

# Humpbacks in Credit Spreads\*

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## Abstract

Models of credit valuation generally predict a hump-shaped spread term structure for low quality issuers. This is understood to be driven by the shape of the underlying conditional default probabilities curve. We show that (a) recovery assumptions and (b) deviation of bond's price from par can also drive different term structure shapes. On examining a large set of speculative grade bonds and credit default swaps, we find evidence that par-spread term structures are likely to be downward sloping as credit quality deteriorates sufficiently. Our analysis explains the various theoretical results and resolves conflicting empirical evidence on the shape of speculative grade spread curves.

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As the markets for corporate bonds and credit default swaps have expanded and become more liquid, market participants have become more interested in understanding the behavior of the term structure of credit spreads. Term structure of credit spreads is as important in the study of credit risk as term structure of interest rates is in the study of interest rate risk. It contains information about future evolution of credit risk.<sup>1</sup> It is, therefore, a crucial input to quantifying the credit risk of a portfolio, economic capital calculations and pricing of credit contingent claims. While an enormous amount of research has been done on understanding the term structure of default-free interest rates, our understanding of term structure of credit spreads is still in early stages.

Some common notions about the term structure of credit spreads are, (a) its shape essentially reflects the shape of the underlying conditional default probability term structure and (b) it is upward sloping for high quality credits. For low quality credits, there is less agreement on the shape of the term structure. While most theoretical models and some empirical research predict hump shaped or downward sloping term structures, other empirical work suggests upward sloping spread curves.

This paper makes several important contributions towards our understanding of the term structure of credit spreads. We show that the theoretical term structure of credit spreads, particularly for low quality bonds depends not only on the term structure of conditional default hazard rates (as is commonly understood), but also on (a) the recovery assumption used and (b) the deviation of a bond's price from its par value. We find that under the Recovery of Treasury (RT) assumption, zero-coupon spreads as well as par spreads have a humped or substantially downward sloping shape for low quality bonds even when the underlying default hazard rate term structure is upward sloping or flat. Under the Recovery of Market Value (RMV) assumption, both zero-coupon curves and par curves are upward (downward) sloping for an upward (downward) sloping hazard rate function. The most interesting finding is obtained under the Recovery of Face (RF)

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<sup>1</sup>See Krishnan et. al. (2005) for recent evidence on the information content of spread term structure for predicting future spreads and default rates.

assumption. Here, zero-coupon spread curves behave differently from par spread curves. Zero-coupon curves under RF behave like the ones under RT i.e. they are hump-shaped or mostly downward sloping for low quality bonds even when the default hazard rate is upward sloping or flat. On the other hand, par curves under RF behave like those under RMV, i.e. they are upward (downward) sloping for an upward (downward) sloping hazard rate function. Thus, under the RF assumption, we can get very different theoretical shapes of spread curves depending on whether we look at zero-coupon curves (or equivalently deeply discounted coupon bond curves) or par curves.

The above results have important implications for the existing literature on spread term structure. The RF assumption is arguably the most realistic recovery modelling assumption for corporate bonds, loans and credit default swaps because it closely resembles the legal structure of recovery for these instruments in default events. However, it is not the most convenient modelling assumption because it leads to extra complexity in models of credit valuation. Thus, it has been common practice in the literature to adopt either the RT or the RMV assumption and discuss the implications of the model, including the shape of the spread term structure. Our analysis shows that, with the RT assumption, for example, we expect to find hump-shaped spread curves for low quality bonds regardless of the functional form of the default hazard rate generated by the model. Further, the results obtained under RT or RMV assumptions may not always extend to the RF assumption, and, thus may not be an accurate description of the empirical credit spread curves in bond and credit default swap markets.

A second major contribution of this paper is to reconcile the somewhat conflicting empirical results on the slope of speculative grade term structure.<sup>2</sup> Sarig and Warga (1989) (SW89 hereafter), Fons (1994) (F94 hereafter) and Kealhofer (2003b) (K03 hereafter) look at the slope of the term structure of spreads by

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<sup>2</sup>The available evidence is unanimous that for investment grade credits, the term structure of spreads is upward sloping. See, for example, Sarig and Warga (1989), Elton et. al. (2001) and Kealhofer (2003b). This result is also consistent with theoretical models and with our findings as well. The effects of recovery rate on slopes manifest more substantially for low quality credits and hence we focus on them in our empirical analysis.

grouping together bonds of identical credit ratings. They find term structures of spreads to be downward sloping for speculative grade credits. Helwege and Turner (1999) (HT99 hereafter), however, show that the results in studies like those referred to above suffer from a sample selection bias. The bias arises because the credit quality of firms within a rating class is heterogeneous and among firms with the same credit rating, the safer ones tend to issue longer-maturity bonds. Given these circumstances, the average spread of a rating class can decrease as time to maturity increases even if for any given firm within that rating class the spreads might increase with time to maturity. HT99 overcome this problem by examining groups of bonds *issued by the same firm* (rather than groups with just the same rating) with different times to maturity. They find that with this methodology, the credit spread term structure for most speculative grade firms is upward sloping. This result is often interpreted to be contrary to the predictions of most bond-pricing models and hence somewhat puzzling.

Our analysis helps to resolve this puzzle. We avoid the potential selection bias by adopting a methodology similar to that of HT99 and examine the slope of speculative-grade term structures in a much larger sample of secondary market bond transaction data. We find that for a majority of speculative-grade firms, the term structure of credit spreads are *downward* sloping. This outcome is consistent with the predictions of most bond-pricing models and with earlier empirical studies such as SW89 and F94. Thus, it shows that potential selection bias alone cannot explain the downward sloping term structures found in these studies. Further, our findings seem contradictory to the findings of HT99. However, this apparent contradiction is substantially resolved when we find that there is an important difference between bonds used by HT99 and those used by us that would cause the results to be different. Almost all the bonds in HT99's data, being new issues, are either par bonds or bonds priced close to par. On the other hand, the bonds in our secondary market data can trade significantly below par. Since RF is the most appropriate recovery assumption for bonds (based on realized recovery rates in practice,) we expect to see a dichotomous behavior of zero-coupon spreads (or equivalently spreads on deeply discounted bonds) and par spreads i.e. when credit quality is low, par bonds can have upward sloping term structures of spreads even

when discount bonds have hump-shaped or downward sloping term structures. We next divide our data sample by price and find that our data and examine if the deviation of bonds' prices from their par values systematically influences the observed term structures. Our empirical results confirm the dichotomous behavior of par spreads versus discount bond spreads. Since this dichotomy is obtained only under the RF assumption, this analysis also provides evidence that the market prices bonds consistent with the RF assumption.

A final contribution of the paper is to cast light on the slope of the term structure of hazard rates for low quality credits. The slope of this term structure is closely reflected in the slope of related par curves.<sup>3</sup> We find that rating buckets encompass a broad range of credit quality and hence mask variation in slope with respect to credit quality. Using spread level as a finer measure of credit quality, we find that when credit quality deteriorates, the par curves (and hazard rates) become progressively more likely to be downward sloping. This finding holds in both secondary market bond curves and CDS curves.<sup>4</sup>

We begin in section 1 with a discussion of the theoretical background underlying the shape of the term structure of credit spreads. Section 2 provides a detailed analysis of par spread term structure and zero-coupon spread term structures under various recovery assumptions. In section 3, we conduct an empirical examination of speculative-grade spread term structures using secondary market bond transaction data. In section 4 we evaluate whether the differences between par spreads and discount bond spreads are borne out in our data. Section 5 studies the variation in slope of the par curves with credit quality. We conclude in section 6.

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<sup>3</sup>This assumes that the market prices bonds consistent with the RF assumption.

<sup>4</sup>Note that about 90% of the market activity in the corporate CDS market is still concentrated at a single tenor of 5 years, thus CDS prices at other tenors are somewhat less reliable. Consequently, we should exercise caution in our interpretation of term structures of CDS spreads. More research on the term structure of CDS spreads will be needed as the liquidity for other tenors improves.

# 1 Theoretical Background underlying Spread Term Structures

Both structural and reduced-form approaches to credit risky debt valuation have broadly similar implications for the term structure of credit spreads. For example, in the context of structural models, Merton (1974), and Pitts and Selby (1983) analyze the term structure of zero-coupon bond spreads and show the following: First, for a time to maturity of zero, the probability of default and, by extension, credit spreads are zero. Second, the credit spread term structures are upward sloping for high quality credits and become hump-shaped or even strictly downward sloping for low quality credits. Studies like Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995) extend Merton's basic model to allow default at anytime over the life of the debt. They generate similar shapes for the credit spread term structure.

Very short term spreads are zero in most structural models because the firm's value is modelled as a continuous-time diffusion process and default happens when the firm value hits an absorbing barrier. Thus, if we know that at a given point in time, the firm value is higher than the barrier, it cannot immediately hit the barrier – because it cannot jump. Thus, the probability of default over a short time horizon approaching zero is zero, thus implying a credit spread of zero. This behavior of spreads will be reflected in all the diffusion-based models where default is triggered by the first hitting time of a diffusion process. The outcome changes in only those models where the firm's value dynamics or default dynamics are qualitatively different. One example is Zhou (1997) where the firm's value process is allowed to jump. Another example is the incomplete information model of Duffie and Lando (2001), where spreads can be positive even as time to maturity shrinks to zero.<sup>5</sup>

The *strictly downward* sloping curves in Merton (1974) arise only from a rather technical feature of the model and are not seen in its various extensions where the

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<sup>5</sup>Yu (2005) provides some empirical evidence in support of the ideas in Duffie and Lando (2001).

low quality curves tend to be *hump-shaped*. In Merton’s model, strictly downward sloping curves arise when the “quasi-leverage ratio”  $d \equiv Be^{-rT}/V$  is greater than 1. Here  $V$  is the value of the firm’s assets,  $r$  is the default-risk-free interest rate (assumed constant),  $T$  is the time to maturity of the debt issue and  $B$  is the face value of the debt. Thus  $d > 1$  represents the situation when the (risk-free) present value of a firm’s liabilities is already greater than that firm’s asset value, meaning that the firm is effectively already in default. In Merton’s model this situation is not considered a default, since default is allowed to happen only *at* time  $T$ . Technically, the model allows for the possibility that the firm’s value may at some  $t < T$  fall below its liabilities and then bounce back to be higher than its liabilities on the maturity date, thereby avoiding default. In most extensions of Merton’s basic model, default is assumed to occur when the firm’s value hits an absorbing barrier at anytime up to maturity date  $T$ .<sup>6</sup> Thus, a condition equivalent to  $d > 1$  can not arise and hence term structures of low quality credits cannot become *strictly downward* sloping.

Reduced-form models, in general, have similar implications for the slope of spread term structures as the Structural models. The main difference from structural models is in their predictions about spread behavior as time to maturity shrinks to zero. Intensity-based reduced-form models predict positive spreads for zero maturity because default is assumed to arrive as a “surprise.” Many reduced-form models predict an upward sloping spread term structure for good quality credits and a downward sloping or humped term structure for poor quality credits. For example, Jarrow et. al. (1997) look at spread term structures for various rating classes using empirical rating transition matrices to model the behavior of conditional default probabilities over time. They find that term structures are upward sloping for credits rated A or better. They become humped for BBB rated credits and are downward sloping for credits rated BB and worse.

The intuition commonly offered behind these results is that the shape of the credit spread curve is reflective of the conditional default probability term struc-

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<sup>6</sup>See Black and Cox (1976), Longstaff and Schwartz (1995), Kealhofer (2003a), and Vasicek (1985) for examples.

ture. High quality credits have small conditional default probabilities over short horizons and hence short horizon spreads are small. As sufficient time elapses, they have little room to improve (because they are already high quality) but can deteriorate significantly because of the volatility of the firm's value or other factors driving default risk. Thus, the credit spread curve is upward sloping for high quality credits. For low quality credits, the situation is reversed. These credits are low quality to begin with; thus their short horizon conditional default probabilities are high and so are their credit spreads. As time elapses, conditional on survival, they do not have much room to deteriorate further. On the other hand, a longer time horizon gives them a greater chance for moving to a higher quality category, leading to a smaller spread over longer time horizons. These borrowers benefit from the underlying volatility of their firm value when looking at longer time horizons. This explains the downward sloping nature of spread curves for low quality credits. Since the spread at time to maturity of zero is anchored at zero in most structural models, we expect to see an initial upward slope in these curves, leading to an overall humped structure. The initial upward sloping portion is less pronounced for models that allow jumps to default. Both structural and reduced form models attempt to capture this general dynamics of conditional default probabilities and hence predict similar slopes of spread term structures.<sup>7</sup>

Hump shaped or mostly downward-sloping spread term-structures for low quality bonds predicted by theory are often seen in the data, too. Fons (1994) and Elton et. al. (2001), among others, explain the downward sloping term structure of spreads using the fact that the term structure of conditional default probabilities implied from empirical default rates is downward sloping for low quality

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<sup>7</sup>Some models, like Collin-Dufresne and Goldstein (2001) enrich this dynamics by incorporating the impact of dynamic leverage ratios and stochastic interest rate that is correlated with the default arrival. Depending on the parameters assumed, they may get different outcome for slope of the spread term structure. Other studies like Kealhofer (2003a) and Vasicek (1984) suggest elaborate modelling of the term structure of liabilities e.g. short term liabilities, long term liabilities, convertible debt, etc. The term structure of liabilities, which can widely differ across firms, affects conditional default probabilities and hence causes the shape of the spread term structure to be different across firms. Our results are complementary to these results. We show that while conditional default probabilities are significant drivers of the shape of spread term structure, there are other important influences as well.

bonds. While the role of conditional default probability term structures has been well articulated, the role of (a) recovery assumptions and (b) deviation of a bond's price from par in influencing the shape of the term structure has received almost no attention in the literature. The next section delves into these issues in detail.

## 2 Recovery Rates and Spread Term Structure

In this section, we show that the modelling assumption about recovery in default has a significant bearing on the shape of the term structure of spreads— particularly for low quality credits. For the same term structure of conditional default probabilities, we can get different shapes of spread term structures under different recovery assumptions. To examine the impact of deviation of bond price from par value, we will first compare yield spreads of par bonds against those of *zero-coupon* bonds, which are extreme cases of discount bonds and much simpler to analyze. Examining zero-coupon spreads is also important to establish a connection with existing theoretical literature on credit-spread term structures. Later on, we will include the analysis of coupon-bearing discount bonds.

Following the definitions laid out in Duffie and Singleton (1999), we will consider three types of recovery assumptions: recovery of treasury (RT), recovery of face (RF) and recovery of market value (RMV). If  $L$  is the *fractional* loss given default (assumed to be constant here, for simplicity), then the dollar recovery  $\phi_t$ , conditional on default at time  $t$  is given by,

$$\mathbf{RT} \quad \phi_t = (1 - L)P_t$$

$$\mathbf{RF} \quad \phi_t = (1 - L)F$$

$$\mathbf{RMV} \quad \phi_t = (1 - L)V_{t-}$$

where  $P_t$  is the value at time  $t$  of a default-free bond that is equivalent to the defaultable bond being considered except that it is default free,<sup>8</sup>  $F$  is the face

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<sup>8</sup>Treasury bonds are commonly considered as a proxy for default-free bonds, hence the name “Recovery of Treasury.”

value of the bond (assumed to be one hereafter) and  $V_{t-}$  is the value of the defaultable bond just prior to default at  $t$ .<sup>9</sup> These assumptions about recovery lead to different expressions for bond values and credit spreads.

## 2.1 Bond Values and Spreads

Let us consider the following notation.  $P_{0,T}$  is the value at time 0 of a default-free, zero-coupon bond with maturity  $T$ . For simplicity, we will assume deterministic default-free rates with a flat term structure. The default probabilities under the risk-neutral measure can be described in many different ways. We will assume that  $Q_{0,T}$  is the risk-neutral cumulative default probability to time  $T$ ,  $q_t$  is the unconditional default probability density at time  $t$ ,  $h_t$  is the default hazard rate at time  $t$  and  $H_{0,T}$  is the integrated hazard function from time 0 to time  $T$ .<sup>10</sup> Consider a  $T$  period defaultable coupon bond with coupon rate of  $C$  per period. Its values at time 0 under the various recovery assumptions are given by,<sup>11</sup>

**RT**

$$V_{0,T}^C = P_{0,T}(1 - LQ_{0,T}) + C \sum_{t=1}^T P_{0,t}(1 - LQ_{0,t}) \quad (1)$$

**RF**

$$V_{0,T}^C = P_{0,T}(1 - Q_{0,T}) + C \sum_{t=1}^T P_{0,t}(1 - Q_{0,t}) + (1 - L) \int_0^T A_t P_{0,t} q_t dt \quad (2)$$

**RMV**

$$V_{0,T}^C = P_{0,T} \exp(-H_{0,T}L) + C \sum_{t=1}^T P_{0,t} \exp(-H_{0,t}L) \quad (3)$$

Values of zero-coupon bonds are easily obtained by setting  $C = 0$  in the above expressions. We have to be more careful when we use the RF assumption with

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<sup>9</sup>See Duffie and Singleton (1999) for more details about different recovery assumptions.

<sup>10</sup>See Appendix A for more details on these different approaches to specifying the default probabilities and the relationships among them.

<sup>11</sup>See Appendix B for more details regarding these expressions.

zero-coupon bonds. Defaultable zero-coupon bonds are rare, but when they exist, they have a claim amount accrual schedule, which determines the fraction of the face value that becomes the legal claim of the bondholder over the life of the bond. Such accrual schedules are usually not specified for coupon-paying, plain vanilla bonds. In expression (2) for bond value under the RF assumption,  $A_t$  represents the accrued contractual claim (as a fraction of Face Value) at time  $t \leq T$ . For coupon bonds,  $A_t = 1$  for all  $t \leq T$ . In the absence of bond specific information about such schedules for zero-coupon bonds, we will assume a linear accrual of claim from 0 at time 0 to 1 at time  $T$ . Recovery in the event of default at time  $t$  ( $0 <= t <= T$ ) is thus  $(1 - L)A_t$ , where  $A_t = \frac{t}{T}$ . Thus, the possible values of a zero-coupon bond under the various recovery assumptions are given by,

**RT**

$$V_{0,T}^Z = P_{0,T}(1 - LQ_{0,T}) \quad (4)$$

**RF**

$$V_{0,T}^Z = P_{0,T}(1 - Q_{0,T}) + (1 - L) \int_0^T A_t P_{0,t} q_t dt \quad (5)$$

**RMV**

$$V_{0,T}^Z = P_{0,T} \exp(-H_{0,T}L) \quad (6)$$

The continuously compounded zero-coupon spread  $S_{0,T}^Z$  to tenor  $T$  is obtained by writing  $V_{0,T} = P_{0,T} \exp(-S_{0,T}^Z T)$ .

**RT**

$$S_{0,T}^Z = -\frac{1}{T} \ln(1 - LQ_{0,T}) \quad (7)$$

**RF**

$$S_{0,T}^Z = -\frac{1}{T} \ln \left( (1 - Q_{0,T}) + \frac{1 - L}{P_{0,T}} \int_0^T A_t P_{0,t} q_t dt \right) \quad (8)$$

## RMV

$$S_{0,T}^Z = \frac{H_{0,T}L}{T} \quad (9)$$

Par spreads can also be computed using the expressions for the valuation of coupon bonds above. A par spread is the spread between the par yield of a defaultable bond and the par yield of an identical maturity default-free bond. The par yield is computed as the coupon rate at which the value of a bond equals its par value. Thus we set the value of the bond to one in expressions (1), (2) and (3) and solve for the coupon rate to calculate the par yields. The default-free par yield is computed by setting all default probabilities to 0. Thus, we get the following expressions for par spreads,

## RT

$$S^{Par} = \frac{1 - P_{0,T}(1 - LQ_{0,T})}{\sum_{t=1}^T P_{0,t}(1 - LQ_{0,t})} - \frac{1 - P_{0,T}}{\sum_{t=1}^T P_{0,t}} \quad (10)$$

## RF

$$S^{Par} = \frac{1 - P_{0,T}(1 - Q_{0,T}) - (1 - L) \int_0^T P_{0,t} q_t dt}{\sum_{t=1}^T P_{0,t}(1 - Q_{0,t})} - \frac{1 - P_{0,T}}{\sum_{t=1}^T P_{0,t}} \quad (11)$$

## RMV

$$S^{Par} = \frac{1 - P_{0,T} \exp(-H_{0,T}L)}{\sum_{t=1}^T P_{0,t} \exp(-H_{0,t}L)} - \frac{1 - P_{0,T}}{\sum_{t=1}^T P_{0,t}} \quad (12)$$

We will now use the above expressions to develop intuition regarding the term structures of spreads.

## 2.2 Term Structures of Zero-Coupon Spreads

First, let us consider zero-coupon spreads using the RT assumption. The shape will depend on the functional form of  $Q_{0,T}$ , the cumulative (risk-neutral) probability of default to maturity  $T$ .  $Q_{0,T}$ , being a cumulative probability, is non-decreasing in  $T$  as is its transformation  $-\ln(1 - LQ_{0,T})$ . The zero-coupon spread is thus a

ratio of two non-decreasing functions of  $T$  and can be increasing, decreasing or non-monotone in  $T$ . In models that do not allow a sudden jump to default (e.g. most structural models),  $Q_{0,T} \rightarrow 0$  as  $T \rightarrow 0$ , thus  $S^{ZC} \rightarrow 0$  as  $T \rightarrow 0$ .<sup>12</sup> Low quality bonds have a high probability of default at a short horizon, thus spreads will rise steeply from 0 as  $T$  increases initially. Since  $Q_{0,T}$  has an upper bound of one, this probability does not increase as fast as  $T$  increases. So long as  $L < 1$ , the function  $-\ln(1 - LQ_{0,T})$  mirrors the behavior of  $Q_{0,T}$  and converges to an upper bound as  $T$  increases. On the other hand,  $T$  can increase without limit. Thus, the spread will eventually go to zero as  $T$  increases without limit. Since the spread is initially increasing and then starts decreasing at some maturity, it has a humped term structure. Notice that this will hold *regardless* of the credit quality of the bonds. For high quality bonds, however, the peak of the hump occurs at sufficiently long tenors that are typically not observed in traded bonds. Thus, for tenors generally observed, high quality bonds will show an upward sloping spread term structure. The lower the credit quality, the shorter the tenor at which the peak of the hump occurs. Thus, low quality bonds will show a downward-sloping term structure for most tenors.

We now illustrate these ideas using a parameterized form of  $Q_{0,T}$ . This is achieved by assuming the default hazard rate  $h_t$  is linear in time to maturity i.e.  $h_t = h_1 + h_2t$  where  $h_1$  and  $h_2$  have the interpretation of level and slope of the hazard rate function, respectively.<sup>13</sup> Fractional loss given default  $L$  is assumed to be constant at 0.6. Figure 1(A) shows the term structures of zero-coupon spreads under the RT assumption. Different curves correspond to different levels of  $h_1$ , with  $h_2$  always set to 0.02. Even though the default hazard rate function is always upward sloping, the term structures become hump shaped. The peak of the hump occurs at shorter tenors when hazard rate levels are high.

Assuming RF with linear amortization yields qualitatively similar results. This can be seen in Figure 1(B).

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<sup>12</sup>For example, see Pitts and Selby (1983) for a rigorous proof of this property for the zero-coupon spread in the Merton (1974) model.

<sup>13</sup>See Appendix A for more details on this parameterization.

*Insert Figure 1 here.*

In contrast, the RMV assumption leads to a very different shape of the spread term structure. For example, expression (9) shows that for a constant hazard rate, the spread is a constant i.e. the term structure is flat. For a hazard function linear in  $T$ , the spread is linear in  $T$  as well, i.e. the spread reflects the shape of the hazard function. While the zero-coupon spread under RMV reflects the shape of the hazard function, this recovery assumption is hard to justify for corporate bonds and credit default swaps where the legal structure of the contracts closely matches the RF assumption.<sup>14</sup> In the next section we will examine par spreads under different recovery assumptions.

### 2.3 Term Structure of Par Spreads

First, we consider the term structure of par spreads under the RT assumption. This is given by expression (10). We can again illustrate the shape of the par-spread curve by using a parameterized form of  $Q_{0,T}$ . As before, we assume a risk neutral default hazard rate  $h_t$  that is linear in time to maturity i.e.  $h_t = h_1 + h_2t$ . Fractional loss given default  $L$  is assumed to be constant at 0.6. The resulting par-spread curves under the RT assumption are shown in figure 2, Panel (A). Even though the default hazard rate is upward sloping, these spreads have a humped shape, just like their zero-coupon counterparts.

Panel (B) of figure 2 shows that par spread curves under RF and RMV are (a) close to each other and (b) upward sloping for upward sloping hazard-rate functions. For par bonds, the market value of the bond just before default  $V(t-)$  is close to par, and thus the RF and RMV assumptions imply similar recoveries in default. Hence the par spread curves under the two assumptions behave similar to each other.<sup>15</sup>

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<sup>14</sup>Duffie and Singleton (1999) point out that RMV is a reasonable description of recoveries in certain contracts such as interest rate swaps.

<sup>15</sup>Duffie and Singleton (1999) find similar results using a different specification of hazard rates,

*Insert Figure 2 here.*

## 2.4 Implications for Observed Term-structure of Spreads

The analysis in the preceding paragraphs shows that the zero-coupon curves and par curves have slopes similar to each other under either the RT assumption or the RMV assumption. Under the RT assumption, both curves are humped, even when the hazard rate term structure is upward sloping. Under the RMV assumption, both the curves reflect the slope of the hazard rate function. In contrast, zero-coupon curves and par curves are dramatically different from each other under the RF assumption. While zero curves are humped, par curves reflect the slope of the hazard rate function. Figure 3 shows this contrast graphically, using the assumption of an upward sloping hazard rate function. As credit quality worsens, the par curve remains upward sloping while the zero curve starts becoming hump shaped and eventually becomes downward-sloping for most maturities.

*Insert Figure 3 here.*

This contrast under the RF assumption is important in practice because the recovery in the event of default for both bonds and credit default swaps corresponds closely to the RF assumption. This analysis suggests the upward-sloping par curves for high yield bonds can be consistent with the hump-shaped or downward-shaped zero curves predicted by many theoretical models and empirically seen in SW89 (who analyze zero-coupon spreads). In practice, however, defaultable zero-coupon bonds are rare. We can, however, find a large number of speculative grade coupon bonds trading at significant discounts to their par values. Will our results comparing zero-coupon spreads with par spreads extend to discounted coupon bonds as well? Figure 4 shows that this should hold true. This figure shows the spread curves generated by assuming an upward-sloping hazard function  $h_t = h_1 + h_2t$ , with  $h_1 = 0.03$  and  $h_2 = 0.02$  for various coupon rates under

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e.g. see their figure 3.

the RF assumption. Panel (B) shows the corresponding price curves. It is clear that the spread term structure of discounted coupon bonds is also humped. Thus, our results suggest that upward-sloping par curves can be consistent with hump-shaped spread curves for discounted coupon bonds as well. In the next section, we relate these findings to the existing theoretical literature.

*Insert Figure 4 here.*

## 2.5 Relationship with Existing Literature

The above discussion helps us relate back to various theoretical and empirical results regarding spread term structures. In the context of structural models, Longstaff and Schwartz (1995) assume RT in their spread calculations. Thus, in their model, both the zero-coupon spreads and coupon-bond yield spreads have a hump-shaped term structure for low credit quality bonds. Their analysis also shows that this shape of the term structure is essentially the same even with (a) different values for the risk-free rate and (b) different values of correlation between the interest rate process and the asset value process. Similar results can be seen in Kim et. al. (1993). Merton (1974) considers the term structure of zero-coupon spreads. The term structure is strictly downward-sloping when the quasi-leverage ratio is greater than one. This case has not been covered in our discussion above. However, as pointed out earlier, technically, the firm is already in default if the quasi-leverage ratio is greater than one at any time before the debt matures. The default does not happen because it is assumed to happen only at the time of maturity. Thus, this case is not relevant in a practical sense and does not arise in other first-passage-time structural models. When the firm is solvent, i.e. the quasi-leverage ratio is less than one, the term structure closely corresponds to that obtained under the RT assumption.<sup>16</sup>

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<sup>16</sup>In Merton's model, the recovery is also stochastic and correlated with the default risk. This complicates the shape of the zero-coupon term structure. While the spread is still zero as time to maturity goes to zero, the spread does not converge to zero as time to maturity goes to infinity (as would be the case in RT model discussed above). Instead it converges to a positive value. This can be seen in, e.g. Pitts and Selby (1983).

Reduced-form models have qualitatively similar results except that here, the spread at very short maturities can be positive because these models allow defaults to arrive as surprises. Jarrow et. al. (1997) assume RT in their analysis and generate hump-shaped term structures for zero-coupon bonds of low credit quality. Duffie and Singleton (1999), on the other hand, assume RMV in their analysis. They examine the term structures of par spreads under the RF and RMV assumptions and find that (a) the RF and RMV spread curves are close to each other and (b) the slope of these curves is positive (negative) when the hazard function has a positive (negative) slope. The results of Duffie and Singleton (1999) are obtained using a stochastic default intensity. Our analysis above highlights that the similarity of spreads under the RF and RMV assumptions shown in Duffie and Singleton (1999) would not hold if we considered zero-coupon spreads or spreads on discounted coupon bonds. Thus, it is clear that not only the term structure of default hazard rates but also (a) the recovery assumption and (b) the deviation of a bond's price from its par value, make a material difference to the shape of the spread curves.

### **3 Empirical Analysis of Speculative-Grade Spread Term Structure**

In this section, we will examine in detail if our results can help reconcile seemingly conflicting empirical results in the literature regarding the slope of speculative grade spread term structures. As mentioned in the introduction, one set of results comes from studies of SW89, F94 and K03, who all group together bonds by rating categories and then examine the term structure of spreads within each rating bucket. They find that spread curves are humped or downward-sloping for speculative grade credits. These findings are also consistent with most of the theoretical literature reviewed earlier. However, the analysis in HT99 suggests that the above results are potentially driven by a sample selection bias. The credit quality in each rating category is quite heterogeneous and safer firms in each rating category tend to issue longer dated bonds. Thus, when we estimate spread curves for each

rating category, they tend to be downward-sloping. HT99 exercise a more precise control for credit quality by examining the slope of the spread curve in sets of bonds issued by the *same firm* (rather than just the same credit rating), having the same seniority, rating and pricing date but different tenors. They find that, with this improved methodology, the spread curves for speculative grade credits are pre-dominantly upward-sloping.

We re-examine the spread term structures for speculative grade credits using a methodology similar to that of HT99 but in a much larger, transaction price data sample of corporate bonds. The primary results in HT99 are obtained using new issue bond prices (and spreads) from the SDC Platinum database. Since it contains only the new issues' prices, this data set yields only a small test sample in which the slope of the speculative-grade spread term structure can be examined. Further, the speculative grade bonds in this test sample come from only two rating categories— BB and B, thus one cannot examine spread term structures for lower credit categories. Finally, the bond prices in this data set, being new issue prices, are set by bond underwriters rather than the market.<sup>17</sup> How accurately do such prices reflect the bond market's view of each bond's economic value? Fridson and Garman (1998) report that underwriter-determined, new-issue prices of high-yield bonds can significantly deviate from the underlying fundamentals and that wide discrepancies have been observed between the underwriters' promotions and actual pricing at issuance.<sup>18</sup> While the question of new issue versus secondary market pricing may require further work, which we leave for future research, we believe an analysis of term structure slopes in secondary market transaction data will yield new insights.

We use a database of secondary market transactions of corporate bonds maintained by Capital Access International (CAI) as our main source of information

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<sup>17</sup>HT99 also examine a sample of bond quotations from Lehman Brothers' Fixed Income database, but this sample is also tiny. The prices in this sample are non-binding trader quotes rather than actual transactions.

<sup>18</sup>Fridson and Garman (1998) go on to conclude that "...by orchestrating an effective roadshow, a lead manager may obtain a narrower spread for one issue than is given to another issue that, based on the numbers alone, ought to be priced identically."

on bond prices. These data have been used by several authors like Chakravarty and Sarkar (1999), Hong and Warga (1998) and Schultz (2001). The main sources of these data are schedule-D filings by insurance companies. The manner in which these data are collected improves the reliability of these prices. Schultz (2001) describes these data in greater detail. We compute bond yields from these transaction prices. We obtain data on U.S. treasury yields for various maturity buckets from the FRED database maintained by the Federal Reserve Bank at St. Louis. Credit yield spreads on each bond are computed as the difference between yield-to-maturity and the interpolated treasury yield at the time to maturity of the bond.<sup>19</sup> We use data covering the period 1995-2003. A number of filters are applied before any bond prices are selected for analysis. For example, we remove all bonds with options other than simple call options. All secured bonds are also omitted. We compute yield-to-maturity corresponding to each transaction price. If a bond transacts more than once during a day, we use the average yield to maturity in our analysis of spreads.

We determine the spread term structure by looking at bond yield spreads on a given day for bonds that are issued by the same firm and have identical seniority and credit rating, but different times to maturity. For ease of reference, these sets of bond spreads that are identical in every respect except time to maturity are termed “bond sets.” The slope of each obligor’s spread term structure is examined by looking at the relative spreads in each bond set. This methodology, by imposing the condition that all bonds in a “bond set” be issues of the same firm rather than just have the same rating, achieves a far more stringent control for default risk, thus facilitating a straightforward view of the term effect on spreads.

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<sup>19</sup>Credit yield spread is defined here using the Treasury yield as the reference rate in order to be consistent with the earlier studies. As shown by Elton et. al. (2001) and others, this yield spread also incorporates tax effects and liquidity effects. For speculative grade bonds, these effects should be only a small fraction of the total spread. In addition, as we discuss later, our analysis using swap curves as the reference curve generates qualitatively similar results.

### 3.1 Results

Table 1 shows the results of our analysis. Our bond sets can have 2 bonds (pairs), 3 bonds (triplets) or 4 bonds (quadruplets). In this first set of results, we let callable and straight bonds be mixed together in constructing our bond sets. According to the estimates in Crabbe (1991), call options contribute only about nine basis points to the spread on average, which is a tiny fraction of the yield spread of a speculative grade bond. Thus, ignoring the call option should not have material impact on the results. Call options may however, be more valuable for speculative-grade firms expected to improve over time or in economic environments where interest rates have been falling. Thus, in the next section, we conduct a robustness check in which we remove the callable bonds and analyze just the straight bonds.

<i>Insert Table 1 here.</i>
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A large majority of our sets are pairs and for these sets, we can only have two shapes of slope, upward (U) or downward (D). We found no cases where the slope was exactly flat. We do have some triplets and quadruplets and here the spreads can change with time to maturity in a monotone or non-monotone way.<sup>20</sup> Overall, our results are quite different from those of HT99. While 82 to 94 percent of bond pairs in their sample had *upward* sloping term structure, in our sample 60 to 77 percent of bond pairs have a *downward* sloping term structure. The fraction of pairs with downward-sloping term structure seems to be slightly higher for lower ratings (Caa and below) though Ca is an exception.<sup>21</sup> Results from triplets and quadruplets again show a tendency for downward-sloping term structures.

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<sup>20</sup>Notice that our sample is far bigger than the HT99 sample. For example, they had 44 pairs of bonds across two speculative-grade rating categories, while we have 2042 pairs spread across five rating categories.

<sup>21</sup>Here we use Moody's rating scale. In some of the earlier discussions of previous research, we refer to S&P's rating scale because that is the scale used by those papers. The two scales are approximately comparable.

### 3.2 Robustness Checks

We conduct several robustness checks on our results. First, we remove the callable bonds from our sample and retain only the bonds with no option features. The results on this set of bonds are shown in Table 2. The results are similar to what we show earlier in Table 1.

<i>Insert Table 2 here.</i>
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We then examine the slope of the credit-spread term structure when spreads are computed over swap rates rather than treasury rates. Use of swap rates as reference rates is quite common among practitioners and this practice is increasingly popular among academics. We find that the results are similar even when we use swap curves as the reference curves. These results are not reported, but available upon request.

We complete a third analysis to check if the occurrence of downward slopes is concentrated in a particular time period. We divide our data of bond sets by year and examine the fraction with downward slopes by year. The results are shown in Table 3. The incidence of downward slopes varies by year, but in general, the majority of slopes are downward in all years studied. Thus, our results are not specific to a particular time period.

<i>Table 3 here.</i>
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## 4 Slope and Bond Price

Our empirical results are obtained using a methodology similar to that of HT99 that avoids the potential selection bias of earlier empirical studies evident in SW89 and F94. However, our results are qualitatively different from those of the HT99 and similar to those of SW89 and F94. Thus, potential selection bias alone cannot account for producing a downward-sloping term structure. How do we reconcile these findings? Differences between our and HT99 data alone are unlikely to ac-

count for such significantly different results. Our analysis in this section reveals that the differences are primarily driven by the differences in the nature of bond spreads being considered. The primary results in the HT99 study are based on a sample of newly issued bonds where coupons are so chosen that bonds are mostly priced at par or close to par. This can be seen from figure 5(A), which shows the distribution of speculative-grade bond prices in the SDC Platinum new-issues database used by HT99. Our sample of speculative-grade bonds, on the other hand, comes from the secondary bond market. As a result, it contains a substantial number of bonds that trade at a discount to their par values, as can be seen from figure 5(B).

*Insert Figure 5 here.*

The analysis in the previous sections suggests that the differences between par curves and discount bond curves is a likely explanation for why our findings differ from those of HT99. In the paragraphs below, we investigate whether the theoretical findings are vindicated in our data. In other words, we examine if the slopes of spread curves in our data vary systematically with the bond price as suggested by the analytical results.

As before, we construct bond pairs, each containing two bonds with different times to maturity but identical issuer, credit rating, seniority and pricing date. To ensure that the two bonds in any bond pair do not have prices that are too far apart, we omit all bond pairs for which the price differential is greater than ten dollars.<sup>22</sup> The average price of the two bonds in each pair is called the “group price.” We use the group price of each bond pair to put them into price buckets. Our buckets are  $[P \leq \$70]$ ,  $[\$70 < P \leq \$80]$ ,  $[\$80 < P \leq \$90]$ ,  $[\$90 < P \leq \$100]$ ,  $[\$100 < P \leq \$110]$  and  $[\$110 < P]$ . We investigate whether the slope of the term structure varies systematically with respect to the price bucket. This analysis is done by looking at the slope of the spread term structure for each bond pair and aggregating these results by rating category and the relevant price bucket of each

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<sup>22</sup>We also try a price differential cutoff of five dollars, which reduces the sample size but does not change the qualitative nature of the results.

pair. The results are summarized in table 4.

These results suggest that the fraction of the bond pairs with downward-sloping term structures indeed tends to be the highest in the most deeply discounted bonds and tends to decrease as the bond prices move up to par and above. This result seem to hold across all the speculative grade rating categories (except in Ca and C categories where our sample size is small).

The above approach to bucketing by price may not produce unambiguous results because of the differences in sample size across buckets and sampling errors. We therefore investigate the impact of price on the slope of the term structure using a logit model. Let  $\pi$  be the probability of observing a downward-sloping term structure in a bond pair. If this probability depends on the group price  $P$  of the bond pair, we can express this relationship as the following logit model,

$$\ln\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta P \quad (13)$$

where  $\alpha$  and  $\beta$  are logit coefficients to be estimated. We estimate this model for each rating category using bond pairs for that category. The dependent variable is a dummy variable  $y$  that takes the value 1 if the slope of the spread term structure for the bond pair is negative and 0 if the slope is positive. Using the estimated coefficients  $\alpha$  and  $\beta$ , we reconstruct  $\pi = \frac{\exp(\alpha+\beta P)}{(1+\exp(\alpha+\beta P))}$ , the probability of observing a downward-sloping spread term structure. Figure 6 shows how the estimated probabilities of downward-sloping spread term structures vary with group price.

*Insert Figure 6 here.*

We can see that the estimated probabilities of downward-sloping spread term structures  $\pi$  are close to 1 for deeply discounted bonds for almost all the rating categories (except Caa where our sample is small) and steadily decreases as the group price increases. For bonds near par, this probability is close to 0.5, implying that those term structures are equally likely to be upward or downward-sloping. This behavior of  $\pi$  is in line with our theoretical analysis above that the spread

curves for low-quality par bonds can be upward-sloping or flat when the same curves for discount bonds are downward-sloping. This dichotomous behavior is seen in model spreads with the RF assumption, only. (Under either the RT or RMV assumptions, slopes of zero curves and par curves are similar to each other.) The fact that we see similar dichotomous behavior in our data provides indirect evidence that the pricing of defaultable bonds in the marketplace is more consistent with the RF assumption rather than the RT or RMV assumptions. We leave this question to future research as further analysis is required for us to draw strong conclusions.

## 5 Term Structure of Par Spreads

The above analysis has highlighted the difference between par spreads and discount spreads. However, we have not yet considered in detail the slope of the par curves. The slope of par curves is interesting to examine because it reflects the slope of the underlying default hazard rate function.<sup>23</sup> Figure 6 already gives us some sense of how the slope of the spread curve for bonds trading close to par may vary with credit rating. In this figure, for Ba rated bonds, the probability of a downward-sloping term structure is about 0.65 for bonds trading close to par. This probability declines to 0.60 for B rated bonds and to 0.50 for Caa rated bonds. However, it rises back to 0.55 for the Ca category and to 0.70 for the C category. Thus, overall, if we use credit ratings as our measure of credit quality, we do not see a strong and monotonic variation in slope of the par curves with credit quality.

This behavior is in contrast with the commonly understood behavior of default hazard rates, highlighted in, for example Fons (1994), Elton et. al. (2001) and Kealhofer (2003b). The evidence in these papers implies that the empirical default hazard rates are upward-sloping for good quality credits and become humped or downward-sloping as credit quality worsens. Slopes of par curves are expected

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<sup>23</sup>Here, we assume that the market prices bonds and credit default swaps consistent with the RF model of recovery. This is consistent with the legal structure of claims in default and also supported by the results in the previous section.

to show similar systematic behavior with respect to credit quality. One reason we may not observe this behavior could be that each credit rating encompasses a broad range of credit quality. We use mean spread of a bond pair as a finer measure of credit quality and examine how the slope of the spread curve varies with this measure. To focus on par curves, we choose only those bond pairs whose group price is in the range of 95 dollars to 105 dollars. As before, we look at speculative grade bonds only and analyze their slopes using the following logit model,

$$\ln\left(\frac{\pi}{1-\pi}\right) = \alpha + \beta S \quad (14)$$

where  $S$  is a measure of credit quality (mean spread of a bond pair in this case) and other terms have the same meaning as before. The estimated probabilities of downward-sloping par curves are plotted in figure 7, Panel (A). When mean spread is used as a measure of credit quality, we find that the probability of downward-sloping par curve increases as credit quality deteriorates. For very low quality credits (spread level approaching 1000 bp), par spreads have an estimated probability of downward-sloping term structure that exceeds 80%.

*Insert Figure 7 here.*

In theory, credit default swap (CDS) spreads are closely comparable to par bond spreads (see, for example, Duffie(1999)). Thus, we also examine the CDS spreads to evaluate another view on the slope of the par bond curves. One issue with the CDS market is that over 90% of trading is concentrated at a single tenor of 5 years. This can be seen in figure 8 which shows the distribution of tenors in CDS data obtained from GFINet, one of the largest CDS broker-dealers.<sup>24</sup> We construct CDS pairs for speculative-grade corporate credits from GFINet data covering the 2001-2004 period and examine the variation in slope with credit quality using the logit model in expression (14). The results are shown in figure 7, Panel (B). These

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<sup>24</sup>There are some CDS data providers that provide full CDS curves for many credits. Since there is hardly any market activity in most names for tenors other than 5 years, these data points likely reflect the model curves used by the data providers and therefore may not be suitable for meaningful term structure analysis.

results are somewhat different from the bond results in the sense that the majority of CDS curves are upward-sloping for speculative-grade credits. However, as the credit quality deteriorates sufficiently, the estimated probability of downward sloping curves increases substantially, similar to what we observed for bond curves.

*Insert Figure 8 here.*

Since HT99 looked at mostly par curves (of speculative grade new issue bonds), it may be useful to compare our findings with their results. They used ratings as measures of credit quality and found that most of their par curves were upward-sloping. Our analysis using CDS data shows similar findings. It shows that a majority of speculative grade CDS curves are also upward-sloping, however they become predominantly downward-sloping if we go sufficiently down the quality spectrum. Interestingly enough, our bond results are somewhat different. In our sample of par bonds, we estimate approximately a 50% to 65% probability of a downward-sloping term structure compared to about 30% in HT99. There could be several reasons for such a difference. The differences could reflect different nature of prices in new issue versus secondary market pricing. The time period covered by our sample is different from theirs and factors like interest rates or even the spread levels can cause the term structures to be different across time periods. In addition, the cross-section of firms that we analyze can be systematically different from theirs. As pointed out earlier, studies like Kealhofer (2003a) and Vasicek (1984) advocate detailed modelling of liabilities term structure to capture cross-sectional differences in slopes of spread curves. A more detailed analysis of determinants of the slopes of par spreads is an interesting topic that we leave for future research.

## 6 Conclusion

Term structure of credit spreads is an important dimension of credit risk. It is well understood that the slope of spread term structures obtained in theoretical models as also in the observed data depends on the shape of the underlying de-

fault hazard rate term structure. We show that the slope of the spread curve also depends crucially on the recovery assumption. Different assumptions about the recovery model, i.e. Recovery of Face (RF), Recovery of Treasury (RT), and Recovery of Market Value (RMV) can lead to very different spread term structures, particularly for low quality credits, even when the hazard rate term structure is identical. These findings help us in explaining the shapes of the term structure obtained in the extant theoretical literature.

Deviation of a bond's price from its par value is found to be another important factor that has a major influence on the shape of the credit spread curve. In particular, we find that under the RF assumption, which is the one relevant for bonds and credit default swaps, an upward-sloping default hazard rate function is consistent with upward-sloping par spread curves but hump-shaped discount bond curves for low quality credits. This finding helps us in reconciling the somewhat conflicting empirical results in the literature on the slope of speculative-grade credit spread curves. We find that studies like SW89 and F94 examine discount bond spreads and find downward-sloping term structures while HT99 look at new issue par spreads and determine that the term structures are upward-sloping. The dichotomy between par spreads and discount spreads can explain the differences between their results. Since most of the theoretical literature considers the spread term structures for either zero-coupon bonds or coupon bonds that are likely to be discount bonds, our analysis also explains why the results of HT99 do not necessarily contradict the predictions of the theoretical models.

Finally, we examine the slopes of par-spread curves (for speculative-grade credits), which should reflect the slopes of the underlying hazard rate functions. Using bond as well as credit default swap data, we find that par curves become progressively downward-sloping as credit quality deteriorates. Using ratings as a credit quality indicator may not yield this clear pattern because of the wide range of credit qualities encompassed in each rating. Using spread levels as a quality measure yields a better resolution.

As emphasized in the recent work by Krishnan et. al. (2005), research on

the information content of the credit spread term structure is still in early stages. Our results should improve our collective understanding of how best to analyze and interpret this information content for forecasting future credit dynamics and pricing of credit contingent claims.

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## A Different Approaches to specifying Default Probabilities

Default probabilities can be described in many different ways. Assume that using any model of default timing,  $Q_{0,t}$  is the risk-neutral cumulative default probability to time  $t$  and  $\pi_{0,t} = 1 - Q_{0,t}$  is the survival probability to time  $t$ . Some alternative ways to describe the same default probability term structure is to specify one of the following:  $q_t$ , the unconditional default probability density at time  $t$ ,  $h_t$ , the default hazard rate at time  $t$  or  $H_{0,t}$ , the integrated hazard function from time 0 to time  $t$ . These quantities are easily related to  $Q_{0,t}$ .

Default density  $q_t$  describes the local probability of default at time  $t$ . It relates to the cumulative default probability as below,

$$Q_{0,T} = \int_0^T q_t dt. \quad (15)$$

Now, assuming that the survival probability  $\pi_{0,t}$  is strictly positive and differentiable in  $t$ , the hazard rate at  $t$  is defined as,

$$h_t = -\frac{\partial}{\partial t} \ln \pi_{0,t}. \quad (16)$$

Hazard rate  $h_t$  is thus the rate of default arrival at time  $t$  conditional only on survival to time  $t$ . It is also called the *forward default rate*. If  $h_t$  is continuous, then  $h_t \Delta t$  approximates the probability of default between  $t$  and  $t + \Delta t$  conditional on survival to  $t$ . Often times, we see the terms *hazard rate* and *intensity* used interchangeably, particularly in the reduced-form modelling literature. Intensity or hazard rate at time  $t$  in those models is the arrival rate of default conditional on *all* information at time  $t$ , while the term hazard rate as used in this paper is the mean arrival rate of default at  $t$ , conditional only on survival to  $t$ . Intensity may not exist in all the models of default arrival e.g. it does not exist for the perfect information first passage time structural models, but a hazard rate or forward

default rate as defined here can be specified for both structural and reduced-form models.

From the above definition of hazard rate, it follows that,

$$Q_{0,T} = 1 - \exp\left(-\int_0^T h_t dt\right) \quad (17)$$

$$= 1 - \exp(-H_{0,T}) \quad (18)$$

where  $H_{0,T} \equiv \int_0^T h_t dt$  defines the *integrated hazard function*.

Since the term structure of spreads is known to be closely related to the conditional default probability term structure, we find it convenient to parameterize the default probabilities by assuming a flexible functional form for the default hazard rate  $h_t$  under the risk-neutral measure. This will help us examine the shape of spread term structures for a given fixed hazard rate term structure but different recovery assumptions. One convenient, but still quite general way to do this parameterization is to assume that the integrated hazard function  $H_{0,T}$  is a polynomial function of time to maturity  $T$ ,

$$H_{0,T} = \sum_{d=1}^D \frac{h_d}{d} T^d \quad (19)$$

where  $D$  is the degree of the polynomial and  $h_d$  are unknown parameters. Note that this function places the required restriction  $H_{0,0} = 0$  by omitting the constant term. For  $D = 1$ ,  $H_{0,T} = h_1 T$  and  $h_t = h_1$ . Thus,  $h_1$  has the interpretation of the average hazard rate. For  $D = 2$ ,  $H_{0,T} = h_1 T + \frac{h_2}{2} T^2$  and  $h_t = h_1 + h_2 t$ , implying that  $h_1$  and  $h_2$  represent the level and the slope of the hazard rate function respectively.

## B Valuation of Defaultable Coupon Bonds

Consider a  $T$  period coupon bond with face value 1 and per period coupon of

*C.* Its valuation under various recovery assumptions can be done using the risk-neutral valuation approach. For simplicity, we will assume deterministic interest rates given by  $r_t$ . This assumption will not affect our general results about the term structure of spreads.  $L$  is the fractional loss given default.

### B.1 Valuation under Recovery of Treasury

Jarrow et. al. (1997) have a detailed analysis of valuation under the RT assumption. The value of a defaultable zero-coupon bond is given by,

$$V_{0,T}^Z = E_0^Q \left[ \exp \left( - \int_0^T r_t dt \right) 1_{\tau > T} + (1 - L) 1_{\tau \leq T} \right] \quad (20)$$

$$= P_{0,T}(1 - LQ_{0,T}) \quad (21)$$

where  $P_{0,T}$  is the value at time 0 of a default-free zero-coupon bond with maturity  $T$ . A coupon bond under this recovery assumption can be valued as a portfolio of zero-coupon bonds. The value of the coupon bond is thus the sum of the values of individual zero-coupon bonds.

$$V_{0,T}^C = P_{0,T}(1 - LQ_{0,T}) + C \sum_{t=1}^T P_{0,t}(1 - LQ_{0,t}) \quad (22)$$

### B.2 Valuation under Recovery of Face

Valuation under the RF assumption can be easily obtained by valuing two cash flow streams separately. The first cash flow stream is the cash flows realized if there is no default over the life of the bond. The value of these cash flows is given by the first two terms in the expression (23) below. The second cash flow is the recovery cash flows in the event of a default by the maturity date of the bond. The present value of these cash flows is given by the third term in expression (23) below. Note that under the RF assumption, the coupons have zero recovery in the event of a default.

$$V_{0,T}^C = P_{0,T}(1 - Q_{0,T}) + C \sum_{t=1}^T P_{0,t}(1 - Q_{0,t}) + (1 - L) \int_0^T A_t P_{0,t} q_t dt \quad (23)$$

Here  $A_t$  is the accrued claim at time  $t$ . For coupon bonds  $A_t = 1$  for all  $t \leq T$ , i.e. the contractual claim is equal to the full face value of the bond. For zero-coupon bonds,  $A_t$  is usually specified by an accrual schedule as discussed in the text.

### B.3 Valuation under Recovery of Market Value

Duffie and Singleton (1999) show that the value of a defaultable zero-coupon bond under this assumption is given by,

$$V_{0,T}^Z = E_0^Q \left[ \exp \left( - \int_0^T (r_t + \lambda_t L_t) dt \right) \right] \quad (24)$$

where  $E_0^Q$  denotes the risk-neutral expectation at date 0,  $r_t$  is the default-free short term interest rate,  $\lambda_t$  is the default intensity at time  $t$  and  $L_t$  is the expected fractional loss in market value if default were to occur at time  $t$ , conditional on the information available up to time  $t$ . For simplicity we assume deterministic interest rates and default intensity and a constant expected loss. When default intensity  $\lambda_t$  varies deterministically, it is identical to the default hazard rate or forward default rate  $h_t$ . With this assumption, the valuation expression simplifies to,

$$V_{0,T}^Z = \exp \left( - \int_0^T (r_t + h_t L) dt \right) \quad (25)$$

$$= P_{0,T} \exp(-H_{0,T}L) \quad (26)$$

Duffie and Singleton (1999) also show that a coupon bond under the RMV assumption can be valued as a portfolio of zero-coupon bonds. Thus, the value of a coupon bond is given by,

$$V_{0,T}^C = P_{0,T} \exp(-H_{0,T}L) + C \sum_{t=1}^T P_{0,t} \exp(-H_{0,t}L). \quad (27)$$

Table 1: Slope of Spread Term Structure : All Bonds

This table shows the slopes of spread term structure for speculative-grade bonds. Slopes are computed from “bond-sets” which could contain two (pairs), three (triplets) or four (quadruplets) bonds with different tenors but identical issuer, credit rating, seniority and price (or spread) date. Upward slopes are represented by 'U' and downward slopes by 'D'. The results are based on our secondary market data obtained from Capital Access International. The analysis shown in this table does not differentiate between callable and straight bonds in constructing the bond sets or calculating the spreads.

Outcomes	Rating Classes				
	Ba	B	Caa	Ca	C
Pairs					
Number of Sets	1168	568	177	96	33
U	34%	37%	23%	40%	27%
D	66%	63%	77%	60%	73%
Triplets					
Number of Sets	177	91	26	10	16
UU (2U)	7%	7%	0%	0%	19%
UD (1U)	21%	22%	31%	20%	25%
DU (1U)	35%	34%	23%	10%	13%
DD (2D)	37%	37%	46%	70%	44%
Quadruplets					
Number of Sets	49	14	7	1	18
UUU (3U)	0%	7%	14%	0%	0%
UUD (2U)	6%	0%	0%	0%	6%
UDU (2U)	8%	0%	0%	0%	0%
DUU (2U)	8%	21%	0%	0%	0%
UDD (2D)	8%	21%	0%	0%	17%
DUD (2D)	12%	7%	14%	0%	22%
DDU (2D)	29%	0%	29%	100%	0%
DDD (3D)	29%	43%	43%	0%	56%

Table 2: Slope of Spread Term Structure : Straight Bonds Only

This table shows the slopes of spread term structure for speculative-grade bonds. Slopes are computed from “bond-sets” which could contain two (pairs), three (triplets) or four (quadruplets) bonds with different tenors but identical issuer, credit rating, seniority and price (or spread) date. Upward slopes are represented by 'U' and downward slopes by 'D'. These results are based on our secondary market data obtained from Capital Access International. The analysis shown in this table retains only bonds without any options.

Outcomes	Rating Classes				
	Ba	B	Caa	Ca	C
	Pairs				
Number of Sets	932	301	106	45	29
U	35%	33%	22%	38%	28%
D	65%	67%	78%	62%	72%
	Triplets				
Number of Sets	167	73	19	4	13
UU (2U)	8%	5%	0%	0%	23%
UD (1U)	21%	19%	21%	0%	23%
DU (1U)	34%	33%	26%	0%	15%
DD (2D)	37%	42%	53%	100%	38%
	Quadruplets				
Number of Sets	43	10	5	0	18
UUU (3U)	0%	0%	20%	0%	0%
UUD (2U)	7%	0%	0%	0%	6%
UDU (2U)	9%	0%	0%	0%	0%
DUU (2U)	7%	30%	0%	0%	0%
UDD (2D)	9%	20%	0%	0%	17%
DUD (2D)	14%	10%	0%	0%	22%
DDU (2D)	21%	0%	40%	0%	0%
DDD (3D)	33%	40%	40%	0%	56%

Table 3: Term structure slopes by year

This table shows the number of bond pairs and the fraction of pairs with downward-sloping term structures of spreads, analyzed by rating category and year. Each bond pair consists of two bond prices (spreads) with different tenors but identical issuer, credit rating, seniority and pricing date. Only bonds with no call and other options are used in this analysis.

Panel (A) : Number of Bond Pairs					
year	Ba	B	Caa	Ca	C
1995	36	3	0	0	0
1996	39	4	1	0	0
1997	55	4	1	0	0
1998	54	1	1	0	0
1999	149	39	17	7	0
2000	145	44	33	13	0
2001	290	133	35	26	0
2002	354	121	20	0	46
2003	131	49	4	2	5

Panel (B) : Fraction with downward slopes					
year	Ba	B	Caa	Ca	C
1995	47%	67%	-	-	-
1996	51%	75%	100%	-	-
1997	53%	75%	100%	-	-
1998	52%	100%	100%	-	-
1999	60%	74%	88%	57%	-
2000	48%	57%	64%	54%	-
2001	64%	65%	80%	62%	-
2002	82%	76%	70%	-	63%
2003	79%	86%	100%	0%	100%

Table 4: Slopes grouped by bond prices

This table shows the number of bond pairs and the fraction of pairs with downward-sloping term structures of spreads, analyzed by rating category and bond price. Each bond pair consists of two bond prices (and spreads) with different tenors but identical issuer, credit rating, seniority and pricing date. To ensure that we do not pair up bonds with very different prices, we impose the condition that the two bond prices in a pair should not differ by more than \$10.

Panel (A): Number of Bond Pairs					
price bucket	Ba	B	Caa	Ca	C
20 to 70	25	50	94	39	13
70 to 80	56	42	18	6	–
80 to 90	147	80	28	9	6
90 to 100	547	277	29	16	6
100 to 110	566	195	13	–	6
110+	17	8	–	–	–
Panel(B): Percentage of Pairs with downward-sloping term structure					
20 to 70	80%	86%	87%	56%	92%
70 to 80	80%	93%	72%	67%	–
80 to 90	71%	81%	61%	78%	67%
90 to 100	67%	64%	52%	44%	100%
100 to 110	68%	62%	54%	–	50%
110+	41%	88%	–	–	–

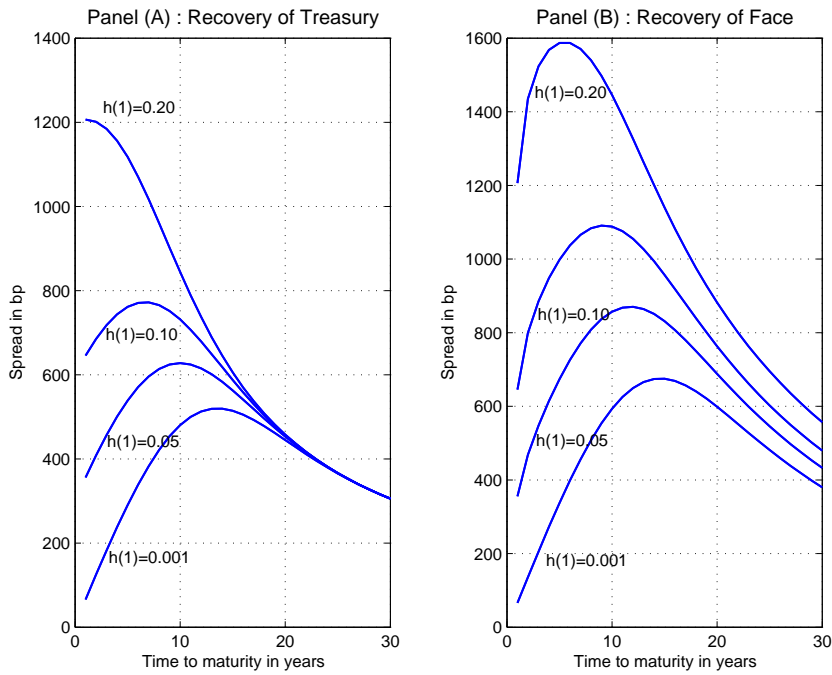


Figure 1: Term Structure of zero-coupon Spreads

This figure illustrates the term structure of zero-coupon spreads for an upward sloping hazard rate function  $h_t = h_1 + h_2 t$ , with  $h_2 = 0.02$ . Panel (A) assumes Recovery of Treasury (RT), while Panel (B) assumes Recovery of Face (RF) with a linear accrual of the claim amount. Different curves on each panel correspond to different levels of hazard rate (i.e. different values of  $h_1$ ), each time keeping the slope (i.e. value of  $h_2$ ) the same.

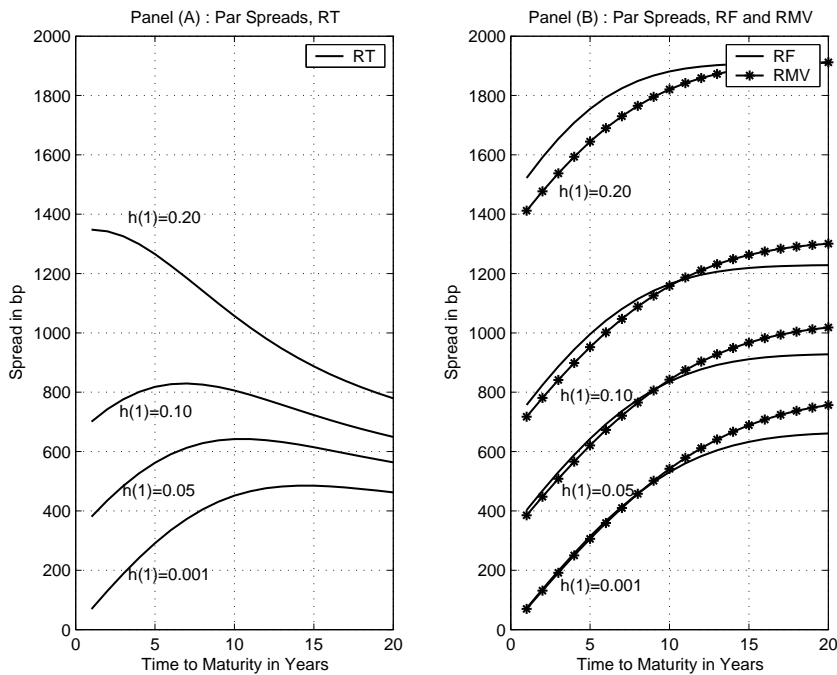


Figure 2: Term Structure of Par Spreads

This figure illustrates the term structure of par spreads for an upward sloping hazard rate function  $h_t = h_1 + h_2 t$  with  $h_2 = 0.02$ . Panel (A) shows the term structures under Recovery of Treasury (RT) assumption while Panel (B) shows the spread term structures under Recovery of Face (RF) and Recovery of Market Value (RMV) assumptions. Different curves on each panel correspond to different levels of hazard rate (i.e. different values of  $h_1$ ), each time keeping the slope (i.e. value of  $h_2$ ) the same.

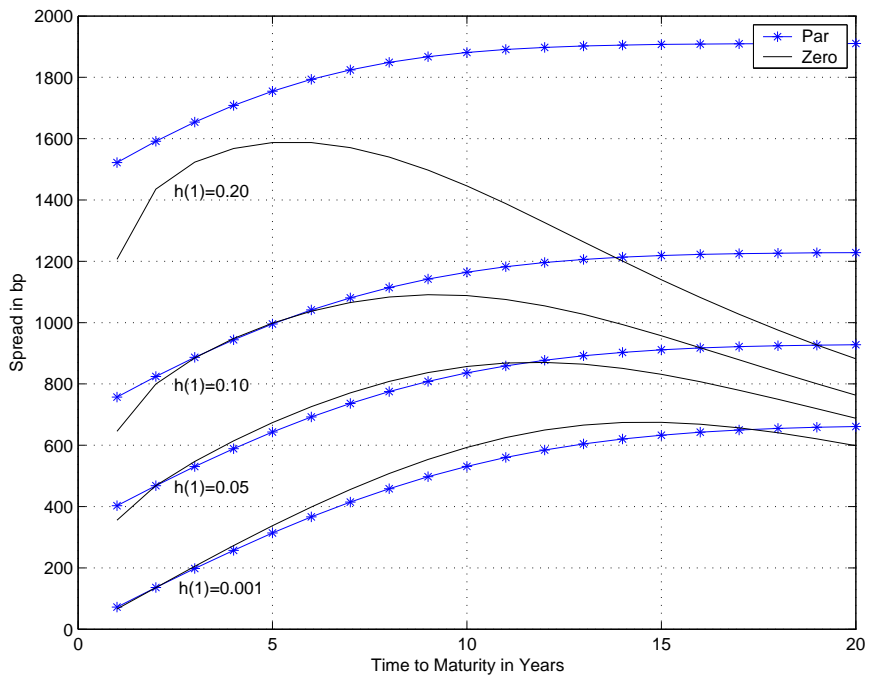


Figure 3: zero-coupon Spread and Par Spread term structures

This figure compares the zero-coupon spreads with par spreads under the RF assumption. All the curves assume an upward sloping hazard rate function  $h_t = h_1 + h_2 t$  with  $h_2 = 0.02$ . Different curves correspond to different levels of hazard rates (i.e. different values of  $h_1$ ).

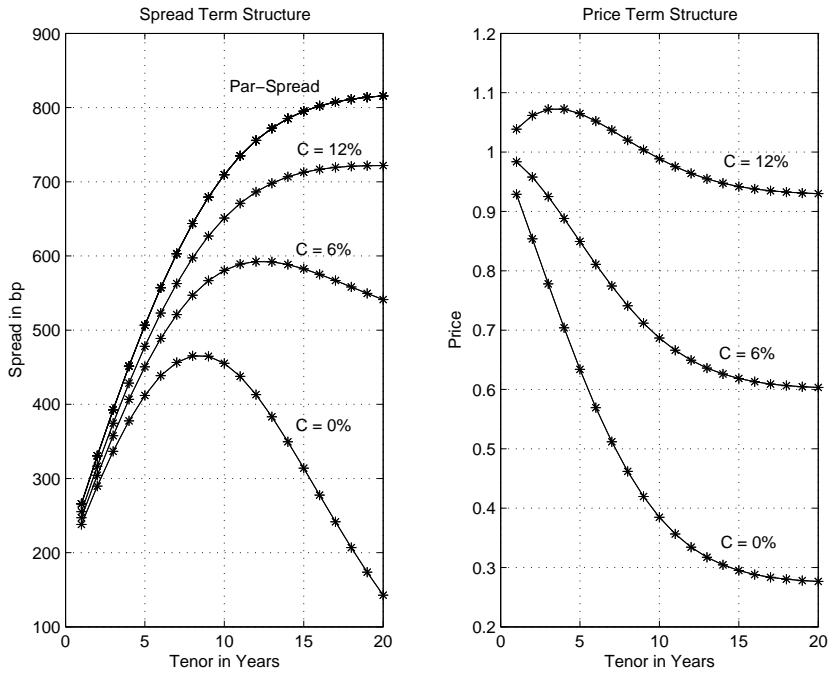


Figure 4: Term structures of zero-coupon spread, coupon spread and par spread  
 Panel (A) shows the variation in spread term structure with the coupon rate. All the spread curves are generated by assuming an upward-sloping hazard function  $h_t = h_1 + h_2t$ , with  $h_1 = 0.03$  and  $h_2 = 0.02$ . The coupons rates assumed are shown. The curves assume RF with fractional LGD of 0.6. Panel (B) shows the corresponding price term structure.

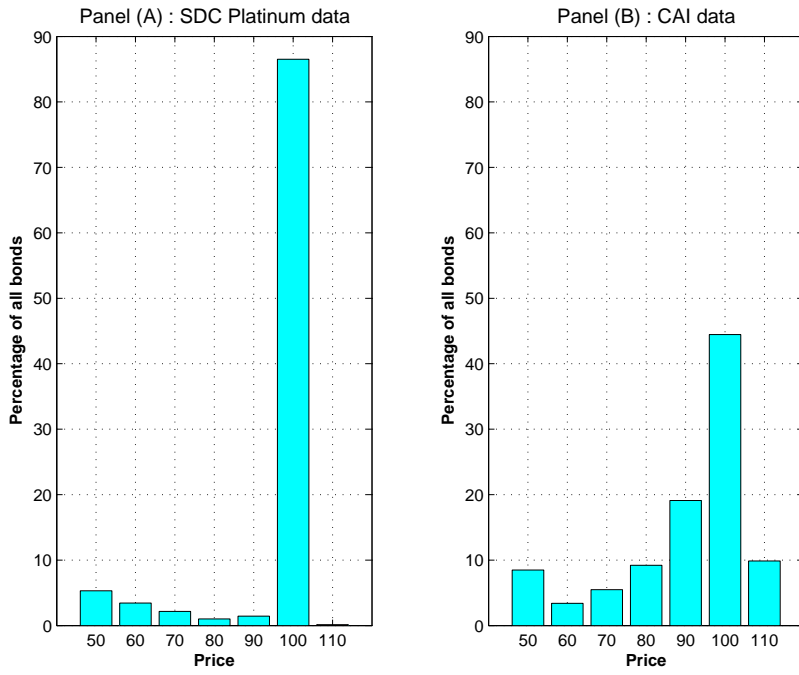


Figure 5: Distribution of Speculative Grade Bond Prices

This figure shows the distribution of speculative grade bond prices in the SDC Platinum new issues database (Panel (A)) and Capital Access International (CAI) secondary market database (Panel (B)). SDC Platinum database is used in HT99 study, thus the above price distribution shows that their bonds are mostly par bonds. CAI data is used in producing our Tables I and II. The price distribution shows that a significant fraction of our bonds trade below their par values.

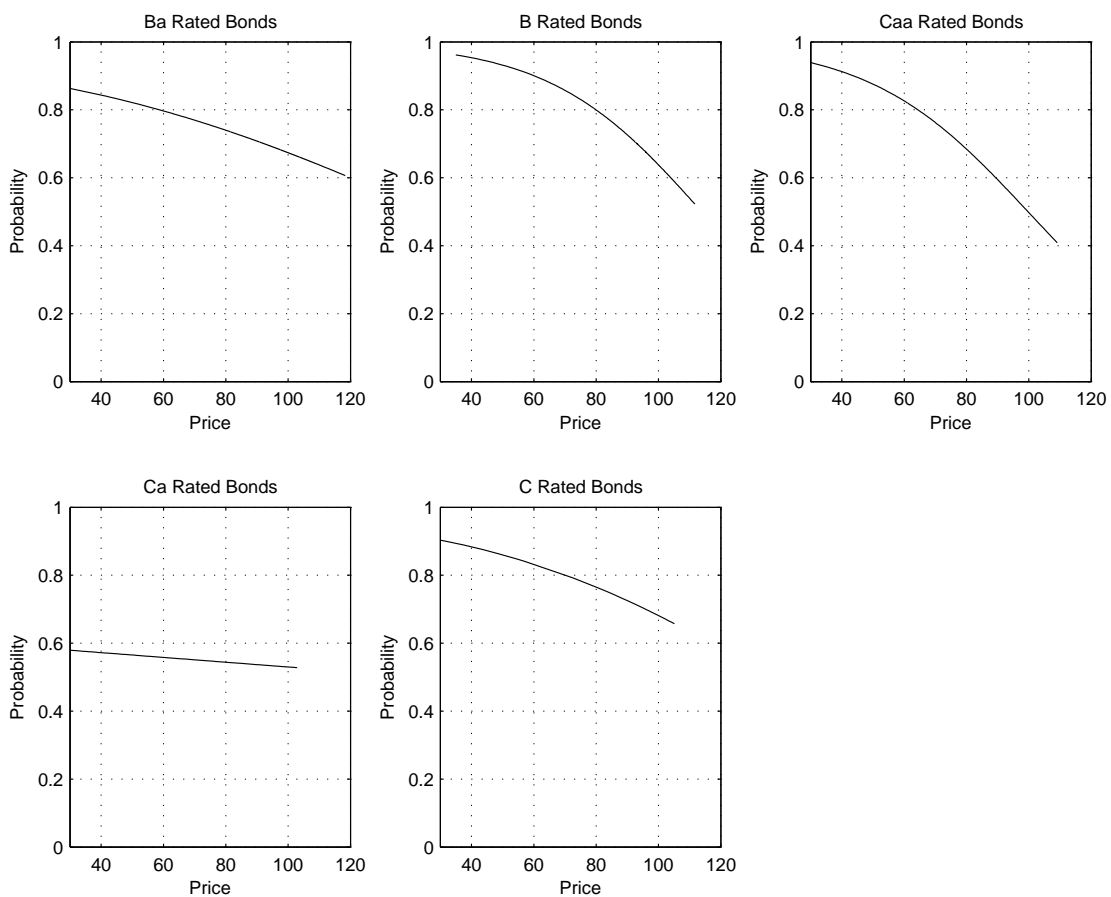


Figure 6: Estimated Probabilities of downward-sloping term structure

This figure shows the estimated probability of a downward-sloping term structure of credit spreads as a function of bond price for various speculative grade rating classes. The slopes are computed from bond pairs containing bonds with identical issuer, rating and seniority but different tenors. For each bond pair, we impose the additional condition that the prices of the two bonds in the pair should be within ten dollars of each other. The mean price of the two bonds is called the “group price”. The probabilities shown are estimated by running a logit model in each rating class, where dependent variable is the slope dummy and independent variable is the group price.

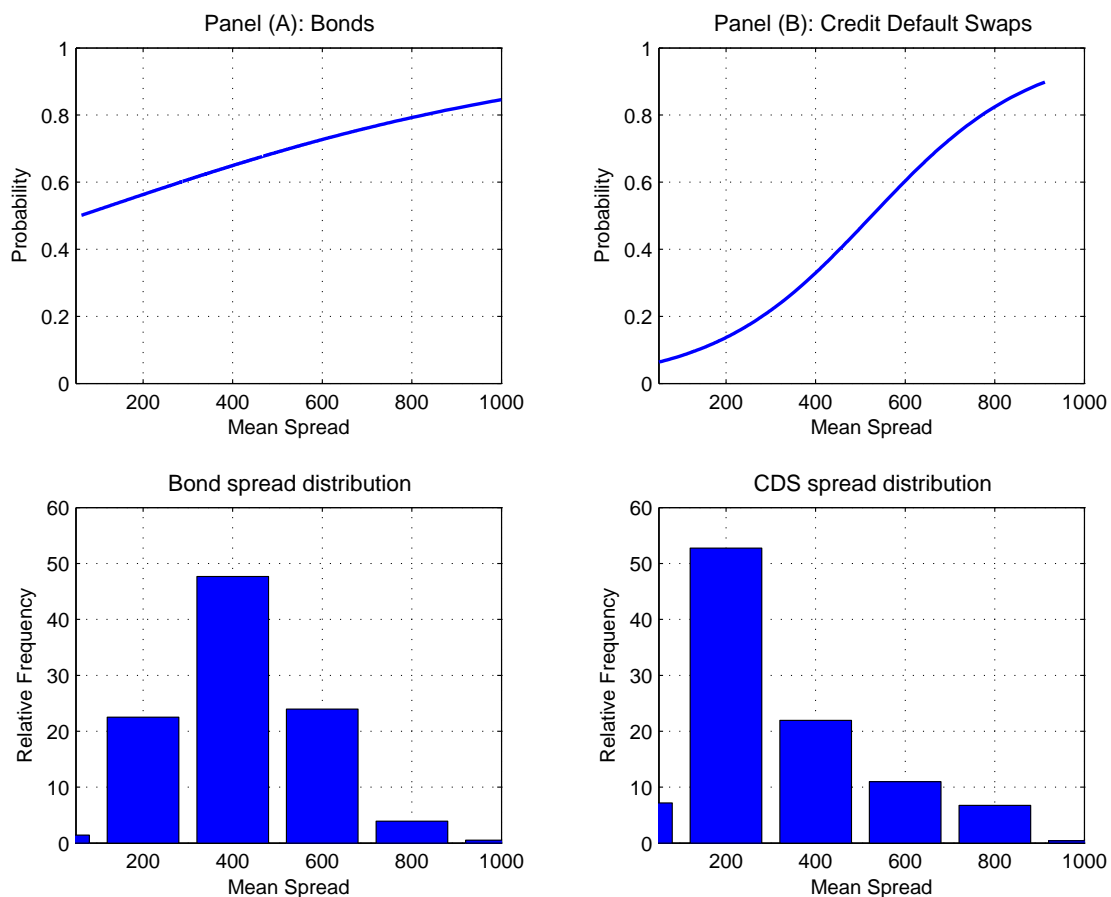


Figure 7: Variation in Slope of Par Curve with Credit Quality

The top panels show the estimated probability of a downward-sloping term structure of par credit spreads as a function of credit quality (measured by the mean spread of a pair of bonds or Credit Default Swaps) for speculative grade credits. The slopes are computed from bond (CDS) pairs with identical issuer (reference entity), rating and seniority but different tenors. For each bond pair, we impose the additional condition that the prices of the two bonds in the pair should be within ten dollars of each other and the average price of the pair should be between 95 and 105 dollars. The probabilities shown are estimated by running a logit model where dependent variable is the slope dummy and independent variable is the mean spread of the pair. The bottom panel shows the distributions of the mean spreads for bonds and CDS pairs used in this analysis.

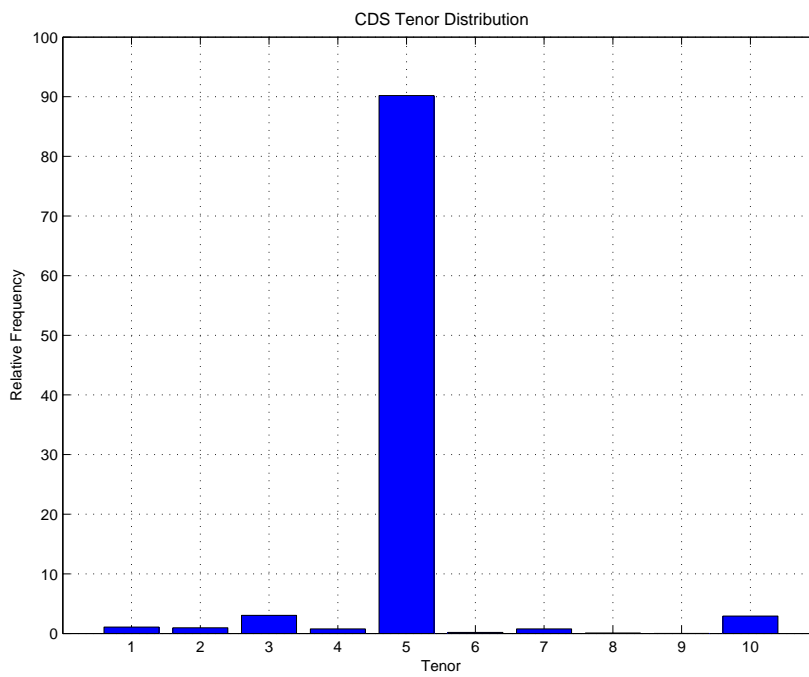


Figure 8: Distribution of tenors in GFINet CDS data

This figure shows the distribution of tenors in CDS price data obtained from GFINet, one of the biggest broker-dealers in the CDS market. The figure shows that about 90% of the activity is concentrated at 5 year tenor. CDS on Sovereign credits are excluded.