Discussion of
F. Longstaff and A. Rajan: An Empirical Analysis of
the Pricing of Collateralized Debt Obligations

David Lando
Copenhagen Business School
(Visiting Princeton University)

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The Problem

• Consider 125 firms on which CDS contracts trade

• $\tau_i$ is the default time of firm $i$,
  $l_i$ is the loss per unit of principal in default for a CDS protection seller.

• Define loss process $L_t = \frac{1}{125} \sum_{i=1}^{125} l_i 1\{\tau_i \leq t\}$

• Synthetic CDO on the 125 names define contracts whose payouts are functions of $L_t$
• Loss on tranches is defined as follows

\[
\Lambda_{0,3}(t) = L_t - \max(L_t - 0.03, 0) \\
\Lambda_{3,7}(t) = \max(L_t - 0.03, 0) - \max(L_t - 0.07, 0) \\
\ldots \\
\Lambda_{30,100} = \max(L_t - 0.3, 0)
\]

• Protection seller compensates protection buyer for these losses - receives a premium of remaining principal

• The value of the sum is the value of \( L_t \) but the distribution of value between the tranches depends critically on entire distribution of \( L_t \).
• This distribution (i.e. the price of each tranche) is affected by dependence between the default events (and recoveries, but we ignore that here)
The correlation effect

![Graph showing probability distribution for number of defaults with mixed binomial and binomial models.](image)
The clear effects

• An increase in the arrival rate of losses decreases all tranche prices

• Increasing correlation for a given default intensity acts like a ‘mean preserving’ spread - increasing value of equity tranche, decreasing value of senior tranche.

• May 2005 saw increasing value of equity protection, decreasing mezzanine tranche, despite rise in index.
• Chain of events (?): Downgrade of Ford/GM hurts equity value

• 'Hedges' consisting of selling equity protection and buying mezz protection (in certain proportion) were unwound
Factor loadings in PCA on normalized data.

Bloomberg data, series 5.

<table>
<thead>
<tr>
<th>Tranche/comp</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
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</thead>
<tbody>
<tr>
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<td>0.890</td>
<td>0.149</td>
<td>-0.307</td>
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<td>3-7</td>
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<td>0</td>
<td>0.616</td>
<td>0.608</td>
<td>0.162</td>
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<td>7-10</td>
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<td>-0.677</td>
<td>0.330</td>
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<tr>
<td>10-15</td>
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<td>-0.175</td>
<td>-0.830</td>
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<td>15-30</td>
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<td>0</td>
<td>-0.730</td>
<td>0.279</td>
<td>0.420</td>
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Factor loadings in PCA on covariances.

Bloomberg data, series 5.

<table>
<thead>
<tr>
<th>Tranche/comp</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
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<tbody>
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<tr>
<td>15-30</td>
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<td>-0.129</td>
<td>0.411</td>
<td>0.632</td>
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</table>
Different intensity approaches to correlation

- Conditional independence

- 'Mild' contagion: Defaults cause increases in other firm’s default intensities (either because of a modelled observed effect, or because of updating of latent variables)

- Strong contagion - firms default at the same time

- Duffie and Garleanu (2001) is of the first kind. Longstaff and Rajan (2006) is of the third kind.
The conditional independence approach

- Duffie and Garleanu (2001), see also Mortensen (2006)
  \[ \lambda_i(t) = \nu_g(t) + \nu_{\text{ind}}(t) + \nu_i(t) \]

- \(\nu\)-processes are independent. Default times are conditionally independent given evolution of these processes.

- Disadvantage: Numerical price to be paid for inhomogeneity, but see Mortensen (2006) for combination of affine technology and recursive algorithm.
• Advantage: Consistency in treatment of hedges and bespoke tranches

• Fact: fairly high variability in intensities - best achieved through jumps - required to achieve level of dependence (for given marginals) fitting market prices.
Assuming homogeneous firms

- Letting all firms have the same default intensity brings us back to binomial or - if we ignore diminishing pool - Poisson distribution type results - possibly with a mixture element

- Without hacks, it becomes difficult to answer how much tranche prices should change due to changes in single names

- It is easy to impose strong dependence structures under homogeneity. In LR this is done through three intensities: One controls single defaults, two others control (industry,
widespread) defaults. But note the latter two control simultaneous defaults.

\[ L_t = 1 - \exp(-\gamma_1 N_1 t) \exp(-\gamma_2 N_2 t) \exp(-\gamma_3 N_3 t) \]

- Advantage: Computationally much easier.

- Disadvantage: How to handle bespoke tranches, hedging?
A few technical issues

- The choice of intensity processes $d\lambda_t = \sigma \sqrt{\lambda_t} dW_t$ implies absorption of the intensity at 0 when it hits there.

- Estimation of model does not involve the likelihood under the real world measure of intensity dynamics.
A few technical issues

• The choice of intensity processes \( d\lambda_t = \sigma \sqrt{\lambda_t} dW_t \) implies absorption of the intensity at 0 when it hits there.

• Intensity is not changed by firm leaving the pool

• Estimation of model does not involve the likelihood under the real world measure of intensity dynamics
• Argument for choice of factors - equates choice of one factor with independence.

• But I would argue that the DG model with one common factor is a one factor model. Stochastic intensity induces correlation (with or without residuals)
Concluding remarks

- Model and its estimates very specific to structure - and even to series? Large variation in volatility parameter estimates (See tables 3 and 4).

- In reality fit 6 parameters, choose levels of intensities without likelihood considerations, and allow for 5 different regimes.

- Looking at bespoke tranches: Do we assume same catastrophic proportion of defaults. If we think 35% default hits ’universe’ of CDS-firms, then wouldn’t any sub-selection show random variation around that frequency?
• Hard to know if fit is really good. Probably easy to get small absolute variations on senior tranch

• Are we fitting supply/demand effects and therefore finding intensity estimates which are very structure dependent?