{0T} has an upper bound of $-\frac{1}{T} \ln(V_{R,T})$.

Under reasonable parameter values, as T increases, spread can decrease.

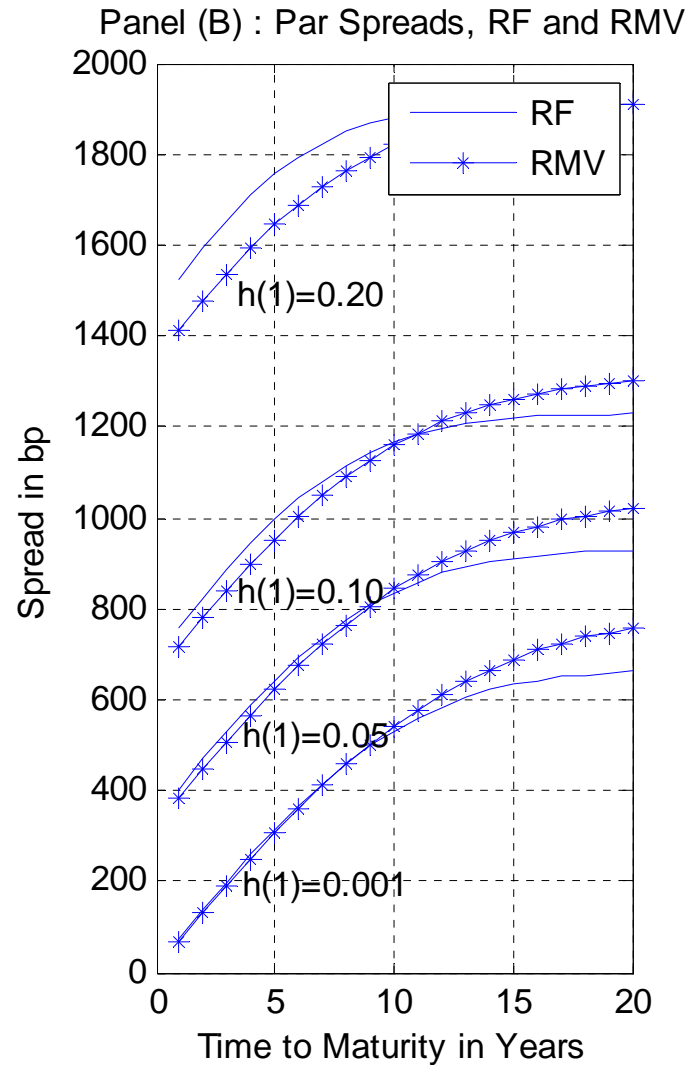
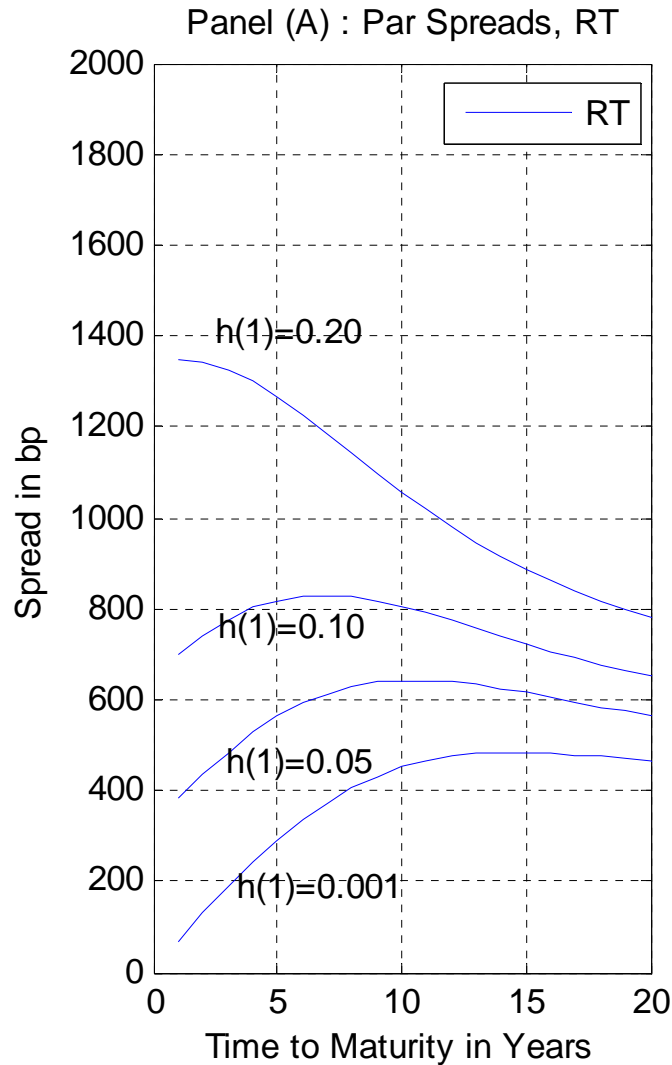
Term structure of zero coupon spreads

RMV:

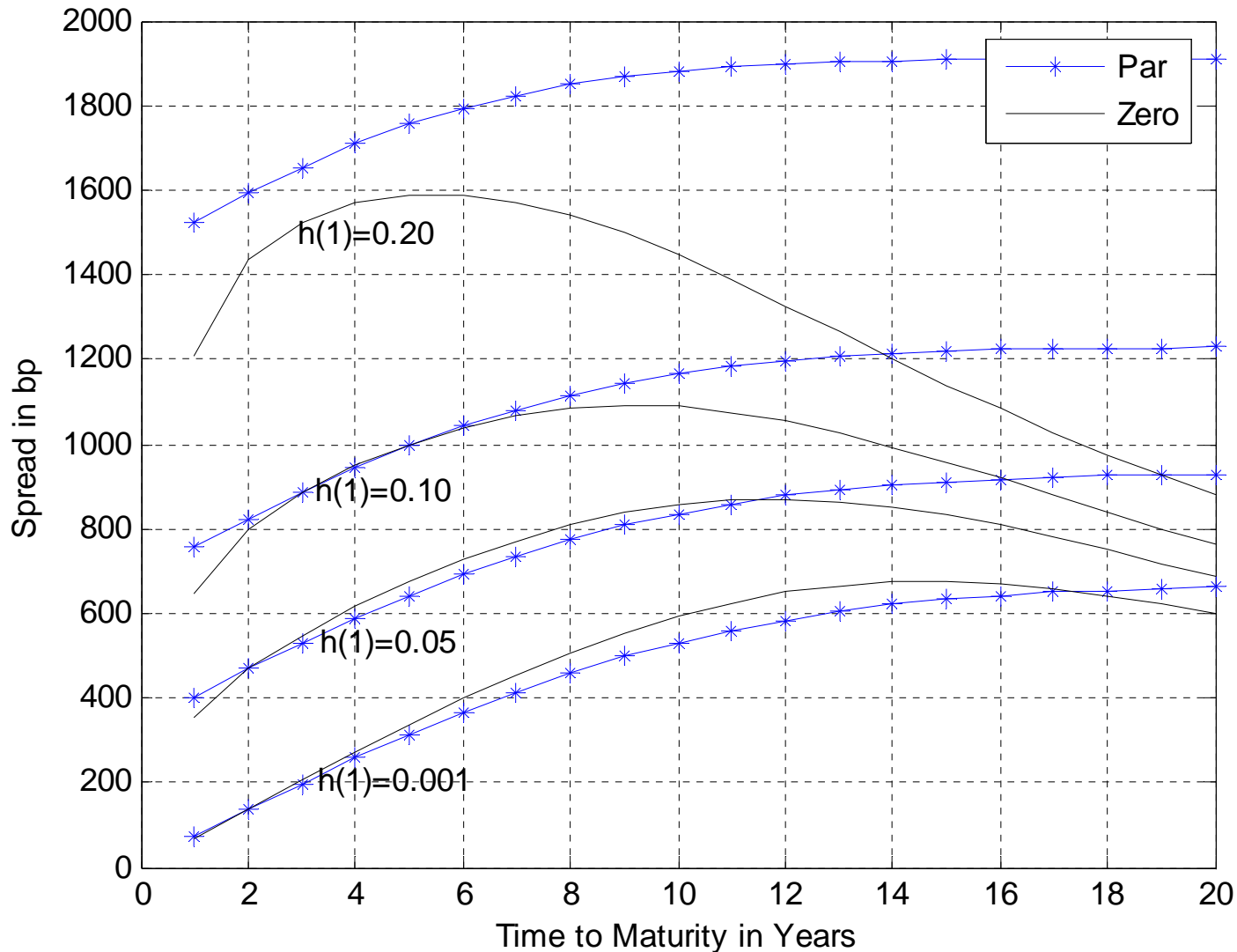
$$S_{0T} = \frac{H_{0T}L}{T} = \frac{L \int_0^T h_t dt}{T} = L(h_1 + h_2T)$$

Spread curve is upward sloping like the hazard function.

Par spreads with RT, RF and RMV models



Zero coupon versus Par Spreads with the RF assumption



Divergence between par and zero coupon curves

Under RF model,

- Only recover a fraction of face value, no recovery of coupons.
- Low quality par bond has high coupons, suffers higher effective loss compared to a deeply discounted bond or zero coupon bond.
- Par spread > zero coupon spread.
- If hazard function is upward sloping, expected loss on par bond rises more rapidly with tenor (more and more coupon payments are lost), than the expected loss on a zero.
- Drives a wedge between the two spreads.

Under RT and RMV models,

- Same fractional recovery on face value as well as coupons.

HT99 Results versus Our Results

HT99 Results		
	BB	B
No of bond pairs	27	17
Downward slopes	19%	6%

Our Results					
	Ba	B	Caa	Ca	C
No of bond pairs	1168	568	177	96	33
Downward slopes	66%	63%	77%	60%	73%

HT99 Results versus our Results

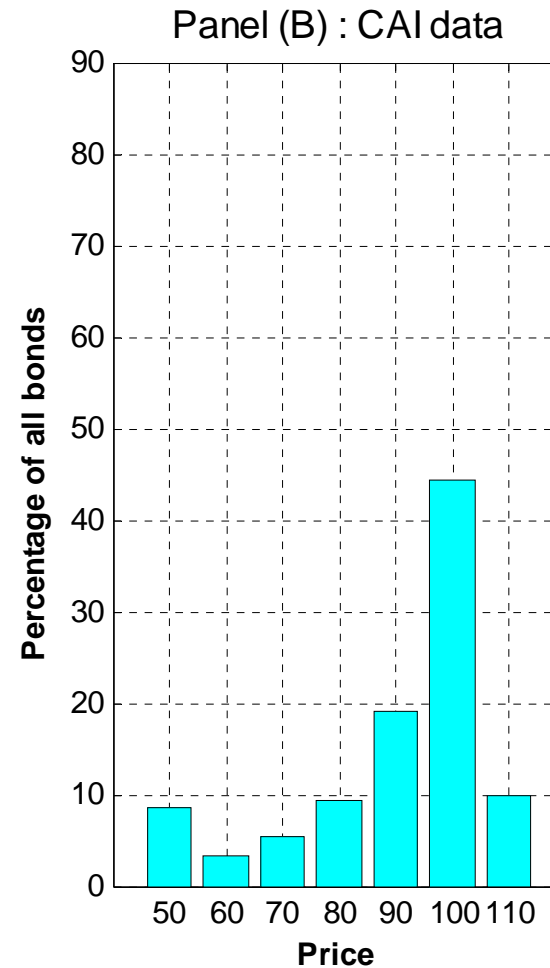
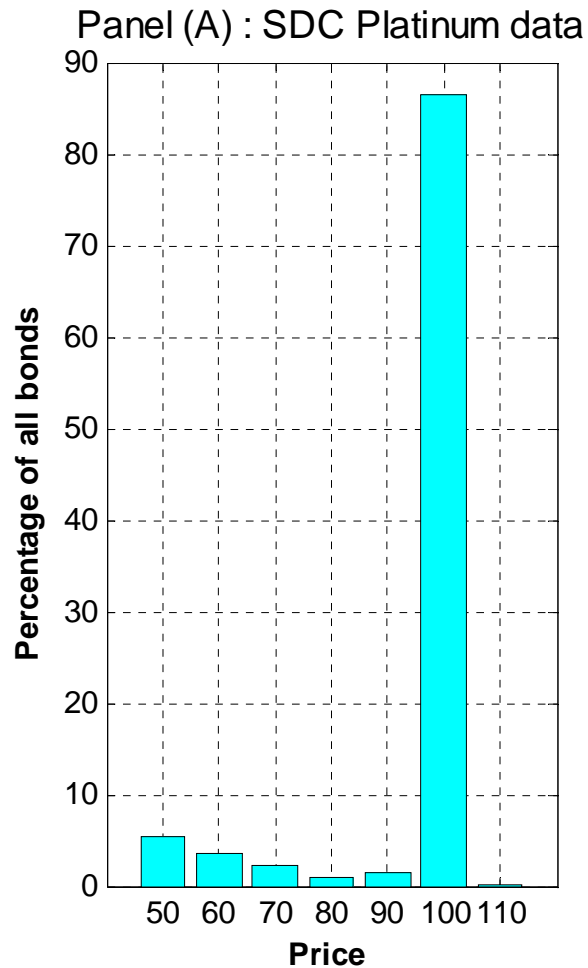
Correcting for sample selection bias is important but, by itself, does not overturn the results of earlier studies.

What gives?

Robustness Checks

- Removed bonds with embedded options
- Spread over swaps versus spread over treasury
- Slope of spread term structure over time

Can price dependence of slope explain the difference between our results and HT99 results?



Empirically, do slopes vary systematically with price?

Number of Bond Pairs

Price bucket	Ba	B	Caa	Ca	C
20 to 70	25	50	94	39	13
70 to 80	56	42	18	6	--
80 to 90	147	80	28	9	6
90 to 100	547	277	29	16	6
100 to 110	566	195	13	--	6
110+	17	8	--	--	--

Fraction with downward sloping term-structure

Price bucket	Ba	B	Caa	Ca	C
20 to 70	80%	86%	87%	56%	92%
70 to 80	80%	93%	72%	67%	--
80 to 90	71%	81%	61%	78%	67%
90 to 100	67%	64%	52%	44%	100%
100 to 110	68%	62%	54%	--	50%
110+	41%	88%	--	--	--

Empirically, do slopes vary systematically with bond price?

Logit Model

π = Probability of observing a downward-sloping term structure in a bond pair

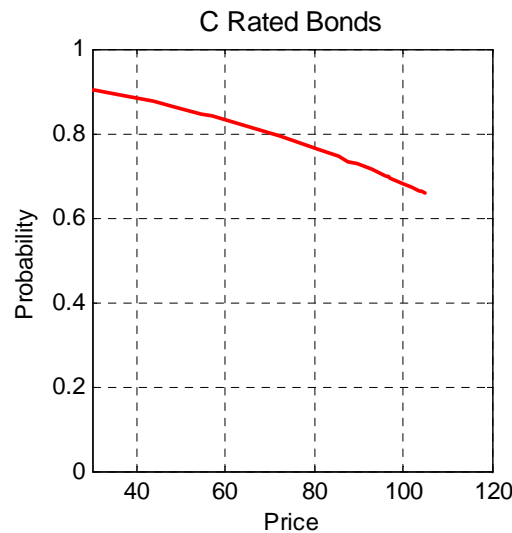
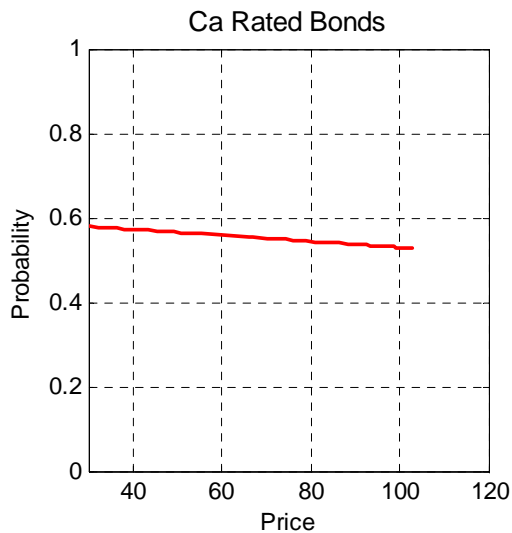
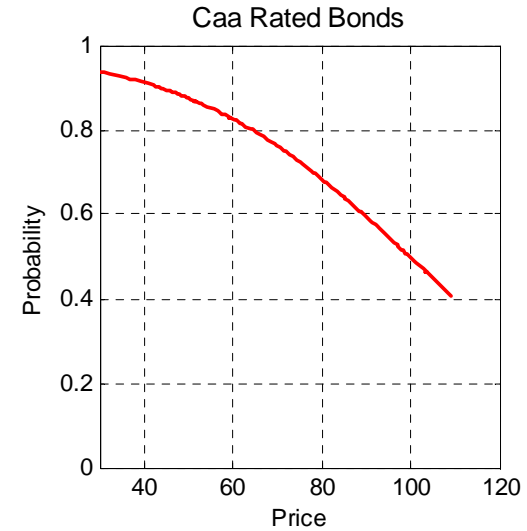
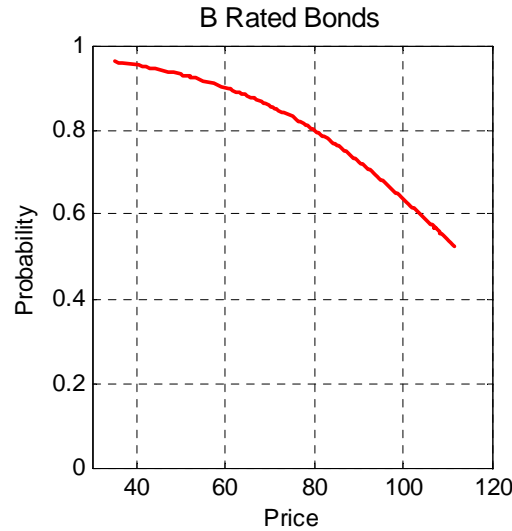
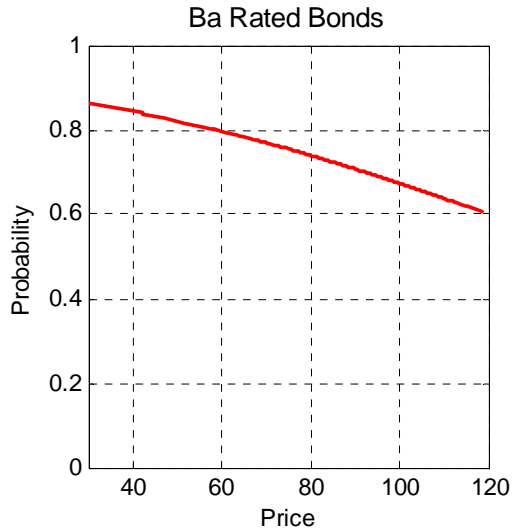
P = Average price of bonds in the pair

$$\ln\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta P$$

Estimate for each rating category.

Plot estimated values of $\pi = \frac{\exp(\alpha + \beta P)}{1 + \exp(\alpha + \beta P)}$ versus P .

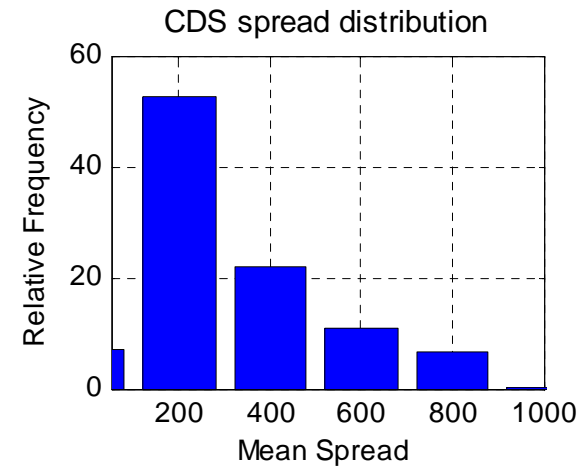
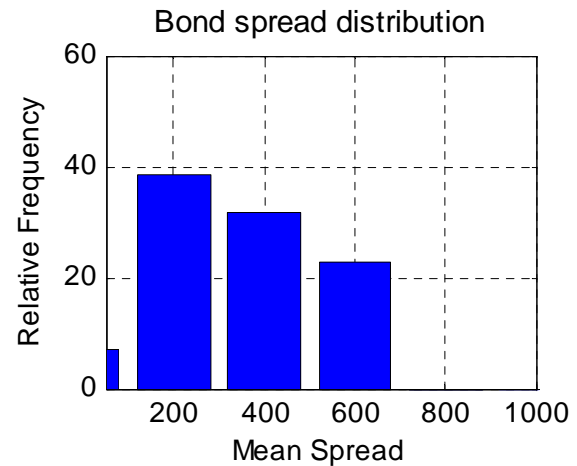
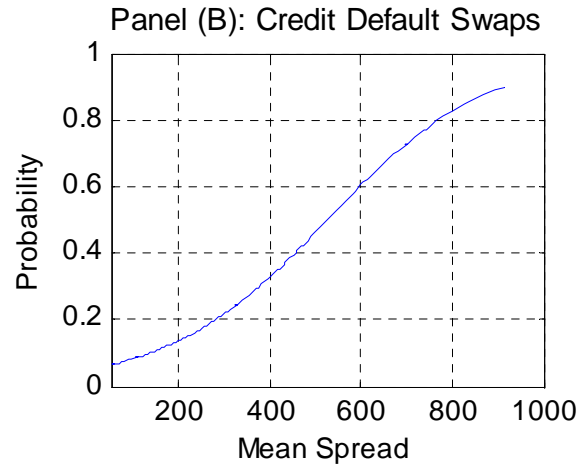
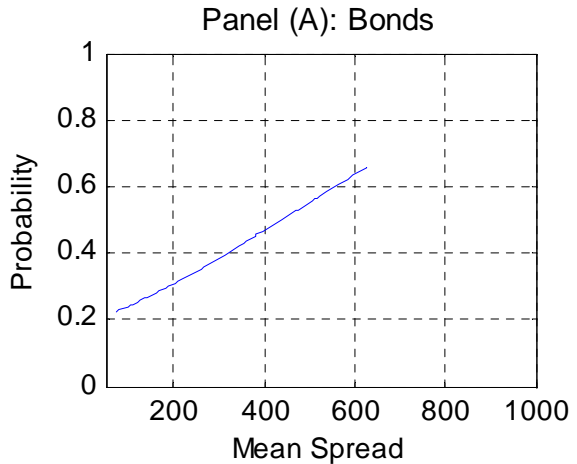
Estimated probabilities of downward sloping term-structure



$$\ln\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta P$$

Slope of speculative grade par curves

$$\ln\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta S$$



Summary

For low quality credits, shape of spread curves is determined by not only hazard rate curve but also recovery assumption and bond price relative to par.

Divergent empirical results on speculative grade term structure can be resolved by considering variation of slope with bond price. Points to RF assumption.

Par curves (hence hazard rates) are upward sloping for good speculative grade credits and become downward sloping at very low credit quality.

Bond valuation under different recovery assumptions

L : Fractional LGD

$P_{0,T}$: Value at $t=0$ of default free zero with maturity T

$Q_{0,T}$: Risk neutral, cumulative probability of default to time T

q_t : Unconditional default probability density at time t

h_t : Default hazard rate at time t :

Forward default rate, not default intensity

Can be defined for both structural and reduced form models.

$H_{0,T}$: Integrated hazard function from time 0 to T , $H_{0T} = \int_0^T h_t dt$

C : Coupon rate per period

Bond valuation under different recovery assumptions

Assume constant fractional LGD, uncorrelated interest rate and default hazard rates.

$$\text{RT: } V_{0,T}^C = P_{0,T} (1 - LQ_{0,T}) + C \sum_{t=1}^T P_{0,t} (1 - LQ_{0,t})$$

$$\text{RF: } V_{0,T}^C = P_{0,T} (1 - Q_{0,T}) + C \sum_{t=1}^T P_{0,t} (1 - Q_{0,t}) + (1 - L) \int_0^T A_t P_{0,t} q_t dt$$

$$\text{RMV: } V_{0,T}^C = P_{0,T} \exp(-H_{0,T}L) + C \sum_{t=1}^T P_{0,t} \exp(-H_{0,t}L)$$

Putting $C=0$ gives expressions for zero coupon bond values. Assume linear accrual schedule for zero coupon bonds under RF.

Spread calculations

Zero coupon spreads

$$V_{0,T} = P_{0,T} \exp(-S_{0,T}T)$$

Par spreads

(Par yield of defaultable bond – par yield of default free bond)

Par yield of defaultable bond : Set bond Value = 1, solve for coupon rate.

Par yield of default free bond : Set default probabilities to zero.