Discussion of F. Longstaff and A. Rajan: An Empirical Analysis of the Pricing of Collateralized Debt Obligations

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The Problem

- Consider 125 firms on which CDS contracts trade
- τ_i is the default time of firm i,
  l_i is the loss pr unit of principal in default for a CDS protection seller.
- Define loss process $L_t = \frac{1}{125} \sum_{i=1}^{125} l_i 1_{\{\tau_i \leq t\}}$
- Synthetic CDO on the 125 names define contracts whose payouts are functions of L_t

- Loss on tranches is defined as follows

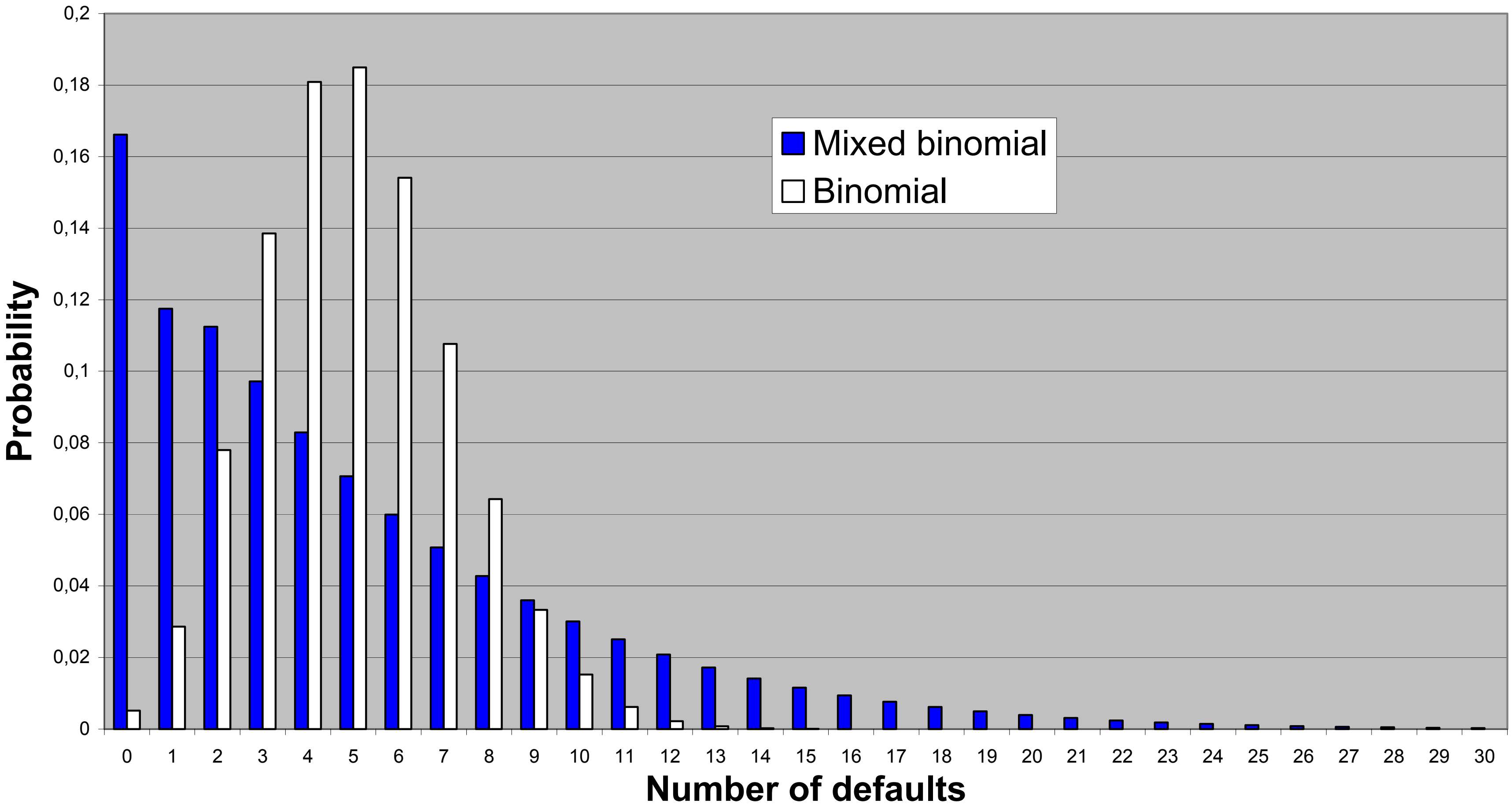
$$
\begin{aligned}
\Lambda_{0,3}(t) &= L_t - \max(L_t - 0.03, 0) \\
\Lambda_{3,7}(t) &= \max(L_t - 0.03, 0) - \max(L_t - 0.07, 0) \\
& \ldots \\
\Lambda_{30,100} &= \max(L_t - 0.3, 0)
\end{aligned}
$$

- Protection seller compensates protection buyer for these losses - receives a premium of remaining principal

- The value of the sum is the value of L_t but the distribution of value between the tranches depends critically on entire distribution of L_t.

- This distribution (i.e. the price of each tranche) is affected by dependence between the default events (and recoveries, but we ignore that here)

The correlation effect



The clear effects

- An increase in the arrival rate of losses decreases all tranche prices

- Increasing correlation for a given default intensity acts like a 'mean preserving' spread - increasing value of equity tranche, decreasing value of senior tranch.

- May 2005 saw increasing value of equity protection, decreasing mezzanine tranch, despite rise in index.

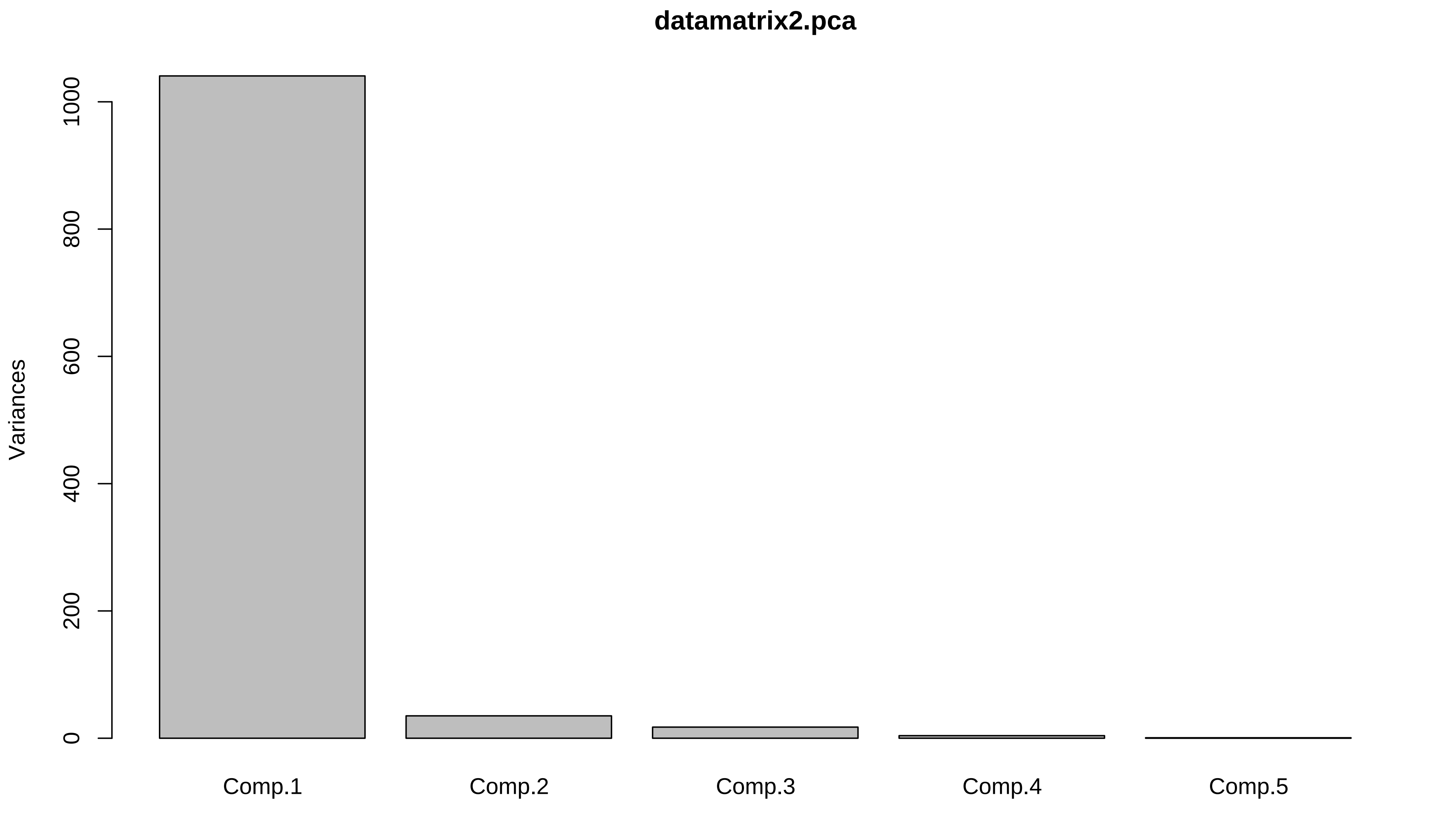
- Chain of events (?): Downgrade of Ford/GM hurts equity value

- 'Hedges' consisting of selling equity protection and buying mezz protection (in certain proportion) were unwound

Factor loadings in PCA on normalized data.

Bloomberg data, series 5.

Tranche/comp	C1	C2	C3	C4	C5
0-3	-0.304	0.890	0.149	-0.307	0
3-7	-0.471	0	0.616	0.608	0.162
7-10	-0.467	-0.424	0.187	-0.677	0.330
10-15	-0.509	-0.148	-0.175	-0.830	0
15-30	-0.457	0	-0.730	0.279	0.420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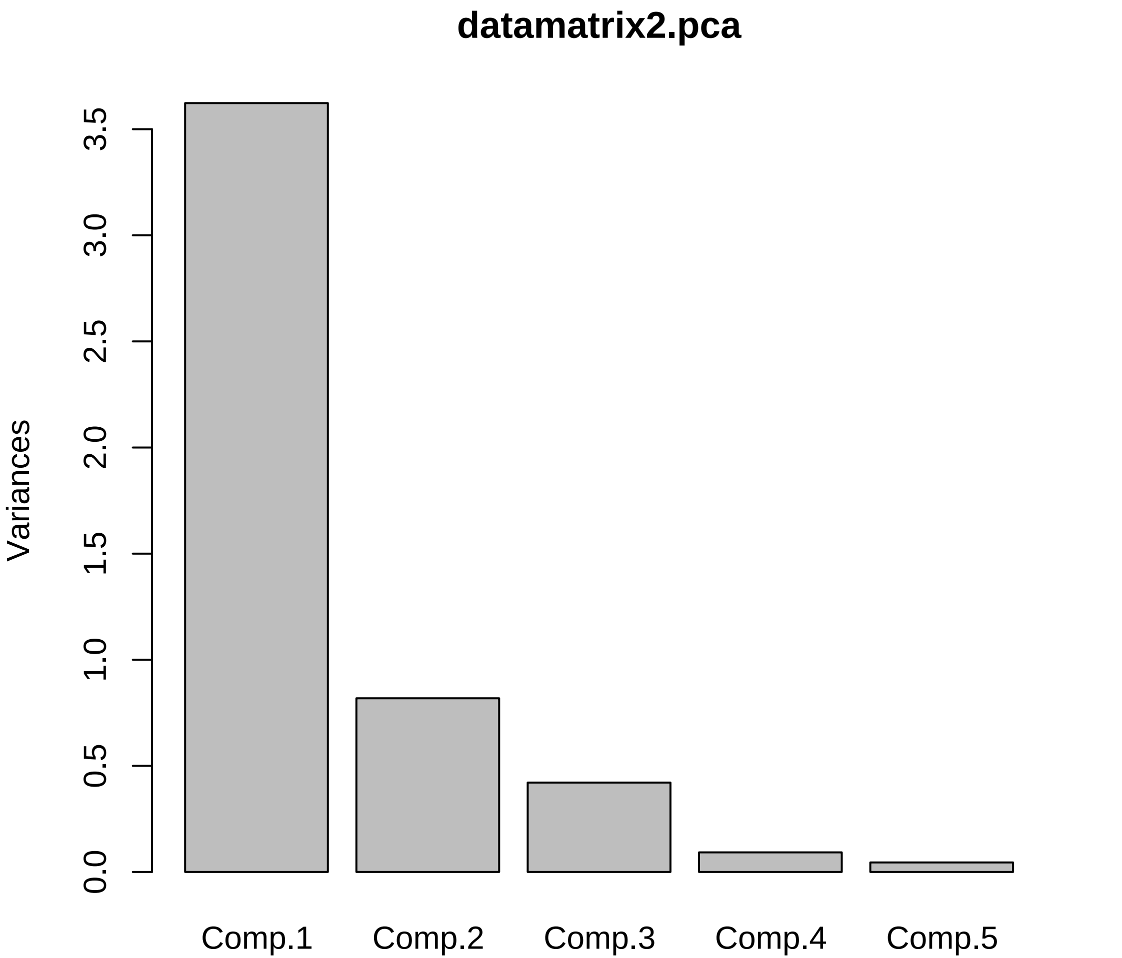
Variances
0
200
400
600
800
1000
Comp.1
Comp.2
Comp.3
Comp.4
Comp.5


datamatrix2.pca
Variances
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Comp.1
Comp.2
Comp.3
Comp.4
Comp.5

Factor loadings in PCA on covariances.

Bloomberg data, series 5.

Tranche/comp	C1	C2	C3	C4	C5
0-3	0	0.434	0.799	-0.408	0
3-7	-0.955	0.226	-0.185	0	0
7-10	-0.259	-0.833	0.211	-0.430	
10-15	-0.111	-0.224	0.337	0.497	-0.760
15-30	0	-0.129	0.411	0.632	0.642

Different intensity approaches to correlation

- Conditional independence

- 'Mild' contagion: Defaults cause increases in other firm's default intensities (either because of a modelled observed effect, or because of updating of latent variables)

- Strong contagion - firms default at the same time

- Duffie and Garleanu (2001) is of the first kind. Longstaff and Rajan (2006) is of the third kind.

The conditional independence approach

- Duffie and Garleanu (2001), see also Mortensen (2006)

$$\lambda_i(t) = \nu_g(t) + \nu_{ind}(t) + \nu_i(t)$$

- $\nu-$processes are independent. Default times are conditionally independent given evolution of these processes

- Disadvantage: Numerical price to be paid for inhomogeneity, but see Mortensen (2006) for combination of affine technology and recursive algorithm

- Advantage: Consistency in treatment of hedges and bespoke tranches

- Fact: fairly high variability in intensities - best achieved through jumps - required to achieve level of dependence (for given marginals) fitting market prices.

Assuming homogeneous firms

- Letting all firms have the same default intensity brings us back to binomial or - if we ignore diminishing pool - Poisson distribution type results - possibly with a mixture element
- Without hacks, it becomes difficult to answer how much tranche prices should change due to changes in single names
- It is easy to impose strong dependence structures under homogeneity. In LR this is done through three intensities: One controls single defaults, two others control (industry,

widespread) defaults. But note the latter two control simultaneous defaults.

$$L_t = 1 - \exp(-\gamma_1 N_{1t}) \exp(-\gamma_2 N_{2t}) \exp(-\gamma_3 N_{3t})$$

- Advantage: Computationally much easier.

- Disadvantage: How to handle bespoke tranches, hedging?

A few technical issues

- The choice of intensity processes $d\lambda_t = \sigma\sqrt{\lambda_t}dW_t$ implies absorption of the intensity at 0 when it hits there.

- Estimation of model does not involve the likelihood under the real world measure of intensity dynamics

A few technical issues

- The choice of intensity processes $d\lambda_t = \sigma\sqrt{\lambda_t}dW_t$ implies absorption of the intensity at 0 when it hits there.

- Intensity is not changed by firm leaving the pool

- Estimation of model does not involve the likelihood under the real world measure of intensity dynamics

- Argument for choice of factors - equates choice of one factor with independence.

- But I would argue that the DG model with one common factor is a one factor model. Stochastic intensity induces correlation (with or without residuals)

Concluding remarks

- Model and its estimates very specific to structure - and even to series? Large variation in volatility parameter estimates (See tables 3 and 4).

- In reality fit 6 parameters, choose levels of intensities without likelihood considerations, and allow for 5 different regimes.

- Looking at bespoke tranches: Do we assume same catastrophic proportion of defaults. If we think 35% default hits 'universe' of CDS-firms, then wouldn't any sub-selection show random variation around that frequency?

- Hard to know if fit is really good. Probably easy to get small absolute variations on senior tranch

- Are we fitting supply/demand effects and therefore finding intensity estimates which are very structure dependent?