Frailty Correlated Default

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Preliminary

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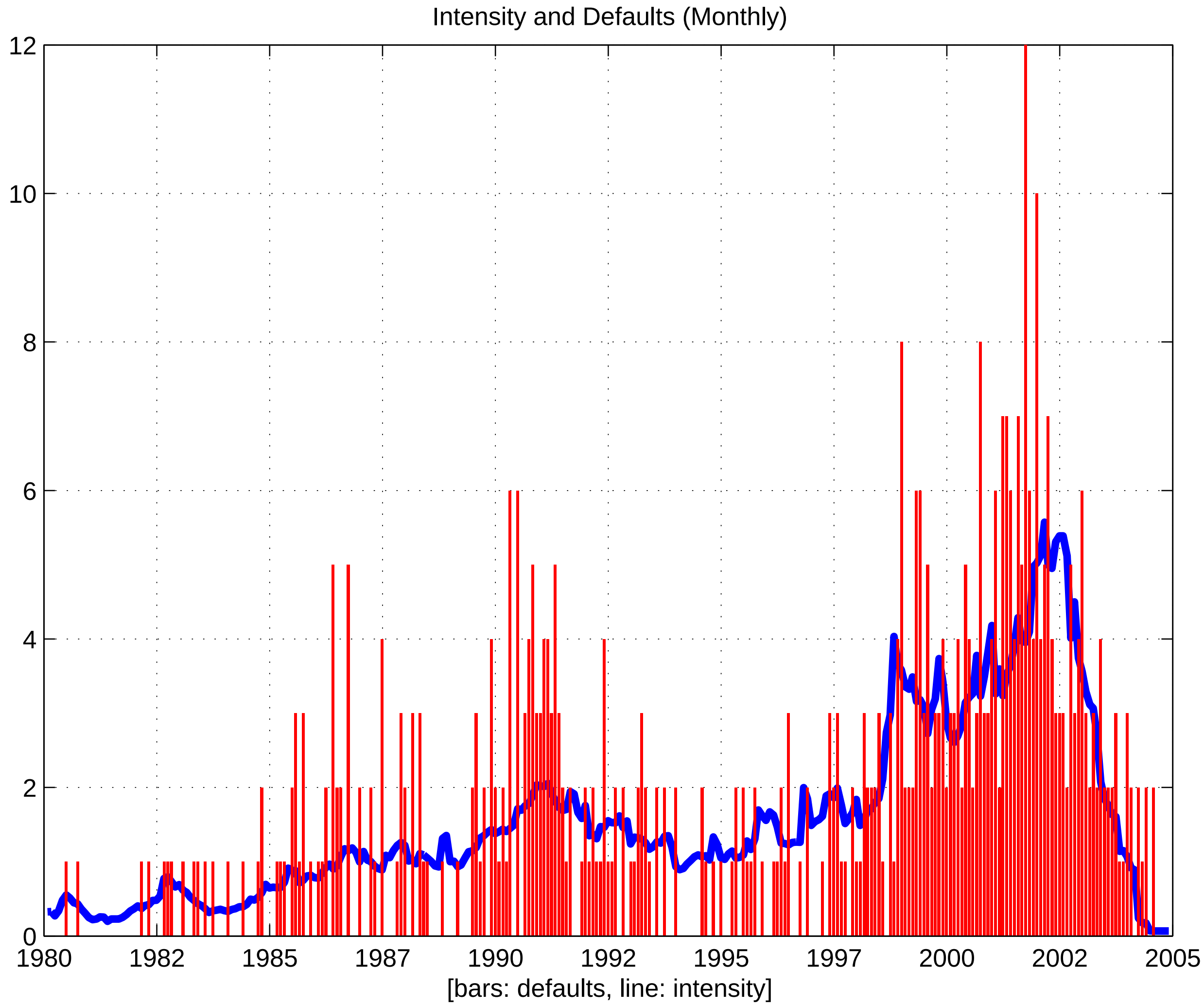


Figure 1: Aggregate default intensities and number of defaults, 1979-2004.

Default Intensity

- Default intensity: the conditional mean arrival rate of default, in events per year.
- Example: An intensity of 0.2% is a mean arrival rate of 2 defaults per 1000 obligor-years, or roughly a probability of default of 0.2% per year.
- We estimate the parameter vector β for default intensity of the form $\Lambda(X_t;\beta)$, where X_t is a list of covariates, including the firm's leverage, volatility, other firm-specific and macro-economic covariates, and "frailty" factors, both common and firm-specific.

Dynamic Default Frailty Model

Firm i default intensity:

$$\lambda_{it} = e^{\alpha + \beta \cdot W_{it} + Y_t + Z_i}$$

where

- W_{it} is a vector of observable covariates.
- Y is a common unobserved Brownian motion.
- and Z_i is unobserved independent-gamma heterogeneity.

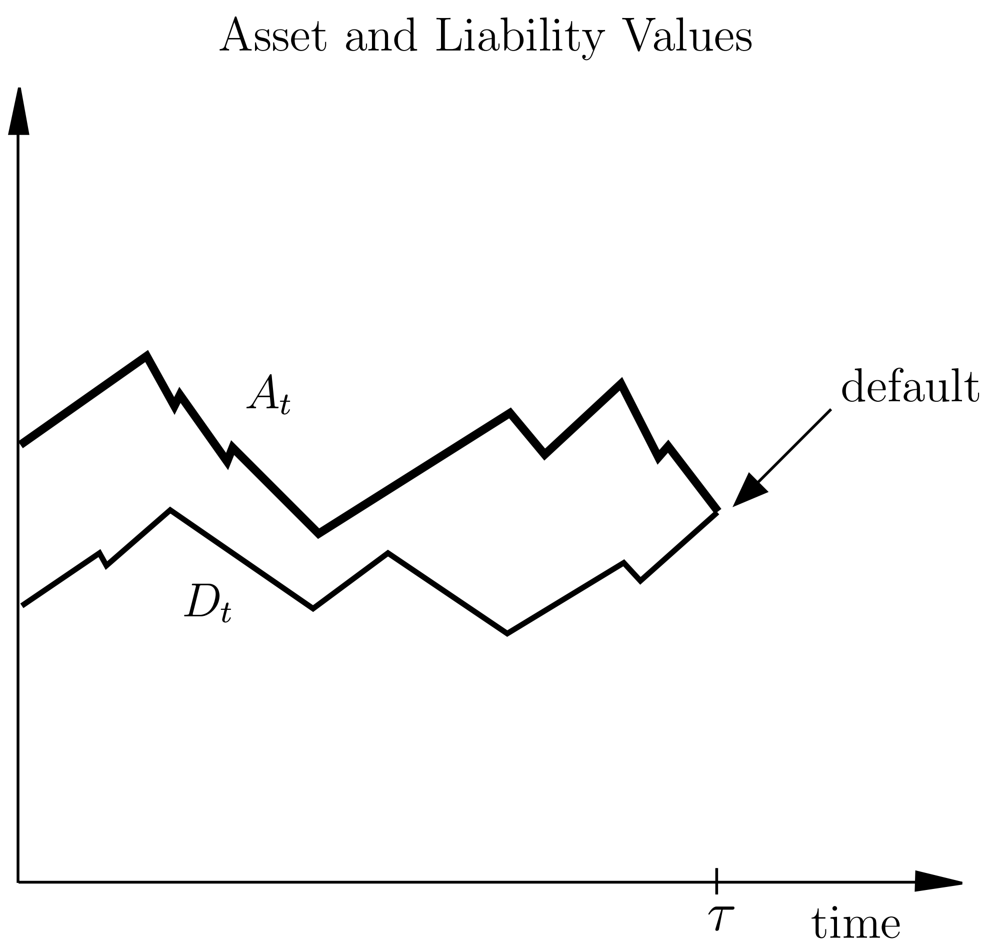
Matchable U.S. Industrials 1979-2004

- 402,000 firm-months of data.
- 2,793 firms.
- 496 defaults.
- 1,072 exits by merger or acquisition.

Some Related Work

- Black and Scholes (1973), Merton (1974), Fischer, Heinkel, Zechner (1989), Leland (1994).
- Beaver (1968), Altman (1968), . . ., Shumway (2001), Beaver, McNichols, and Rhie (2004), Duffie, Saita, Wang (2005), Campbell, Hilscher, and Szilagyi (2005), . . .
- Schönbucher (2003), Collin-Dufrsene, Goldstein, and Huggonier (2004), Delloy, Fermanian, and Sbai (2005).

Key covariate: distance to default



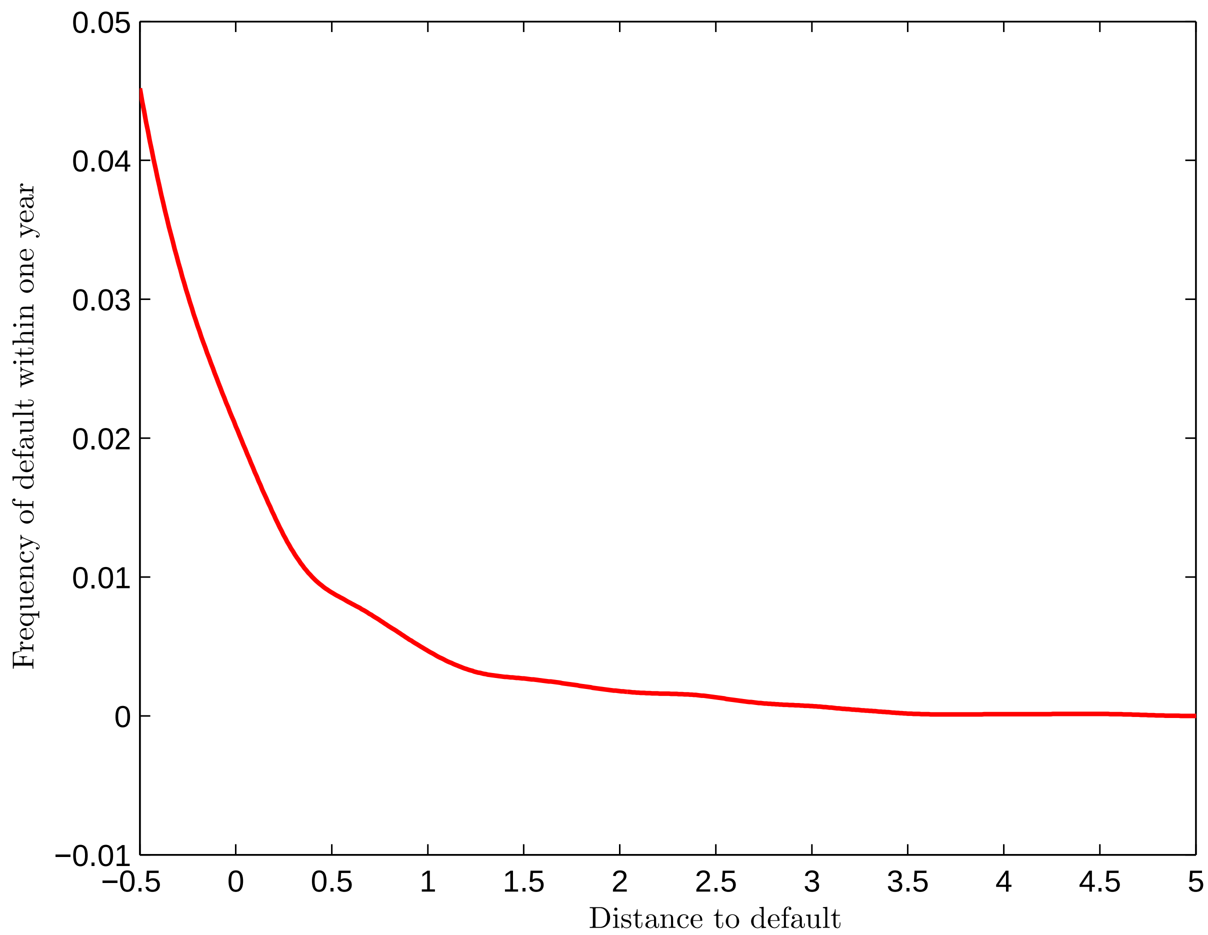


Figure 3: The dependence of empirical default frequency on distance to default. Source: Duffie, Saita, Wang, *J. Financial Economics*, 2006.

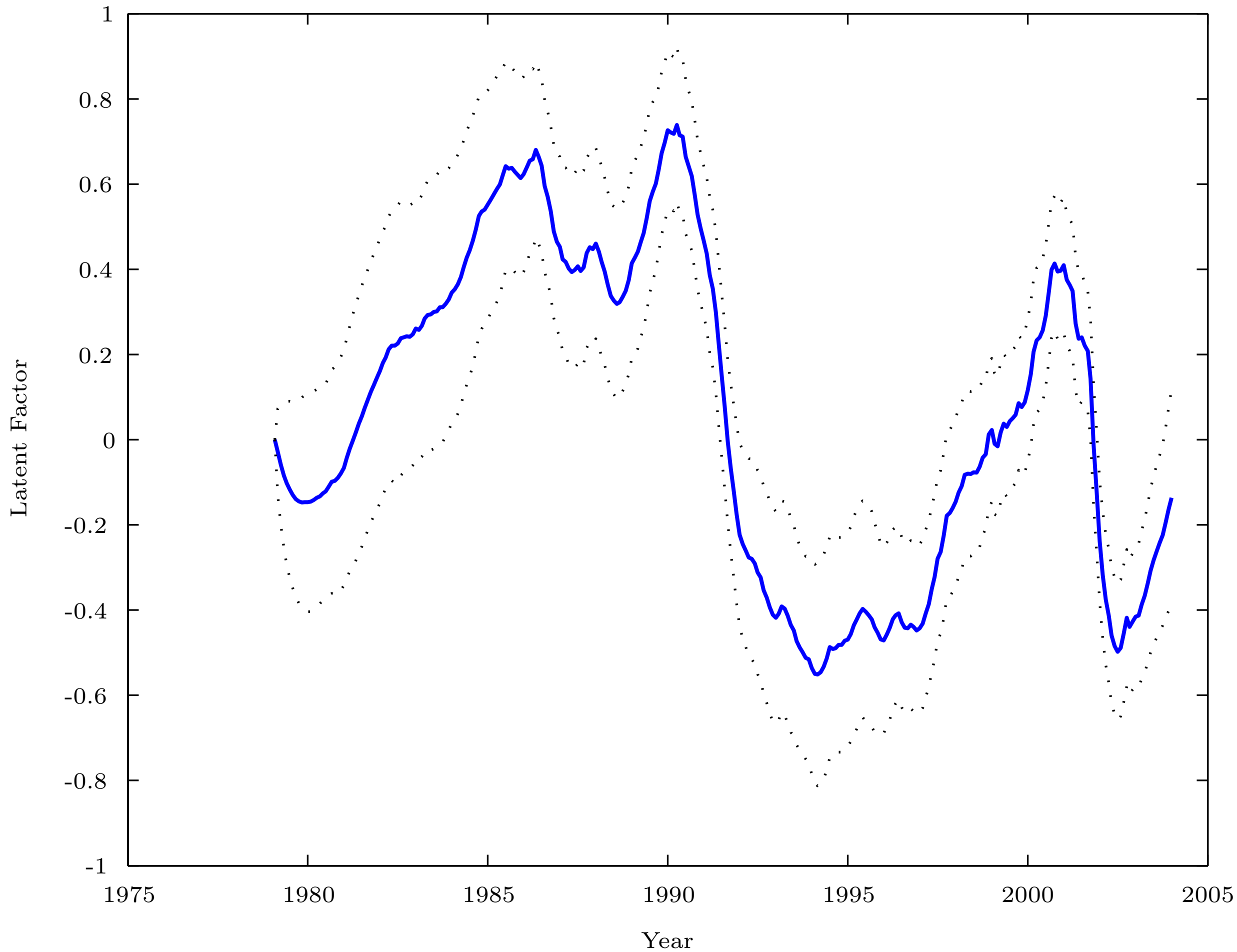


Figure 4: Posterior of the frailty factor, conditioning in 2004.

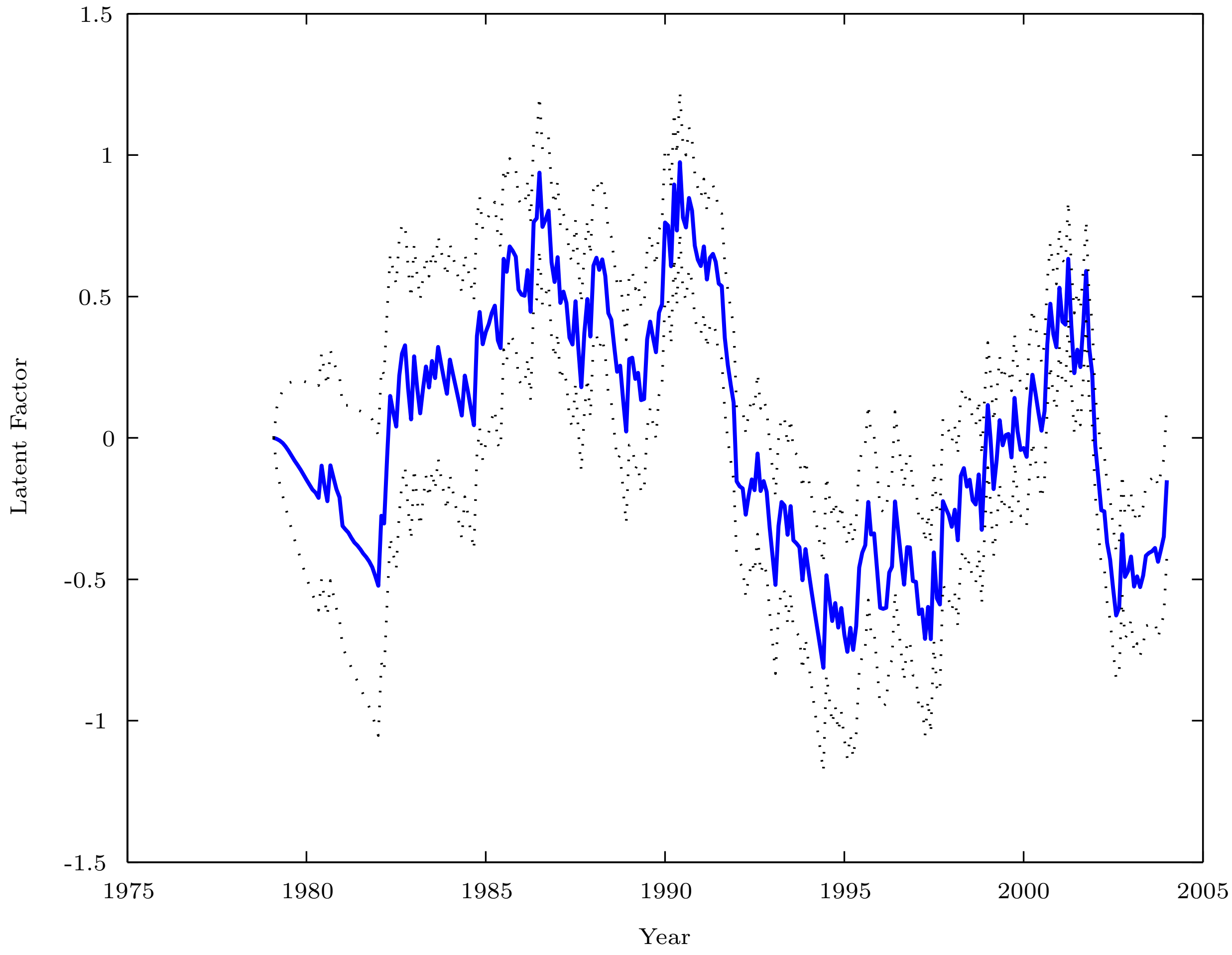


Figure 5: The frailty distribution, conditioning "as you go."

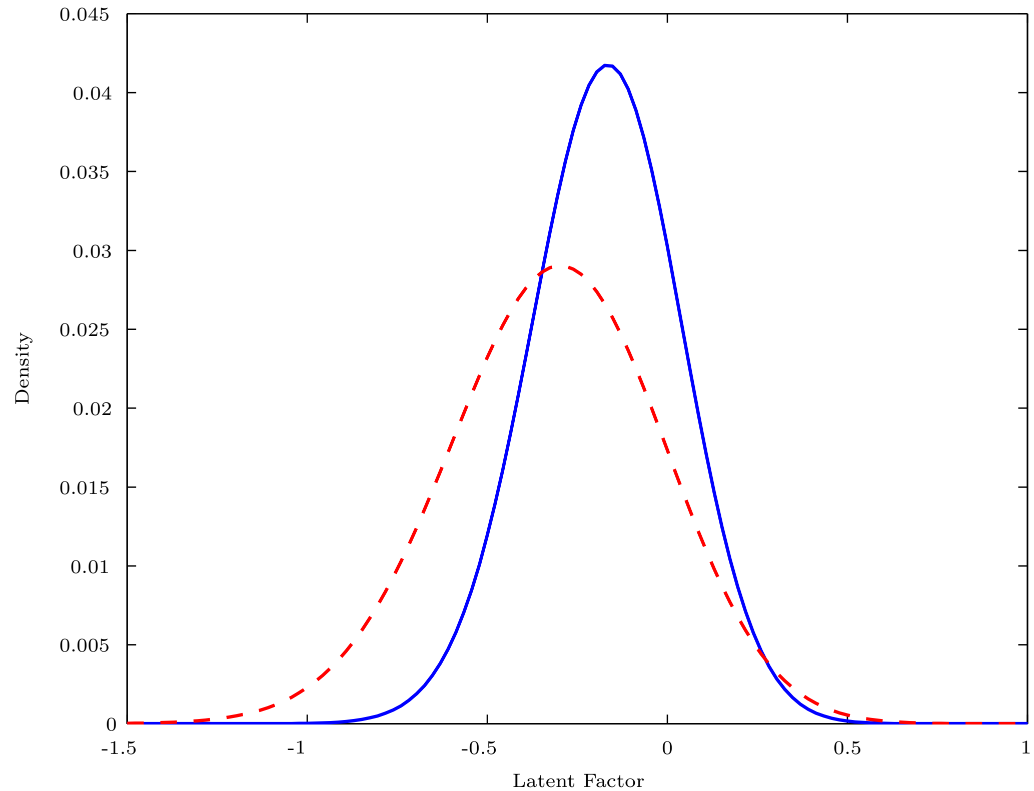


Figure 6: Posterior of the frailty factor in January 1998, conditioning in 2004 (blue), and conditioning in January 1998 (red).

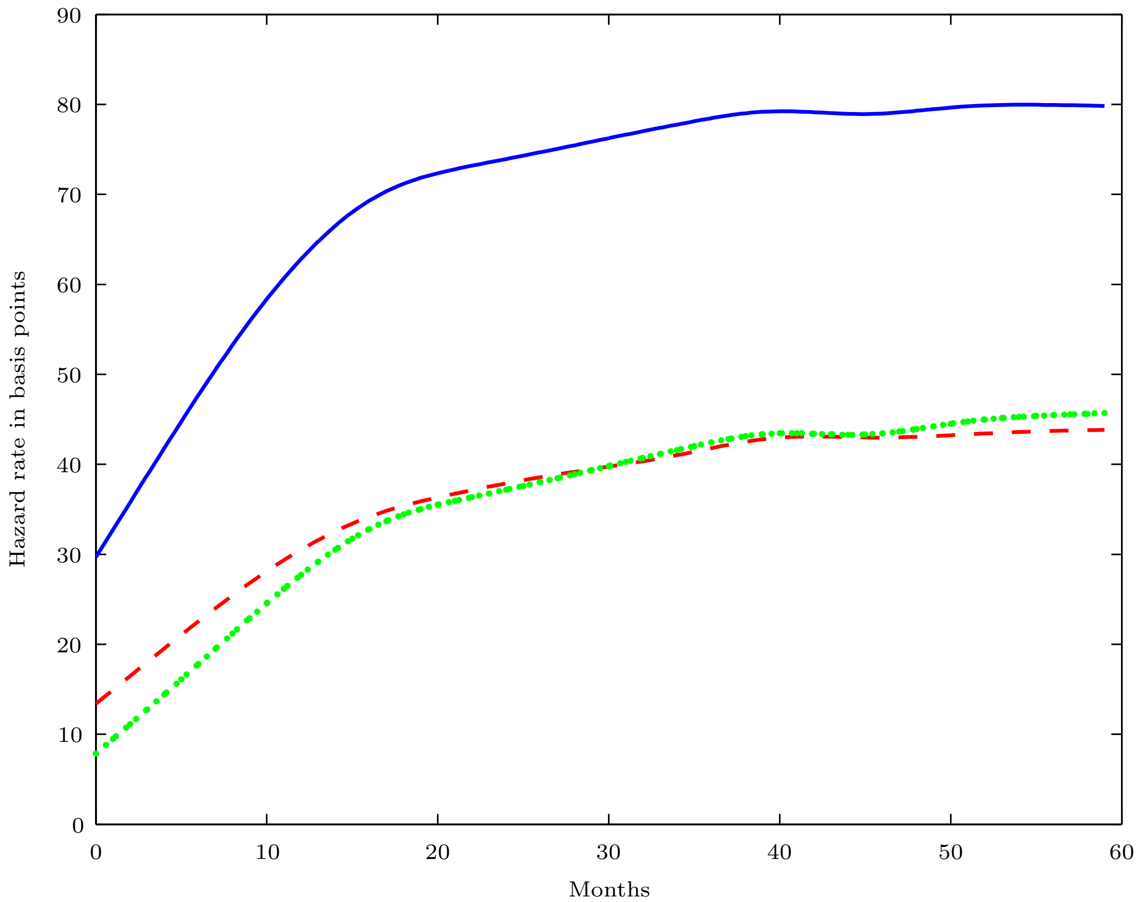


Figure 7: Term structure of hazard rates, Xerox, December 2003: frailty (blue), no frailty (red), unobserved heterogeneity (green).

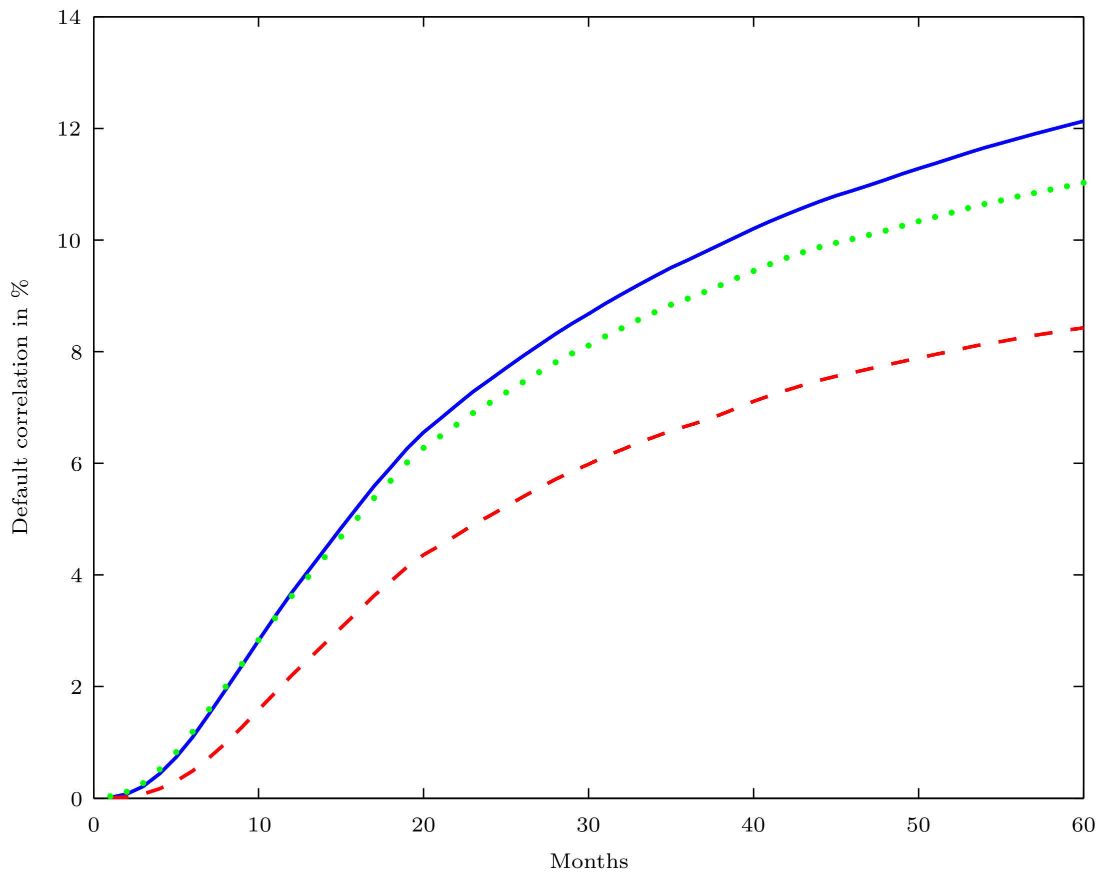


Figure 8: Default correlation of ICO and Xerox with frailty (blue), without frailty (red), and with frailty and unobserved heterogeneity (green).

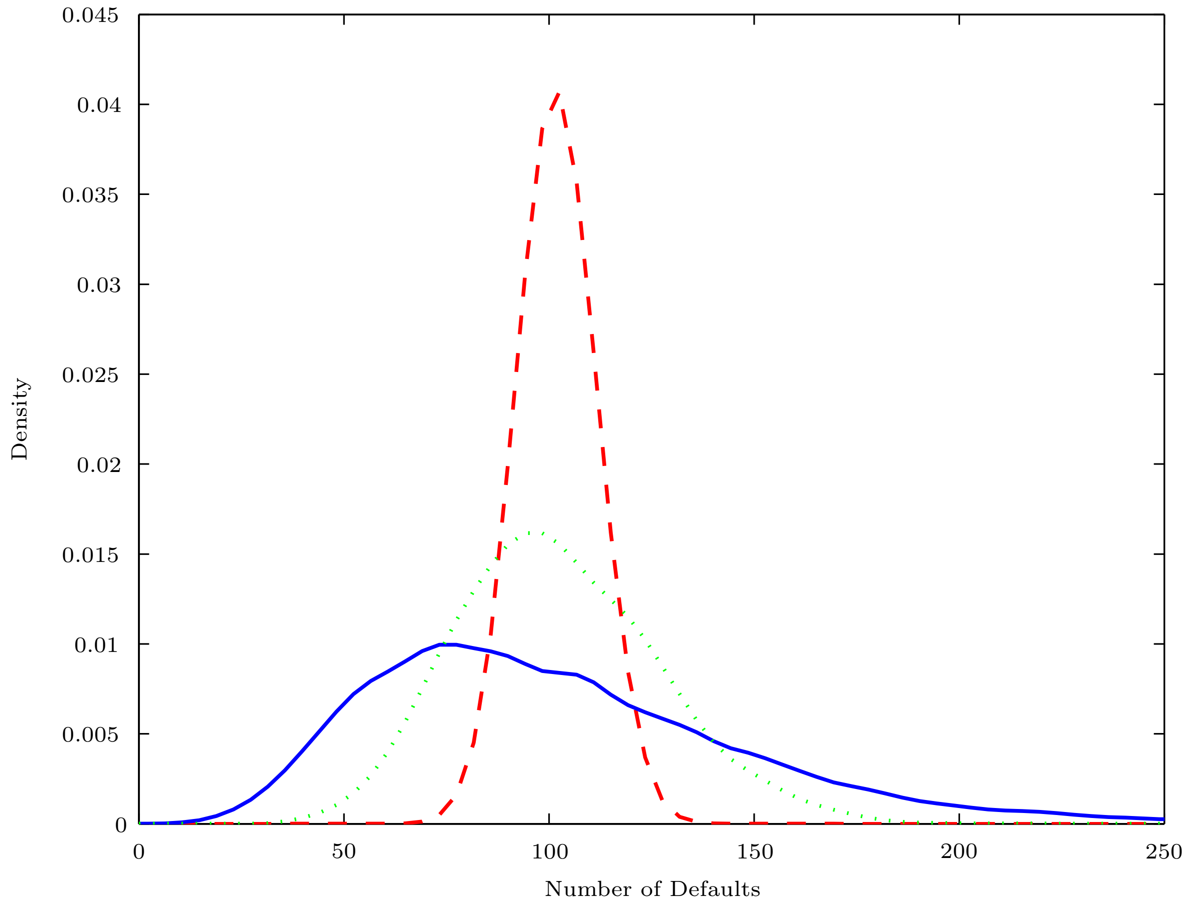


Figure 9: 5-year portfolio defaults: common frailty (blue), independent frailty (red), common start only (green).

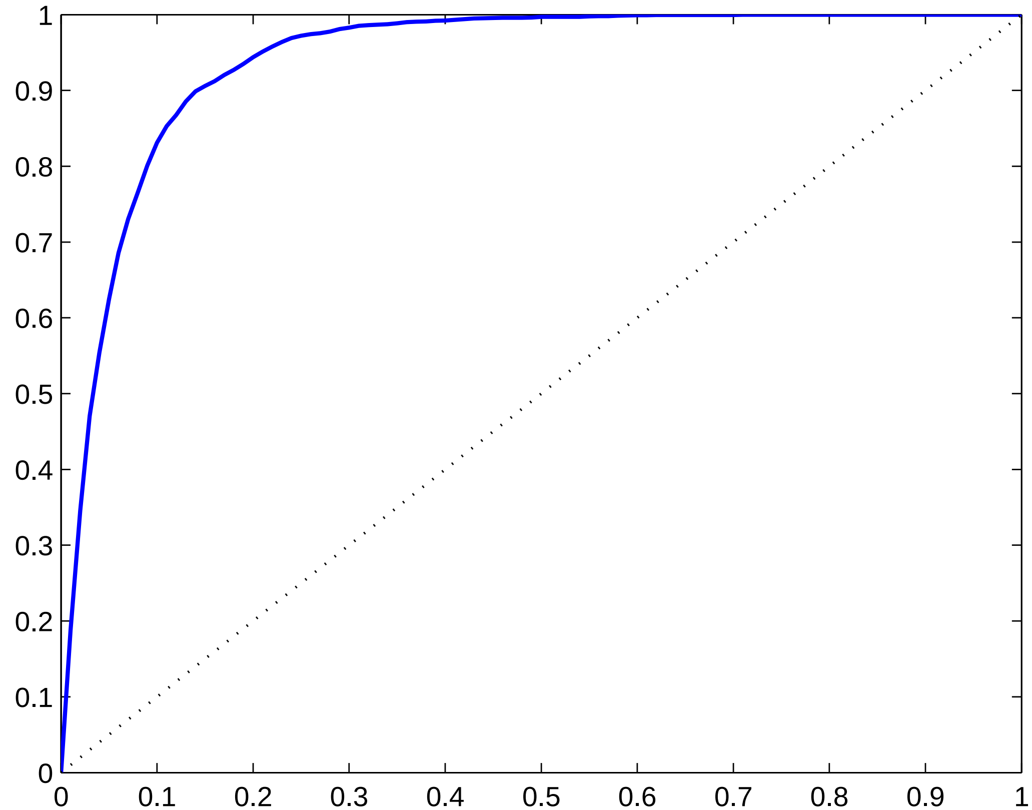


Figure 10: Out-of-sample power curve for 1-year default prediction, January 1993 to December, 2003. Example: The lowest-ranked 10% of firms accounted for 80% of the next year's defaulters. Source: Duffie, Saita, Wang, *J. Financial Economics*, 2006.

Test of Doubly Stochastic Model

From Das, Duffie, Kapadia, and Saita, *J. Finance* (2006).

- Doubly-stochastic: Defaults are independent conditional on the factors determining default intensities.
- Implication: Re-scaling time, with "new time" passing at rate $\sum_i \lambda_i(t)$ per unit of calendar time, total default arrivals are Poisson at rate parameter 1.

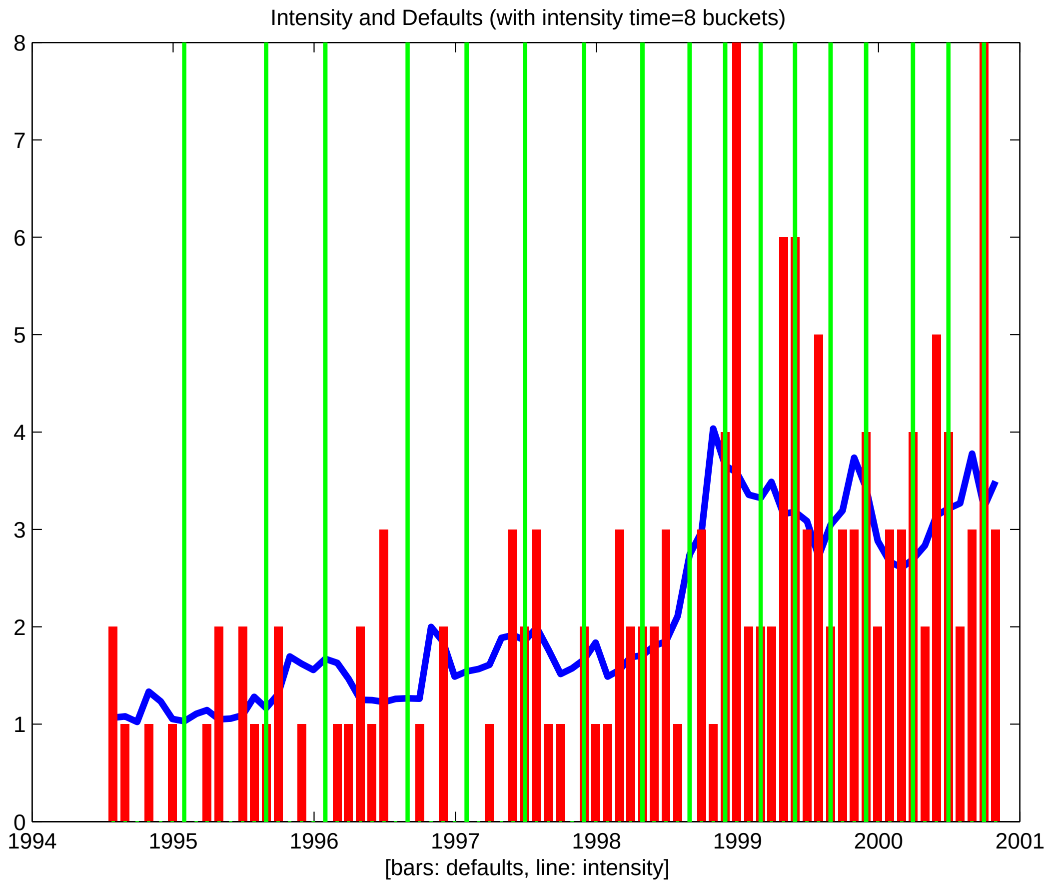


Figure 11: Aggregate intensities and defaults by month, 1996-2001. Bins each include 8 units of total accumulated default intensity. Source: Das, Duffie, Kapadia, and Saita, *J. Finance* (2006).

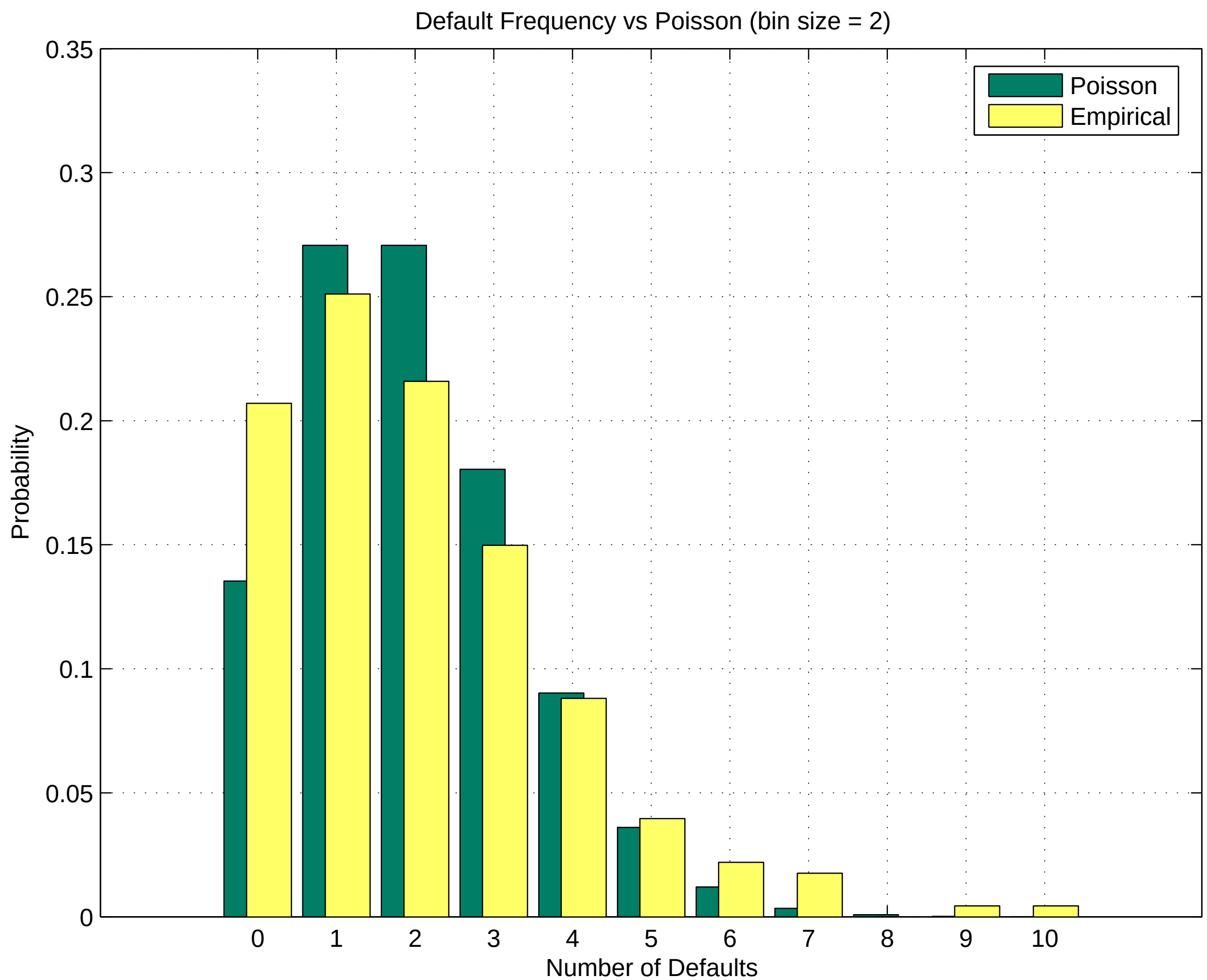


Figure 12: The empirical and Poisson distributions of defaults for bin size 2. Source: Das, Duffie, Kapadia, and Saita, *J. Finance* (2006).

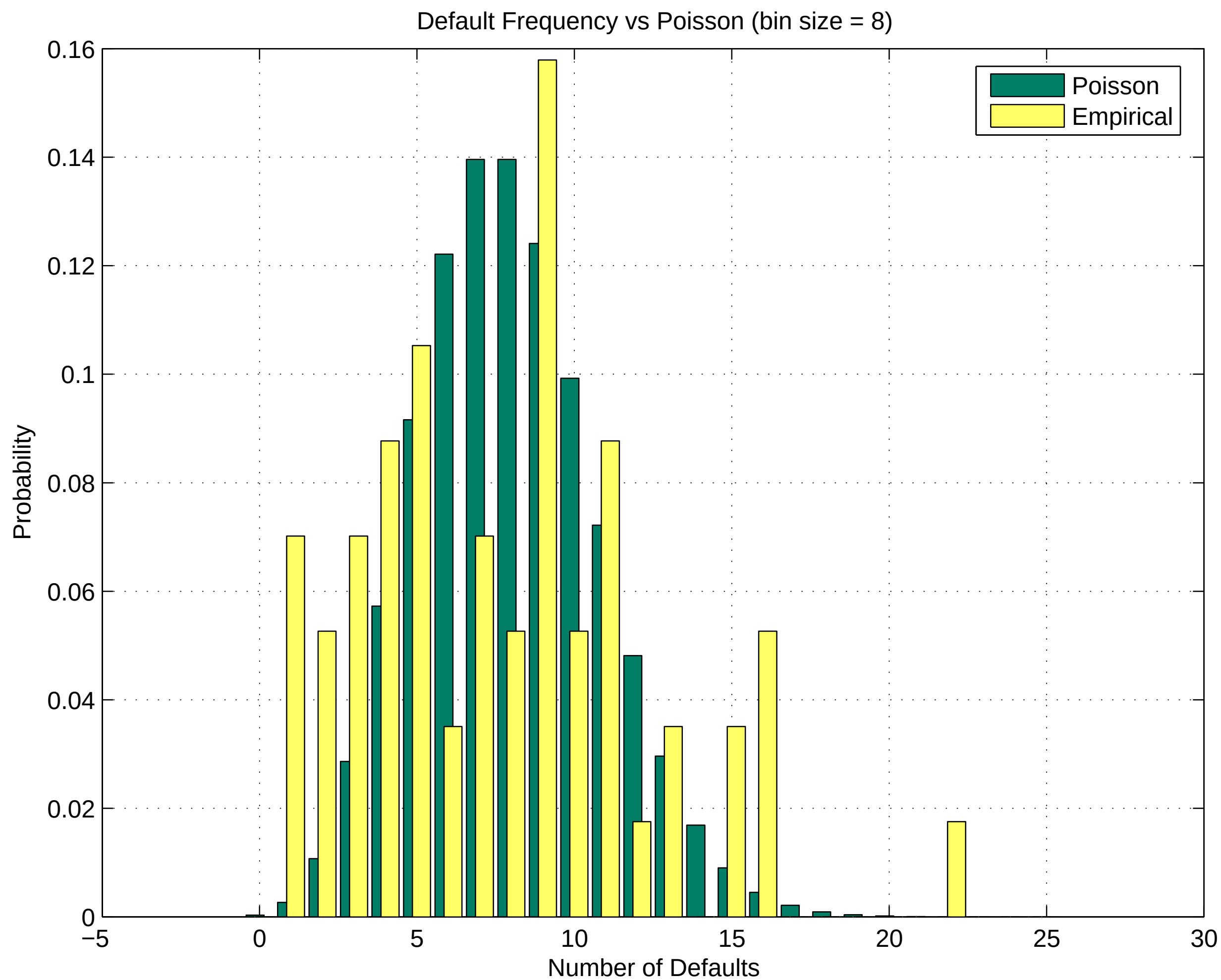


Figure 13: The empirical and Poisson distributions of defaults for bin size 8. Source: Das, Duffie, Kapadia, and Saita, *J. Finance* (2006).

Joint Likelihood with Frailty

- The joint covariate process $X = (W, Y)$, with W observed, Y independent and not observed, is a Markov process whose transition distribution is determined by a parameter vector γ.
- By Bayes' Rule, with observable X, the joint likelihood of X and the censored default times $\tau = (\tau_1, \ldots, \tau_n)$, is

$$\mathcal{L}(\tau, X; \beta, \gamma) = \mathcal{L}(\tau \,|\, X; \beta) \times \mathcal{L}(X; \gamma).$$

- With unobservable Y,

$$\mathcal{L}(\tau, W; \beta, \gamma) = E\left[\mathcal{L}(\tau \,|\, Y, W; \beta) \,|\, W\right] \times \mathcal{L}(W; \gamma),$$

evaluated by Gibbs sampling.

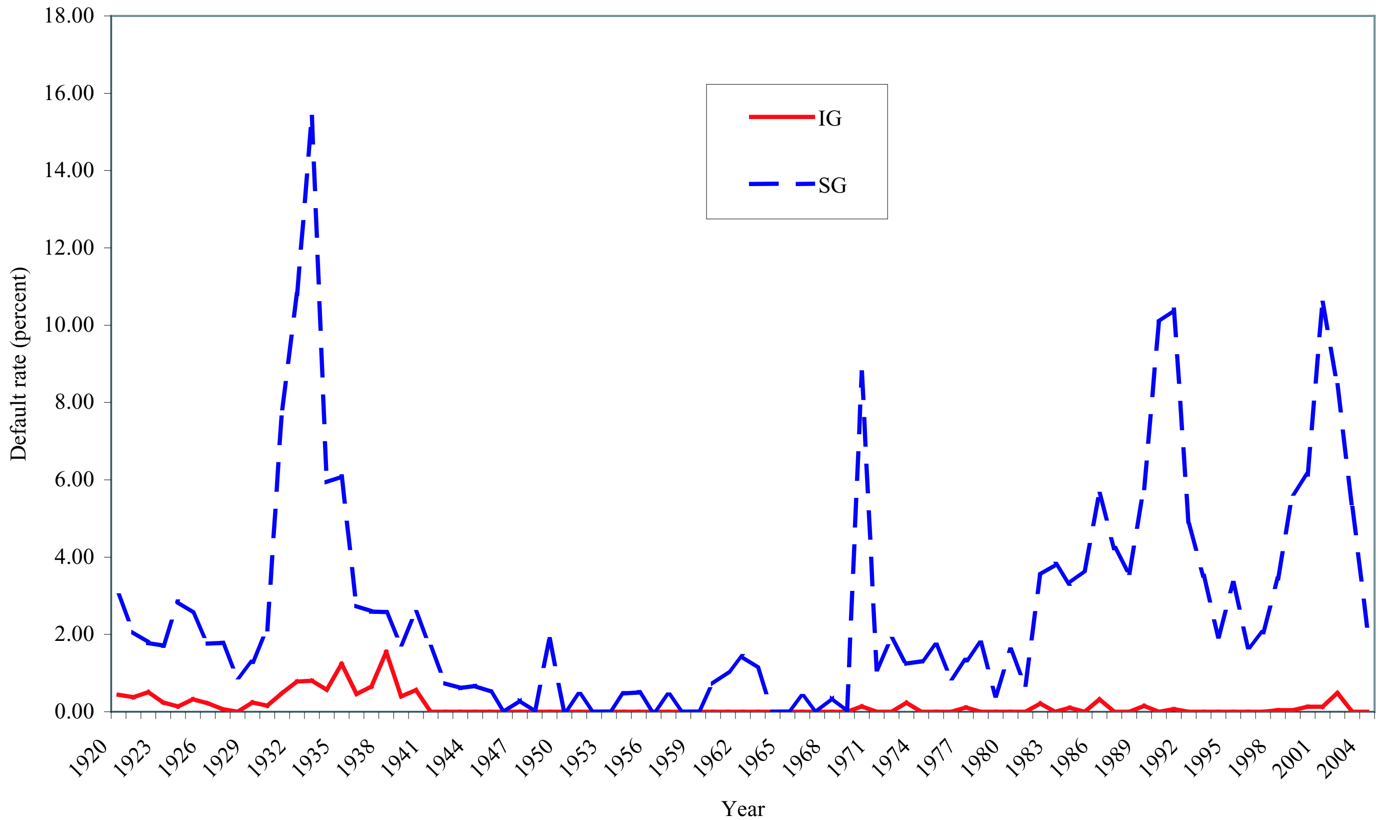


Figure 14: Annual default rates, investment (IG) and speculative (SG). Source: Moodys (2005).

Default Intensity Estimates

$$\lambda(t) = e^{\beta_0 + \beta_1 X_1(t) + \cdots + \beta_n X_n(t)}$$

Maximum Likelihood estimate of β_i (with standard error)

- Distance to default -1.13 (0.036)
- Trailing 1-year stock return -0.694 (0.075)
- 3-month T-Bill rate -0.105 (0.021)
- Trailing 1-year SP500 return 1.20 (0.289)

Conditional Likelihood Function

- For a given firm i, the X-conditional likelihood of survival from $t(i)$ to τ_i, censored at $T(i)$, is

$$\begin{aligned}
\mathcal{L}_i(\tau_i \mid X; \beta) &= q_{t(i)} q_{t(i)+1} \cdots q_{T(i)-1}, & \text{if } \tau_i > T(i), \\
&= q_{t(i)} q_{t(i)+1} \cdots q_{\tau_i - 2}(1 - q_{\tau_i - 1}), & \text{if } \tau_i \leq T(i),
\end{aligned}$$

  where $q_t = e^{-\Lambda(X_{it};\beta)}$ is the one-period X-conditional survival probability.

- The doubly-stochastic assumption implies that the X-conditional likelihood of $\tau_1, \ldots, \tau_n$, as censored, is $\mathcal{L}(\tau \mid X; \beta) = \prod_i \mathcal{L}_i(\tau_i \mid X; \beta)$.