How Profitable Is Capital Structure Arbitrage?

Fan Yu
University of California, Irvine

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Abstract

This paper examines the risk and return of the so-called “capital structure arbitrage,” which exploits the mispricing between a company’s debt and equity. Specifically, a structural model connects a company’s equity price with its credit default swap (CDS) spread. Based on the deviation of CDS market spreads from their theoretical counterparts, a convergence-type trading strategy is proposed and analyzed using 135,759 daily CDS spreads on 261 obligors. At the level of individual trades, the risk of the strategy arises when the arbitrageur shorts CDS and the market spread subsequently skyrockets, forcing the arbitrageur into early liquidation and engendering large losses. An equally-weighted portfolio of all trades produces Sharpe ratios similar to those of other fixed-income arbitrage strategies and hedge fund industry benchmarks. However, the monthly excess returns on this portfolio are not significantly correlated with either equity or bond market factors.
1 Introduction

Capital structure arbitrage has lately become popular among hedge funds and bank proprietary trading desks. Some traders have even touted it as the “next big thing” or “the hottest strategy” in the arbitrage community. A recent Euromoney report by Currie and Morris (2002) contains the following vivid account:

“In early November credit protection on building materials group Hanson was trading at 95bp—while some traders’ debt equity models said the correct valuation was 160bp. Its share held steady. That was the trigger that capital structure arbitrageurs were waiting for. One trader who talked to Euromoney bought €10 million-worth of Hansen’s five-year credit default swap over the course of November 5 and 6 when they were at 95bp. At the same time, using an equity delta of 12% derived from a proprietary debt equity model, he bought €1.2 million-worth of stock at £2.91 (€4.40). Twelves days later it was all over. On November 18, with Hanson’s default spreads at 140bp and the share price at £2.95, the trader sold both positions. Unusually, both sides of the trade were profitable. Sale of credit default swaps returned €195,000. Selling the shares raised €16,000, for a total gain of €211,000.”

In essence, the capital structure arbitrageur uses a structural model to gauge the richness and cheapness of the CDS spread. The model, typically a variant of Merton (1974), predicts spreads based on a company’s liability structure and its market value of equities. When the arbitrageur finds that the market spread is substantially larger than the predicted spread, a number of possibilities can be entertained. He might think that the equity market is more objective in its assessment of the price of credit protection, and the CDS market is instead gripped by fear. Alternatively, he might think that the market spread is “right” and the equity market is slow to react to relevant information. If the first view is correct, the arbitrageur is justified in selling credit protection. If the second view is correct, he should sell equity. In practice, the arbitrageur is probably unsure, so that he does both and profits if the market spread and the model spread converge to each other. The size of the equity position relative to the CDS notional amount is determined by delta-hedging. The logic is that if the CDS spread widens or if the equity price

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1A popular choice among traders appears to be the CreditGrades model. See the CreditGrades Technical Document (2002) for details.
rises, the best one could hope for is that the theoretical relation between the CDS spread and the equity price would prevail, and the equity position can cushion the loss on the CDS position, and vice versa.

In the Hanson episode, the CDS and the equity market did come to the same view regarding Hanson’s default risk. In this case, the market CDS spread increased sharply while the equity-based theoretical spread decreased slightly, realigning themselves. However, is convergence the norm when the strategy is implemented in a large cross-section? In Currie and Morris (2002), traders are quoted saying that the average correlation between the CDS spread and the equity price is only on the order of 5% to 15%. The lack of a close correlation between the two variables suggests that there can be prolonged periods when the two markets hold diverging views on an obligor. If the strategy does not necessarily converge and the equity hedge works poorly, then the strategy can experience large losses when marked to market, triggering margin calls and forcing an early liquidation of the positions.

Imperfections in implementation can also reduce the profitability of capital structure arbitrage. For example, the CDS market spread could be higher than the equity-based model spread because of a sudden increase in asset volatility, newly issued debt, or hidden liabilities that have come to light. These elements are omitted in a simple implementation of structural models using historical volatility and balance sheet information from Compustat quarterly data. In other words, one might enter into a trade when there are, in fact, no profitable opportunities.

Because of a complete lack of evidence in favor of or against this strategy, in this paper I conduct an empirical analysis of the risk and return of capital structure arbitrage as commonly implemented by traders. In asking this question, it is expected that such a strategy is nowhere close to what one calls arbitrage in the textbook sense. Indeed, convergence may never occur during reasonable holding periods, and the CDS spread and the equity price may seem as likely to move in the same direction as in the opposite direction—all the more reasons to worry whether the traders’ enthusiasm is justified. If positive expected returns are found, the next step would be to understand the sources of the profits, i.e., whether the returns are correlated with priced systematic risk factors. From a trading perspective, one may also check if the strategy survives trading costs or constitutes “statistical arbitrage”—one that does not yield sure profit but will do so in the long run.² If the strategy does not yield positive expected returns, one needs to look more carefully, perhaps at refined trading strategies that are more attuned to the

²See Hogan, Jarrow, Teo, and Warachka (2004) for details.
nuances of the markets on a case-by-case basis.

This paper is based on the premise that structural models can price CDS reasonably well. This assumption itself has been the subject of ongoing research. The initial tests of structural models, such as Jones, Mason, and Rosenfeld (1984) and Eom, Helwege, and Huang (2004), focus on corporate bond pricing, and find that structural models generally underpredict spreads. It is now understood that the failure of the models has more to do with corporate bond market peculiarities such as liquidity and tax effects than their ability to explain credit risk. Recent applications to the CDS market has turned out to be more encouraging. For example, Ericsson, Reneby, and Wang (2004) find that several popular structural models fit CDS spreads much better than they do bond yield spreads, and one of the models yields almost unbiased CDS spread predictions. Moreover, industry implementations such as CreditGrades introduce recovery rate uncertainty, which further boosts the predicted spreads. Hence spread underprediction is unlikely to be a persistent problem in the CDS market.

This paper is also related to Duarte, Longstaff, and Yu (2005), who broadly examine the risk and return of popular fixed income arbitrage strategies employed by hedge funds, including swap spread arbitrage, yield curve arbitrage, mortgage arbitrage, volatility arbitrage, and capital structure arbitrage. In contrast, this paper focuses exclusively on capital structure arbitrage and presents a much more detailed analysis at the level of individual trades. Longstaff, Mithal, and Neis (2003) use a vector-autoregression to examine the lead-lag relationship among bond, equity, and CDS markets. They find that equity and CDS markets contain distinct information which can help predict corporate bond yield spread changes. While there is no definitive lead-lag relationship between equity and CDS markets, a convergence-type trading strategy such as capital structure arbitrage can conceivably take advantage of the distinct information content of the two markets.

In practice, capital structure arbitrageurs often place bets on risky bonds in lieu of CDS. They also trade bonds against CDS to take advantage of the misalignment between bond spreads and CDS spreads. However, this paper does not consider bond-equity or bond-CDS trading strategies for the following reasons. First, there is the pragmatic concern for the lack of high frequency data on

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3See Elton, Gruber, Agrawal, and Mann (2001), Ericsson and Reneby (2003), and Longstaff, Mithal, and Neis (2004).

4Zhu (2004) finds the CDS market to lead the bond market in price discovery. Berndt and de Melo (2003) find that the equity price and the option-implied volatility skew Granger-cause CDS spreads, although their analysis is based on just one company, France Telecom.
corporate bonds. Second, CDS is becoming the preferred vehicle in debt-equity trades due to its higher liquidity. Third, there is a simple relationship between CDS and bond spreads, which is supported both in theory and by the data. For example, using arbitrage-based arguments Duffie (1999) shows that CDS spreads are theoretically equivalent to the yield spreads on floating-rate notes. Hull, Predescu, and White (2003), Longstaff, Mithal, and Neis (2003), Houweling and Vorst (2003), and Blanco, Brennan, and Marsh (2003) find a close resemblance between CDS spreads and fixed-rate bond spreads, provided that one selects a proper “risk-free” reference interest rate. Finally, several existing studies already explore the relationship between equity and corporate bond prices, such as Schaefer and Strebulaev (2004) and Chatiras and Mukherjee (2004). In contrast, the model-based relationship between the equity price and the CDS spread is much less well understood.

The rest of the paper is organized as follows. Section 2 dissects the trading strategy. Section 3 outlines the data used in the analysis. Section 4 uses a case study to illustrate the general approach. Section 5 summarizes holding period returns across individual trades. Section 6 constructs a capital structure arbitrage return index from the individual trades and studies the monthly returns. Section 7 contains additional robustness checks. Section 8 concludes.

2 Anatomy of the Trading Strategy

This section dissects the anatomy of capital structure arbitrage. Since the strategy is model-based, I start with an introduction to CDS pricing, and then explore issues of implementation with the help of the analytical framework.

2.1 CDS Pricing

A credit default swap (CDS) is an insurance contract against credit events such as the default on a bond by a specific issuer (the obligor). The buyer pays a premium to the seller once a quarter until the maturity of the contract or the credit event, whichever occurs first. The seller is obligated to take delivery of the underlying bond from the buyer for face value should a credit event take place within the contract maturity. While this is the essence of the contract, there are also a number of practical issues. For example, if the credit event occurs between two payment dates, the buyer owes the seller the premiums that have accrued since the last settlement date. Another complicating issue is that the buyer usually has the
option to substitute the underlying bond with other debt instruments of the obligor of equal priority. This means that the CDS spread has to account for the value of a cheapest-to-deliver option. This option can become particularly valuable when the definition of the credit event includes restructuring, which can cause the deliverable assets to diverge in value. For a simple treatment of CDS pricing, I assume continuous premium payments and ignore the embedded option.\(^5\)

First, the present value of the premium payments is equal to

\[
E \left( c \int_0^T \exp \left( - \int_0^s r_u du \right) 1_{\{\tau > s\}} ds \right),
\]

where \(c\) denotes the CDS spread, \(T\) the CDS contract maturity, \(r\) the risk-free interest rate, and \(\tau\) the default time of the obligor. Assuming independence between the default time and the risk-free interest rate, this can be written as

\[
c \int_0^T P(0, s) q_0(s) ds,
\]

where \(P(0, s)\) is the price of a default-free zero-coupon bond with maturity \(s\), and \(q_0(s)\) is the risk-neutral survival probability of the obligor, \(\Pr(\tau > s)\), at \(t = 0\).\(^6\)

Second, the present value of the credit protection is equal to

\[
E \left( (1 - R) \exp \left( - \int_0^\tau r_u du \right) 1_{\{\tau < T\}} \right),
\]

where \(R\) measures the recovery of bond market value as a percentage of par in the event of default. Assuming a constant \(R\) and maintaining the assumption of independence between default and the risk-free interest rate, this can be written as

\[
-(1 - R) \int_0^T P(0, s) q_0'(s) ds,
\]

where \(-q_0'(t) = -dq_0(t)/dt\) is the probability density function of the default time. The CDS spread is then determined by setting the initial value of the contract

\(^5\)Duffie and Singleton (2003) show that the effect of accrued premiums on the CDS spread is typically small. For detailed discussions on restructuring as a credit event and the cheapest-to-deliver option, see Blanco, Brennan, and Marsh (2003) and Berndt et al. (2004).

\(^6\)Assuming independence between default and the default-free term structure enables one to concentrate on the relationship between the equity price and the CDS spread. The further specialization to constant interest rates below is in the same vein.
to zero:

\[ c = -\frac{(1 - R) \int_{s}^{T} P(0, s) q'_{0}(s) \, ds}{\int_{0}^{T} P(0, s) q_{0}(s) \, ds}. \]  

(5)

The preceding derives the CDS spread on a newly minted contract. If it is subsequently held, the relevant issue is the value of the contract as market conditions change. To someone who holds a long position from time 0 to \( t \), this is equal to

\[ \pi(t, T) = (c(t, T) - c(0, T)) \int_{t}^{T} P(t, s) q_{t}(s) \, ds, \]  

(6)

where \( c(t, T) \) is the CDS spread on a contract initiated at \( t \) and with maturity date \( T \), and \( q_{t}(s) \) is the probability of survival through \( s \) at time \( t \).

To compute the risk-neutral survival probability, I use the structural approach, which assumes that default occurs when the firm’s asset level drops below a certain “default threshold.” Since the equity is treated as the residual claim on the assets of the firm, the structural model can be estimated by fitting equity prices and equity volatilities. Because of the focus on trading strategies, the choice of a particular structural model ought to be less crucial. One model might produce slightly more unbiased spreads than another, but it is the daily change in the spread that is of principal concern here. At this frequency, the only contributor to spread changes is the equity price; other inputs and parameters, which can vary from one model to the next, are essentially fixed. In other words, all structural models can be thought of as merely different nonlinear transformations of the same equity price. Of course, different models will produce different hedge ratios that can impact trading profits. However, Schaefer and Strebulaev (2004) show that even a simple model such as Merton’s (1974) produces hedge ratios for corporate bonds that cannot be rejected in empirical tests.

Consequently, I use the CreditGrades model to implement the trading analysis. This model is jointly developed by RiskMetrics, JP Morgan, Goldman Sachs, and

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7The alternative is the reduced-form approach, which computes the survival probability based on a hazard rate function typically estimated from bond prices. Blanco, Brennan, and Marsh (2003) and Longstaff, Mithal, and Neis (2003) are applications of this approach.

8For example, the Merton (1974) model can be estimated by inverting the equations for equity price and equity volatility for asset level and asset volatility. KMV estimates its model for the expected default frequency using a recursive procedure that involves only the expression for equity value [see Crosbie and Bohn (2002) and Vassalou and Xing (2003)]. The structural model can also be estimated using maximum likelihood applied to the time-series of equity prices [see Ericsson and Reneby (2004)].
Deutsche Bank. It is based on the model of Black and Cox (1976), and contains the additional element of uncertain recovery. This latter feature helps to increase the short-term default probability, which is needed to produce realistic levels of CDS spreads. On the practical side, the model provides closed-form solutions to the survival probability and the CDS spread. It is also reputed as the model used by most capital structure arbitrage professionals. For completeness, the appendix gives an overview of CreditGrades, including the formulas for $q_0(t)$, $c(0, T)$, the contract value $\pi(t, T)$, and the equity delta, defined as

$$\delta(t, T) = \frac{\partial \pi(t, T)}{\partial S_t},$$

where $S_t$ denotes the equity price at $t$.

### 2.2 Implementation

I now turn to the implementation of the trading strategy. Assume that one has available a time-series of observed CDS market spreads $c_t = c(t, t + T)$ on freshly-issued, fixed-maturity contracts. Also available is a time-series of observed equity prices $S_t$, along with information about the capital structure of the obligor. This latter information set allows the trader to calculate a theoretical CDS spread based on his structural model. Denote this prediction $c'_t$ and the difference between the two time-series $e_t = c_t - c'_t$.

If the focus is on pricing, then typically one would like to attain the best possible fit to market data according to some pre-specified metric, for example by minimizing the sum of $e_t^2$ over time. While this would be akin to inverting implied volatility from the Black-Scholes model for equity options, that is not how the model is used in this context. A simple model such as CreditGrades, whose only inputs are the equity price, the historical equity volatility, the debt per share, the interest rate, the recovery rate, and its standard deviation has very little flexibility in fitting the time-series of market spreads precisely. What motivates capital structure arbitrage trading is, instead, the trader’s belief that $e_t$ will move predictably over time.

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9The same effect can be achieved by assuming imperfectly observed asset value, or coarsened information set observed by outside investors compared to managers. See Duffie and Lando (2001), Cetin et al. (2003), and Collin-Dufresne, Goldstein, and Helwege (2003).

10This assumption is consistent with the observation that CDS market quotes are predominantly for newly initiated five-year contracts. Secondary market trading also appears in the data, although only very recently.
Specifically, suppose that the arbitrageur examines the behavior of $e_t$ and finds that it has a mean of $E(e)$ and a standard deviation of $\sigma(e)$. As mentioned before, because of the way the model is implemented, one would not expect the pricing error to be unbiased. However, say that there comes a point at which the deviation becomes unusually large, for example when $e_t > E(e) + 2\sigma(e)$. At this point the arbitrageur sees an opportunity. If he considers the CDS over-priced, then he should sell credit protection. On the other hand, if he considers the equity to be over-priced, then he should sell the equity short. Either way, he is counting on the “normal” relationship between the market spread and the equity-implied spread to re-assert itself. In other words, the trading strategy is based on the assumption that “convergence” will occur.

To see this logic more clearly, assume that the theoretical pricing relation is given by $c_t^{\prime} = f(S_t, \sigma; \theta)$ where $S_t$ is the equity price, $\sigma$ is an estimate of asset volatility, and $\theta$ denotes the other fixed parameters of the model such as the recovery rate. The actual CDS spread is given by $c_t = f(S_t, \sigma^{imp}; \theta)$ where $\sigma^{imp}$ is the implied asset volatility obtained by inverting the pricing equation. When $c_t > c_t^{\prime}$, it may be that the implied volatility $\sigma^{imp}$ is too high and shall decline to a lower level $\sigma$. The correct strategy in this case is to sell CDS and sell equity as a hedge, which is akin to selling over-priced stock options and using delta-hedging to neutralize the effect of a changing stock price. Another possibility is that the CDS is priced fairly, but the equity price reacts too slowly to new information. In this case the equity is over-priced and one should short CDS as a hedge against shorting equity. Both cases therefore give rise to the same trading strategy. A third possibility is that the volatility estimator can underestimate the true asset volatility, sending a false signal of mispricing in the market. Yet a fourth possibility is that other parameters of the model, such as the debt per share, are mis-measured. This can be a problem when using balance sheet variables from financial reports, which are infrequently updated. Finally, the gap between $c_t$ and $c_t^{\prime}$ could simply be due to model misspecification. It is possible to address the last three scenarios, for example, by calibrating the model with option-implied volatility, carefully monitoring the changes in a firm’s capital structure, and simply trying alternative models. As a first attempt to understand capital structure arbitrage trading, I leave these potential improvements to future research.

While delta-hedging is typically invoked in this context, there are differences from the usual practice in trading equity options where one bets on volatility and uses hedging to neutralize the effect of equity price changes. From what traders describe in media accounts, the equity hedge is often “static,” staying unchanged through the duration of the strategy. Moreover, traders often modify the model-
based hedge ratio according to their own opinion of the particular type of convergence that is likely to occur. In the above example, the trader may decide to underhedge if he feels confident about the CDS spread falling, or he may overhedge if he feels confident about the equity price falling. Some traders do not appear to use a model-based hedge ratio at all. Instead, they identify the maximum loss that can occur should the obligor default, and shorts an equity position in order to break even.\textsuperscript{11} When conducting the trading exercise, I set the hedge ratio to correspond to the model CDS spread $c' = f(S_t, \sigma; \theta)$ when entering the trade and fix this hedge ratio throughout the trade. Other hedging schemes are considered in Section 7.

Continuing the description of the trading strategy, now that the arbitrageur has entered the market, he also needs to know when to liquidate his positions. I assume that exit will occur under the following scenarios:

1. The pricing error $e_t$ reverts to its mean value $E(e)$.

2. Convergence has not occurred by the end of a pre-specified holding period or the sample period.

In principle, during the holding period the obligor can also default or be acquired by another company. However, most likely the CDS market will reflect these events long before the actual occurrences and the arbitrageur will have ample time to make exit decisions.

To summarize, the risk involved in capital structure arbitrage can be understood in terms of the subsequent movements of the CDS spread and the equity price. For example, after the arbitrageur has sold credit protection and sold equity short, four likely scenarios can happen:

1. $c_t \downarrow, S_t \downarrow$. This is the case of convergence, allowing the arbitrageur to profit from both positions.

2. $c_t \downarrow, S_t \uparrow$. The arbitrageur loses on the equity but profits from the CDS. He will profit overall if the CDS spread falls more rapidly than the equity price rises, allowing convergence to take place partially.

\textsuperscript{11}In the article by Currie and Morris (2002), Boaz Weinstein of Deutsche Bank bought $10 million-worth of Household Finance bonds at 75 cents on the dollar. He estimated the debt recovery to be 45 cents. Against this potential loss of $3 million, he shorted $2.5 million-worth of equity as a partial hedge.
3. $c_t \uparrow, S_t \downarrow$. The arbitrageur loses on his CDS bet, but the equity position acts as a hedge against this loss. There will be overall profit if the equity price falls more rapidly than the CDS spread rises.

4. $c_t \uparrow, S_t \uparrow$. This is a sure case of divergence. The arbitrageur suffers losses from both positions regardless of the size of the equity hedge.

Clearly, delta-hedging is effective in the second and the third scenarios. The likelihood of the first scenario, however, is critical to the success of capital structure arbitrage.

2.3 Trading Returns

An integral part of this paper is the analysis of trading returns. At the initiation of the strategy, the CDS position has zero market value. Therefore, trading returns must be calculated by assuming that the arbitrageur has a certain level of initial capital.\footnote{I assume that the arbitrageur has a certain amount of initial capital deposited into a margin account. This account is used to finance the initial equity hedge. Subsequent to entering into a trade, any intermediate cash flows from the stock or the CDS positions, such as dividends or CDS premiums, are credited to or deducted from this margin account.} This is a non-trivial parameter of the trading exercise because some of the trades may have to be liquidated early due to large drawdowns. I will experiment with this parameter in the trading exercise.

Through the holding period, the value of the CDS and the equity positions can change. While the latter is trivial, the value of the CDS position has to be calculated according to Eq. (6). Several simplifying assumptions have to be made when implementing this procedure. First, Eq. (6) requires secondary market quotes on an existing contract, while the CDS market predominantly quotes spreads on freshly-issued contracts with a fixed maturity, say 60 months. I circumvent this problem by approximating $c(t, T)$ with $c(t, t+\bar{T})$. This is because the difference between two points on the term structure of CDS spreads six months apart in maturities is likely to be much smaller than the time variation in CDS spreads over six months. Second, the value of the CDS is essentially that of a survival-contingent annuity whose value depends on the term structure of survival probabilities. Consistent with using a hedge ratio that corresponds to the model spread, I compute the survival probabilities by plugging the historical equity volatility into the CG model.

In practice, there was very little secondary trading during the earlier phase of the CDS market. In more recent periods the market is moving toward quoting
spreads on contracts with fixed maturity dates, similar to those in the equity options market, which facilitates secondary market trading. Still, my CDS database does not provide any information on the secondary market value of the contracts. What often occurs in practice is that once the trader buys a CDS from a dealer and then the spread rises, he can try to sell it back to the same dealer. The price for taking the position off his hands is subject to negotiation. If the trader finds the dealer’s offer unacceptable, he can choose to enter an offsetting trade and receive the cash flow of the survival-contingent annuity. He might also receive offers on the existing CDS position from active secondary market participants. In any case, all parties involved are likely to judge the value of the contract based on their own proprietary models. This is why I choose to use the survival probabilities from the CreditGrades model.

2.4 Trading Costs

Except in Section 7, where I explore the impact of transaction costs on trading returns, I assume a 5% bid/ask spread for trading CDS. This implies that for an investment-grade obligor with a spread of 100bp, the buyer of the CDS will be paying 102.5bp per annum while the seller will receive only 97.5bp. Note that a proportional transaction cost is assumed, so that a speculative-grade obligor with a spread of 1,000bp will have a bid/ask spread of 50bp.

Without actual data on bid/ask spreads, this level is assumed for all obligors at all times. Clearly, the level of CDS transaction costs was higher during the earlier phase of the market. But 5% appears to be a reasonable estimate in recent periods.

In addition, since the CDS bid/ask spread is probably the single largest source of trading costs for capital structure arbitrage, I ignore the transaction costs on common stocks. This assumption appears harmless, especially in light of the use of static hedging in the trading strategy.
3 Data

The CDS dataset used in this study is provided by the Markit Group. Specifically, I use daily composite spreads on five-year CDS contracts on senior unsecured debt of North American obligors from 2001 to 2004, with a modified restructuring (MR) clause and currencies denominated in US dollars. I also apply several filters to the data to generate a subset that is suitable for the trading analysis.

First, I merge the CDS data with information on the equity price from CRSP daily stock files and the balance sheet from Compustat quarterly files. The quarterly balance sheet variables are lagged for one month from the end of the quarter to ensure no look-ahead bias. I also obtain five-year constant maturity Treasury yields and three-month Treasury bill rates from Datastream. These additional variables are used to implement a simplified version of the CreditGrades model.

Second, in order to conduct the trading exercise, a reasonably continuous sequence of spreads must be available. For each obligor, I search for the longest string of more than 252 daily spreads that are no more than 14 calendar days apart, which also have available the associated equity and balance sheet information. Presumably, this procedure yields the most liquid portion of coverage for the obligors. It also reflects the reasonable assumption that the arbitrageur will be forced to close out his positions once the liquidity in the obligor vanishes, be it CDS or equity trading.

Applying these two steps to the original dataset yields 135,759 daily spreads on 261 obligors. Figure 1 plots the number of obligors for each month from 2001 to 2003. The steadily increasing coverage is a reflection of the overall growth in the CDS market over the last decade.

Table 1 presents the summary statistics for the 261 obligors. The variables

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13This dataset has also been used by Zhang, Zhou, and Zhu (2004). Briefly, Markit Group takes more than a million CDS quotes each day from over 30 contributing banks. It applies a rigorous screening procedure to eliminate flat curves and stale prices as well as outliers. It computes a daily composite spread only if there are more than three contributors. Also, once Markit starts pricing a credit, more than 75% of the time it will have pricing for the entity on a continuous basis going forward more or less daily. This last feature makes the Markit database ideal for time-series analysis.

14Obviously, no adjustment to the calculation of the model spread is needed for cash dividends or stock splits. In cases of stock dividends or spinoffs, I adjust the balance sheet variables so that the model spread stays unchanged through the event, until they are updated the following quarter.

15The coverage actually spans 2001 to 2004. The number of obligors starts to decline at the beginning of 2004 purely as a result of the above screening procedure for continuous coverage.
presented are average values over time and across obligors in a given credit rating or industry. \( N \) is the number of obligors, SPD is the daily CDS spread in basis points, VOL is the 1,000-day historical equity volatility, SIZE is the equity market capitalization in millions of dollars, LEV is the leverage ratio defined as total liabilities divided by the sum of total liabilities and the equity market capitalization, and CORR is the correlation between daily changes in the CDS spread and the equity price. Overall, the 261 obligors are evenly distributed across the designated industries, and over 80% are rated investment-grade. I exclude financial and utility obligors as their capital structure is more difficult to interpret.

The Markit database provides an average credit rating through the sample period for each obligor. Therefore, it is not possible to classify obligors according to their credit rating at the beginning of the sample period. Nevertheless, segregating the obligors by the provided credit rating allows one to see the impact of structural variables on the pricing of credit risk. For example, there is clearly a positive correlation between the spread and the leverage and volatility variables, which is consistent with the predictions of structural credit risk models. There is also a clear relation between firm size and the leverage and volatility measures.

The most interesting observation from Table 1 is perhaps the pattern of correlations between changes in the CDS spread and the equity price (CORR). First,
the size of the correlation is consistent with the 5%-15% range quoted by traders. Second, they are mostly negative, consistent with structural models such as Merton’s. Third, the magnitude of the correlation seems to be higher the lower the credit rating. This is reminiscent of the finding that structural credit risk models can explain a larger fraction of the variation in speculative-grade credit spreads.\footnote{See, for example, Eom, Helwege, and Huang (2004), and Longstaff, Mithal, and Neis (2004).}

## 4 Case Study of Altria Group

In this section I use the Altria Group as an example to illustrate the general procedure. As is well known, Altria (aka Philip Morris) has been mired in tobacco-related legal problems since the early nineties. In March 2003, a circuit court Judge in Illinois ordered Altria to post a $12 billion bond to appeal a class action lawsuit. This news led to worries that Altria may have had to file for bankruptcy, triggering Moody’s to downgrade Altria from A2 to Baa1 on March 31, 2003. The trading analysis in this section isolates the period from February 28, 2001 to December 31, 2004, which consists of 962 daily observations of CDS market

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<td>12</td>
<td>16</td>
<td>0.32</td>
<td>0.20</td>
<td>81,306</td>
<td>-0.06</td>
</tr>
<tr>
<td>A</td>
<td>68</td>
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<td>0.37</td>
<td>0.32</td>
<td>30,123</td>
<td>-0.04</td>
</tr>
<tr>
<td>BBB</td>
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<td>84</td>
<td>0.39</td>
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<td>-0.05</td>
</tr>
<tr>
<td>BB</td>
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<td>0.51</td>
<td>0.56</td>
<td>7,162</td>
<td>-0.07</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
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<td>0.66</td>
<td>3,627</td>
<td>-0.12</td>
</tr>
<tr>
<td>CCC</td>
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<td>0.82</td>
<td>3,949</td>
<td>-0.23</td>
</tr>
<tr>
<td>NR</td>
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<td>0.73</td>
<td>0.68</td>
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</tr>
<tr>
<td>Comm. and Tech.</td>
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<td>153</td>
<td>0.51</td>
<td>0.42</td>
<td>32,373</td>
<td>-0.07</td>
</tr>
<tr>
<td>Consumer Cyclical</td>
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<td>134</td>
<td>0.43</td>
<td>0.45</td>
<td>15,850</td>
<td>-0.07</td>
</tr>
<tr>
<td>Consumer Stable</td>
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<td>0.36</td>
<td>0.34</td>
<td>34,926</td>
<td>-0.07</td>
</tr>
<tr>
<td>Diversified</td>
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<td>81</td>
<td>0.29</td>
<td>0.40</td>
<td>8,057</td>
<td>-0.02</td>
</tr>
<tr>
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<td>96</td>
<td>0.39</td>
<td>0.46</td>
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<td>-0.03</td>
</tr>
<tr>
<td>Industrial</td>
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<td>102</td>
<td>0.42</td>
<td>0.46</td>
<td>18,542</td>
<td>-0.05</td>
</tr>
<tr>
<td>Materials</td>
<td>38</td>
<td>126</td>
<td>0.39</td>
<td>0.50</td>
<td>7,224</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Table 1: Summary statistics for the 261 obligors across credit rating and industry.
spreads. The first ten daily observations are set aside for use in the model estimation procedure explained below.

In the first step, theoretical CDS spreads are computed from the CreditGrades (CG) model. As shown in the appendix, CG requires the following inputs: the equity price $S$, the debt per share $D$, the mean global recovery rate $\bar{L}$, the standard deviation of the global recovery rate $\lambda$, the bond-specific recovery rate $R$, the equity volatility $\sigma_S$, and the risk-free interest rate $r$. Specifically, I assume that

$D = \frac{\text{total liabilities}}{\text{common shares outstanding}}$,  
$\sigma_S = 1,000\text{-day historical equity volatility}$,  
$r = \text{five-year constant maturity Treasury yield}$,  
$\lambda = 0.3$,  
$R = 0.5$.

The CreditGrades Technical Document (2002, CGTD) motivates the above choice of $\lambda$ and $\sigma_S$. It also has a more complex definition of the debt per share variable, taking into account preferred shares and the differences between long-term and short-term, and financial and non-financial obligations. The value of $R$ is consistent with Moody’s estimated historical recovery rate on senior unsecured debt. The choice of $r$ is consistent with the existing literature that uses Treasury or swap rates to proxy for the risk-free interest rate.

The implementation of the CG model in this section, however, differs from that of the CGTD in one crucial aspect. The CGTD assumes that $\bar{L} = 0.5$ and uses a bond-specific recovery rate $R$ taken from a proprietary database from JP Morgan. In practice, traders usually leave $R$ as a free parameter to fit the level of market spreads. In doing so, they often find that the market implies unreasonable recovery rates, say negative or close to 1. I note that in the CG model, the expected default barrier level is given by $\bar{L}D$, where $\bar{L}$ is exogenously specified. However, the literature on structural models suggests that both $D$ and $\bar{L}$ should depend on the fundamental characteristics of the firm.\textsuperscript{17} For example, low risk firms (characterized by low asset volatility) should take on more debt and in particular more short-term debt. Presumably, a higher proportion of short-term debt in the capital structure should correspond to a higher default barrier, other things being equal. In any case, it seems appealing on both theoretical and practical grounds to assume a fixed debt recovery rate $R$ and let the data speak to the value of $\bar{L}$, which

\textsuperscript{17}For example, see Leland (1994) and Leland and Toft (1996).
determines the default barrier. Following this prescription, I fit the 10 daily CDS spreads at the start of the sample period to the CG model by minimizing the sum of squared pricing errors over $L$. I find that the implied $L$ for Altria is equal to 0.62. Plugging this estimate along with the above assumed parameters into the CG model, the theoretical CDS spread for Altria can be computed.

Figure 2 compares the market and model CDS spreads for the Altria Group. For ease of comparison, it also shows the Altria equity price and equity volatility during the same period. Two key observations are noted from this figure. First, comparing the market spread in the first panel and the equity price in the second panel, there appears to be a negative association between the two. In fact, a simple calculation confirms that the correlation between changes in the CDS spread and
the equity price for Altria is \(-0.25\). In particular, the three episodes of rapidly rising CDS spreads are all accompanied by falling equity prices. Meanwhile, the 1,000-day historical equity volatility presented in the third panel appears quite stable throughout the sample period. Second, despite calibrating the model using only the first ten observations, for the entire sample period of almost four years the model spread stays quite close to the market spread and roughly follows the same trend. One key difference between the two, however, is that the model spread appears much less volatile. During the three episodes of rising market spreads, the predicted spread increases as a result of falling Altria share prices, but the increase pales in comparison to the galloping market spread. For example, when Moody’s downgraded Altria from A2 to Baa1 on March 31, 2003, the market spread rose from 210bp the previous day to 299bp, while the predicted spread went up only 11bp. Four days later, the market spread again rose 171bp in one single day, while the predicted spread increased by only 8bp. These are exactly the sort of trading opportunities that capital structure arbitrageurs feed on.

Next, I conduct a simulated trading exercise following the ideas laid out in Section 2. For each of the 962 days in the Altria sample period, I check whether the market spread and the model spread differ by more than a threshold value. Specifically, the following conditions are verified:

\[ c_t > (1 + \alpha) c'_t \text{ or } c'_t > (1 + \alpha) c_t, \tag{8} \]

where \(\alpha\) is a trading trigger. Recall that \(c_t\) and \(c'_t\) are the market and model spread, respectively. In the first case, a CDS position with a notional amount of $1 and \(-\delta_t\) shares of the common stock are shorted. In the second case, a unit-sized CDS position along with \(-\delta_t\) shares are bought. These positions are held for a fixed holding period or until convergence, where convergence is defined by \(c_t = c'_t\). To ensure that daily returns from the trades are properly defined, I impose a limited liability assumption, such that a trade is liquidated whenever its total value becomes negative. To get a sense of the riskiness of the positions, I also monitor the fraction of trades with negative overall returns or drawdowns exceeding 20%.

Table 2 presents the summary statistics on holding period returns. Twelve strategies are simulated. Among these, I vary the amount of initial capital ($0.1 and $0.5), the length of the holding period (30 days and 180 days), and the trading trigger (\(\alpha = 0.5, 1, \text{ and } 2\)). \(N\) is the total number of trades entered into. \(N_1\)

---

18Hedge funds usually have a threshold for negative returns that would trigger liquidation.
19Understandably, these trades have unequal holding periods that can vary from one day to the pre-specified maximum length (say, 30 or 180 days), depending on how quickly convergence occurs.
Figure 3: Histogram of holding period returns for the Altria Group.
Table 2: Summary statistics of holding period returns for the Altria Group. The return statistics are in percentages. 

<table>
<thead>
<tr>
<th>Capital</th>
<th>HP</th>
<th>$\alpha$</th>
<th>$N$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>30</td>
<td>0.5</td>
<td>550</td>
<td>0</td>
<td>104</td>
<td>260</td>
<td>0.10</td>
<td>-104.56</td>
<td>94.24</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>319</td>
<td>0</td>
<td>53</td>
<td>142</td>
<td></td>
<td>2.46</td>
<td>-44.66</td>
<td>94.24</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>154</td>
<td>0</td>
<td>14</td>
<td>56</td>
<td></td>
<td>2.22</td>
<td>-20.08</td>
<td>18.04</td>
</tr>
<tr>
<td>180</td>
<td>0.5</td>
<td>550</td>
<td>76</td>
<td>235</td>
<td>140</td>
<td></td>
<td>20.20</td>
<td>-104.56</td>
<td>113.37</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>319</td>
<td>8</td>
<td>96</td>
<td>61</td>
<td></td>
<td>24.68</td>
<td>-25.10</td>
<td>113.37</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>154</td>
<td>0</td>
<td>25</td>
<td>19</td>
<td></td>
<td>18.03</td>
<td>-2.68</td>
<td>43.46</td>
</tr>
<tr>
<td>0.5</td>
<td>30</td>
<td>0.5</td>
<td>550</td>
<td>0</td>
<td>1</td>
<td>252</td>
<td>0.14</td>
<td>-10.68</td>
<td>18.92</td>
</tr>
<tr>
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<td>1</td>
<td>319</td>
<td>0</td>
<td>0</td>
<td>138</td>
<td></td>
<td>0.58</td>
<td>-8.87</td>
<td>18.92</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>154</td>
<td>0</td>
<td>0</td>
<td>54</td>
<td></td>
<td>0.55</td>
<td>-3.91</td>
<td>3.75</td>
</tr>
<tr>
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<td>550</td>
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<td>1</td>
<td>134</td>
<td></td>
<td>4.50</td>
<td>-4.63</td>
<td>23.06</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>319</td>
<td>8</td>
<td>0</td>
<td>60</td>
<td></td>
<td>5.37</td>
<td>-4.52</td>
<td>23.06</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<td>0</td>
<td>18</td>
<td></td>
<td>4.04</td>
<td>-5.32</td>
<td>9.30</td>
</tr>
</tbody>
</table>

Table 2: Summary statistics of holding period returns for the Altria Group. The return statistics are in percentages.

is the number of trades ending in convergence. $N_2$ is the number of trades with drawdown exceeding 20%. $N_3$ is the number of trades with negative holding period returns. The mean, minimum, and maximum of the returns in percentages are also presented. The riskiness of the strategies is evident from Table 2. To put things into perspective, Duarte, Longstaff, and Yu (2005) show that their swap spread arbitrage strategy, with close to $0.05$ initial capital for every $1$ notional amount of interest rate swap traded, generates profits and losses similar to the strategies here with ten times more initial capital ($0.5$). Figure 3 plots the histograms for the six strategies with a holding period of 180 days. It is clear that the returns roughly scale with the amount of initial capital.

Upon closer inspection of the trading simulation, most of the trades with positive returns are made during April and July-August 2003, when the Altria market spread shot up but the model spread remained stable (see Figure 2). Ironically, most of the trades with large negative returns are also made during these periods. This is because the market spread may continue to increase after the trades are entered into. For example, with threshold level at $\alpha = 0.5$, a short CDS position and a short equity hedge are initiated on March 31, 2003, when the market spread and the model spread are 299bp and 168bp, respectively. Four days later, when the market spread widened to 529bp, this trade is liquidated because the margin account has been depleted. Over these four days, the market spread increased 230bp while the Altria stock price decreased only $1.66$, rendering a maximum
loss of -100%. On the other hand, trades entered slightly later enjoy positive returns because 529bp turns out to the peak of the spreads.

Having considered the above example, it is not difficult to see the overall risk of the strategy declining when the threshold for trading is raised. Not only does the distribution become tighter (Figure 3), the fraction of trades with negative returns or more than 20% drawdowns declines as well. The mean return initially increases with the threshold level, but tapers off when $a$ reaches 2. This is probably because a higher spread generally requires a larger equity hedge, which may reduce profitability.

Table 2 also shows that as the maximum holding period increases from 30 days to 180 days, more trades end up converging, fewer trades have negative returns, and more trades have drawdowns exceeding 20%. The mean return is also higher with a longer holding period, although this is perhaps related to more trades converging than to longer holding periods per se.

One may rightly be concerned that the evidence in this section pertains to just one obligor. After all, in all three cases where the Altria market spread diverged from the model spread, it eventually converged. What if the market spread had continued to rise in April 2003, possibly leading to bankruptcy and (perhaps more relevantly) the collapse of the CDS market for Altria? This concern can only be addressed by examining a broader sample, as I do in the next section.

5 General Results for Individual Trades

In this section I replicate the preceding trading strategy for all 261 obligors. Specifically, I set an initial capital of $0.5 for each trade, which longs or shorts a CDS position with a $1 notional amount along with its equity hedge, and follow the same trading trigger as Eq. (8). As Table 2 shows, this level of initial capital results in only a very small percentage of trades that exceed the 20% drawdown. This ensures that the Sharpe ratios computed from monthly portfolio returns in Section 6 are relatively insensitive to the choice of initial capital invested. In addition, I separate the 261 obligors into investment-grade (210 obligors) and speculative-grade (51 obligors), which also includes the two obligors without an average rating at the end of their coverage.20 To the extent that speculative-grade spread levels are higher, the trading returns should be larger and more volatile for a fixed level of initial capital. On the other hand, speculative-grade obligors are

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20They are Worldcom and Qwest Communications.
Figure 4: Time-series of market and model CDS spreads for selected obligors. The solid line is the market spread. The dashed line is the model spread.

more likely to default, which can hurt the profitability of the strategy. As a result, one would expect to see different risk and return characteristics across these two groups.

Before analyzing the general trading returns, it is useful to inspect the behavior of the market and model spreads for each individual obligor. I find that they broadly fit into three categories. In the first category, the market and model spreads are closely related. When the market spread deviates from the model spread, they invariably converge to each other. This category includes, among others, AT&T, McDonald’s, and Toys R Us, which are illustrated in the second column of
The feature common to these obligors is that the market spread shows occasional spikes, only to quiet down a short period later. In other words, the market at times becomes concerned about the credit quality of these companies, but they eventually survived. This group is expected to yield positive expected returns for the arbitrageur. Note, however, there are cases where the model spread increases more than the market spread, as in the example of McDonald’s.

For companies in the second category, their market spread and model spread are tightly linked only up to a point, after which they diverge from each other. This group includes, among others, AMR Corp, Delta Airlines, and Worldcom, which are illustrated in the first column of Figure 4. In the case for the two airlines, their market spreads rose rapidly due to the increasing likelihood of Chapter 11 filing. Although both eventually avoided bankruptcy, the episodes clearly point to the vanishing liquidity in CDS trading for these companies, which is likely to force the sellers to liquidate their positions. For Worldcom, the sample period ended with its CDS spread close to 1,500bp and the company led for bankruptcy two months later. Anyone who followed the type of convergence trading described here for these obligors would have suffered huge losses.

Interestingly, the CreditGrades Technical Document (2002) contains a case study on Worldcom, which shows that if one bought CDS and equity (with an appropriate hedge ratio) in September 2001 and terminated the positions in March 2002 he would have boasted a 79 percent return. Indeed, assuming a 10% capital and a trading trigger $\alpha = 0.5$, a long CDS and equity position is entered into on Spetember 7, 2001 when the market spread was 128bp and the CG model spread was 193bp, both of which are very close to the numbers given in the CGTD case study. However, the trade is terminated on October 31 when the market spread rose to 220bp and the model spread remained at 191bp, producing a return of 37 percent. Thereafter, the market spread continued to rise and a large positive deviation from the theoretical spread appeared. Following the strategy presented above, one should short CDS and equity, which would have generated heavy losses given the subsequently skyrocketing market spread. The point is that ex post it is easy to justify buying and holding credit protection before Worldcom ran into trouble. But doing so would be inconsistent with a disciplined application of the basic principles of capital structure arbitrage, i.e., trade only when there is a large deviation between the market and the model spreads and liquidate when the gap disappears. Instead, the strategy presented in the CGTD case study is basically a gamble that Worldcom will not survive.

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21 The Altria Group, which is the subject of Section 4, also falls into this category.
Table 3: Summary statistics of holding period returns for all obligors. The return statistics are in percentages.

<table>
<thead>
<tr>
<th>Rating</th>
<th>HP</th>
<th>α</th>
<th>N</th>
<th>N₁</th>
<th>N₂</th>
<th>N₃</th>
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<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
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</tr>
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<tr>
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<td></td>
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<td></td>
<td>3,912</td>
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<td>94</td>
<td>2,238</td>
<td>1.95</td>
<td>-31.63</td>
<td>70.29</td>
</tr>
<tr>
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<td>0</td>
<td>795</td>
<td>0.05</td>
<td>-11.32</td>
<td>68.72</td>
</tr>
</tbody>
</table>

There are also obligors that seem to fall into a third category, where the trading strategy is perhaps confounded by model misspecification. The third column of Figure 4 illustrates the CDS spreads of Agilent Technologies, Phelps Dodge, and Staples. Here we observe that shortly after the coverage starts, the market and model spreads start to drift apart in a smooth manner in the absence of any major shock to either series. This observation leaves model misspecification as the most likely explanation to the prevalence of negative returns for these obligors.

Turning now to the summary statistics on holding period returns, Table 3 shows results similar to those for the case of Altria. For example, a longer holding period produces more converging trades, fewer trades with negative returns, and better average returns. The effect of a larger trading trigger level, however, is different for the two rating categories. For investment-grade obligors, it leads to higher mean returns and a smaller percentage of trades with negative returns. For speculative-grade obligors, the effect could be the opposite in some cases. This is because raising the trading trigger level biases the sample toward obligors with rising market spreads, which, in the case of speculative-grade obligors, are more likely to stay high and not recover.

Like Figure 3, Figure 5 shows that the distribution of holding period returns becomes tighter as the trading trigger level increases. Moreover, it shows that the speculative-grade distribution is indeed more than twice as dispersed as the investment-grade distribution.
Figure 5: Histogram of holding period returns for all obligors. The left column is for 210 investment-grade obligors. The right column is for 51 speculative-grade obligors.
6 Capital Structure Arbitrage Index Returns

The preceding sections portray capital structure arbitrage as a risky strategy that exploits the divergence between the CDS market spread and the theoretical spread generated by a structural model. The distribution of the holding period returns for individual trades (Figure 5) shows that the strategy is anything but an arbitrage in the textbook sense. In following this strategy, the trader is confounded by a host of potential problems, such as the lack of convergence brought on by model misspecification or mis-measured inputs to the model, and ineffective hedging as a result of the low correlation between changes in the CDS spread and the equity price. Figure 4 also shows that the strategy would work well so long as the divergence is only temporary. In contrast, large and in some cases permanent deviation of the market spread from the model spread can produce dramatic losses.

Regardless of the nature of risk inherent in individual trades, a trader is unlikely to gamble on just a few trades. In this section, I construct a capital structure arbitrage return index from all individual trades and examine the associated monthly returns. This is a more appropriate setting for addressing questions such as whether capital structure arbitrage returns can be explained by well known market-wide risk factors.

Specifically, I compute daily excess returns from all individual trades across the 261 obligors and aggregate them into monthly excess returns. The procedure works as follows. Take for example the investment-grade strategy in Table 3 with a holding period of 180 days and a trading threshold of 0.5. There are a total of 57,621 individual trades. Say on a given day there are \( n \) ongoing trades. I compute an equally-weighted average of the daily excess returns on these \( n \) trades. In essence, I assume that the arbitrageur is always invested in an equally-weighted portfolio of “hedge funds,” where each fund consists of one individual trade. These average daily returns are then compounded into a monthly frequency.

Table 4 presents the summary statistics on the monthly excess returns constructed for a number of strategies. All of these strategies assume a $0.5 initial capital and a 180-day holding period. Since the sample period covers 2001 to 2004, there are a total of 48 monthly returns for each strategy. However, for some strategies there could be no individual trades for some of the months, and this is particularly true for the speculative-grade sample because of its smaller size. If there are no trades for a given month, I assume a zero excess return for that month. Here, \( N \) denotes the number of months with nonzero returns, Neg denotes the fraction of negative returns, Corr denotes the first-order serial correlation of the month returns, and Sharpe denotes the annualized Sharpe ratio for the strat-
Table 4: Summary statistics of monthly excess returns. The return statistics are in percentages.

<table>
<thead>
<tr>
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<th>Corr</th>
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<td>48</td>
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<td>0.05</td>
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<td>3.05</td>
<td>1.13</td>
<td>0.04</td>
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<td>1.27</td>
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<td>0.80</td>
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<td>0.00</td>
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<td>0.40</td>
<td>0.76</td>
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<tr>
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<td>0.00</td>
<td>-2.62</td>
<td>21.87</td>
<td>4.32</td>
<td>0.49</td>
<td>0.29</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table 4 shows that the speculative-grade obligors generally produce higher mean monthly returns and higher Sharpe ratios than the investment-grade obligors. The returns have a positive skewness and in most cases are not significantly autocorrelated. It is noted that the results presented here, including the magnitude of the Sharpe ratio, are quite similar to those of hedge fund industry benchmarks on fixed income arbitrage (see Duarte, Longstaff, and Yu (2005)).

Although the monthly returns are not risk-free, I test whether they give rise to statistical arbitrage following the procedure prescribed by Hogan, Jarrow, Teo, and Warachka (2004). A statistical arbitrage is a zero initial cost self-financing trading strategy with positive expected discounted profits, a probability of a loss converging to zero, and a time-averaged variance converging to zero. I apply their constrained mean test to the 48 monthly returns from capital structure arbitrage. Specifically, denoting the capital structure arbitrage return in month $i$ as $r_i$ and and risk-free rate in month $i$ as $r_i^f$, I start by borrowing one dollar at $r_i^f$ and investing it at $r_i$. This is repeated each month with the profit from the previous month invested at the risk-free rate. The total profit $\Phi_i$ at month $i$ then satisfies $\Phi_i = r_i - r_i^f + \Phi_{i-1} (1 + r_i^f)$. This profit is discounted back to the starting point to produce the incremental discounted profit $\Delta \phi_i$, for which I perform the test of statistical arbitrage. Namely, $\Delta \phi_i$ is assumed to be normally distributed with constant mean $\mu$ and variance $\sigma^2 (\Delta t_i)^{2\lambda}$, and the parameters $(\mu, \sigma, \lambda)$ are estimated by maximum likelihood and the hypotheses of $\mu > 0$ and $\lambda < 0$ are then tested.

Table 5 summarizes the test results for five of the six strategies presented in Table 4. The speculative-grade strategy with a trading trigger of $\alpha = 2$ is excluded because the strategy yields too many months with zero excess returns. With only 48 monthly observations, I find that none of the strategies give rise to statistical ar-
Table 5: Test of statistical arbitrage. Maximum likelihood estimates for $\mu$ and $\sigma$ (in percentages), and $\lambda$ are given. Standard errors are below the estimates.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\lambda$</th>
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</thead>
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<td>1.2</td>
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<td></td>
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<td>0.16</td>
<td>1.4</td>
<td>-0.56</td>
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<td>0.17</td>
<td>0.2</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.23</td>
<td>1.4</td>
<td>-0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.24</td>
<td>0.2</td>
<td>0.17</td>
</tr>
<tr>
<td>Spec</td>
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<td>0.60</td>
<td>2.7</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.46</td>
<td>0.3</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.45</td>
<td>5.5</td>
<td>-0.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.65</td>
<td>0.6</td>
<td>0.16</td>
</tr>
</tbody>
</table>

bitrage at conventional significance levels. However, most strategies yield positive estimates for the mean of the discounted profits (though statistically not significant) and negative estimates for the growth rate of their variances, consistent with statistical arbitrage. As data coverage continues to expand, this test is expected to produce a more meaningful inference on the long-horizon profitability of the trading strategy.

With the monthly portfolio returns, I investigate the relationship between capital structure arbitrage profitability and systematic risk factors. In particular, I use the excess return on the S&P Industrial Index (S&PINDS) to proxy for equity market risk and the excess return on the Lehman Brothers Baa and Ba Intermediate Index (LHIBAAI and LHHYBBI) to proxy for investment-grade and speculative-grade bond market risk. In addition, I include the excess return on the CSFB/Tremont Fixed Income Arbitrage Index (CSTINFA) to capture variations in the monthly returns not related to the market indexes. These additional variables are obtained from Datastream.

Table 6 shows that there is no apparent relationship between capital structure arbitrage monthly returns and any of the included market factors. In particular, neither the investment-grade nor the speculative-grade bond market factor shows up significantly. In fact, the intercept of the regression is close to the original level of the mean monthly excess returns in Table 4, suggesting that none of the risk factors can bid away the “alphas” of the strategies.

To show that this result is robust clearly requires more work. For example, one can always modify how the strategies are implemented (which I will do in the
Table 6: Regression of capital structure arbitrage monthly index returns on common market factors. Standard errors are below the estimates.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>Inv 0.5</th>
<th>S&amp;PINDS 0.22</th>
<th>LHBAAI -0.19</th>
<th>LHYBBI -0.04</th>
<th>CSTINFA -0.18</th>
<th>$R^2$ 0.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inv</td>
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<td>0.17</td>
<td>0.04</td>
<td>0.13</td>
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<td>0.17</td>
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<tr>
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<td>0.05</td>
</tr>
<tr>
<td></td>
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<td>-0.14</td>
<td>-0.01</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>Spec</td>
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<td>0.20</td>
<td>0.04</td>
<td>0.16</td>
<td>0.11</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>0.47</td>
</tr>
<tr>
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<td>-0.41</td>
<td>1.02</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
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<td>0.38</td>
<td>-0.14</td>
<td>-0.23</td>
<td>0.02</td>
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<tr>
<td></td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.68</td>
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</table>

next section to some extent), and search for additional risk factors. However, if one revisits the rationale for capital structure arbitrage, it is not difficult to see that there is no strong reason a priori to believe that the results should be any different. Recall that in this strategy, the trader is essentially buying stocks and shorting bonds (buying CDS) of the same company, or shorting stocks and buying bonds (selling CDS). However, he is not consistently long or short in either market. It is mentioned earlier that the capital structure arbitrageur may suffer losses on individual trades when the obligor approaches default. On the other hand, the success of the strategy depends on the availability of large variations in the spread, which is linked to credit events such as default and bankruptcy. In summary, the finding of no correlation with equity and bond market factors may not be surprising after all.

7 Robustness of the Results

As in any study of trading strategies, one needs to examine the robustness of the returns to a variety of parameters that can influence how the strategies are implemented. Some of these parameters, such as the level of initial capital, the maximum holding period, and the trading trigger level, have already been explored to some extent in earlier parts of the paper. In this section I consider changing
several additional assumptions used in the trading simulation.

First, recall that the value of an existing CDS position is essentially that of a survival-contingent annuity, which requires the modeling of the term structure of survival probabilities for the obligor. Consistent with trading and hedging with the CG model, the previous analysis has used survival probabilities computed from the CG model. In order to reduce the model-dependency of the strategy, I calculate the value of the annuity using the methodology of Duffie and Singleton (1999). Namely, I discount the promised cash flow from the annuity using a risk-adjusted interest rate, which equals the risk-free rate plus the market CDS spread divided by one minus the recovery rate, taken to be 0.5.

Second, the previous analysis assumes a static hedging scheme in which the hedge ratio is determined when a trade is initiated and then maintained until liquidation. Since, as we see from Figure 5, the market spread can undergo dramatic subsequent changes, it is worthwhile to relax this assumption and assume a hedge ratio that is updated more frequently, say daily. While this may improve the effectiveness of the equity hedge, it comes at the disadvantage of higher transaction costs. To see if this indeed presents an improvement to the trading returns, I continue to ignore trading costs from the equity side of the strategy.

Lastly, I vary the level of CDS market bid/ask spreads. Recall that a 5% bid/ask spread has been assumed. I simply increase this level to 10% and check its impact on the monthly excess returns.

Table 7 reports the summary statistics for monthly excess returns from the above variations to the basic trading strategy. All strategies assume an initial capital of $0.5 and a maximum holding period of 180 days. The “All” strategy refers to monthly returns computed from all individual trades, both investment-grade and speculative-grade. Therefore, it represents some type of “average” of the two sets of results reported in Table 4. The “Value” strategy computes the value of CDS positions using the Duffie and Singleton (1999) default risk-adjusted discount rate. The “Hedge” strategy assumes a daily rebalanced hedging position. Finally, the “Trans” strategy assumes a CDS bid/ask spread of 10%.

The results of Table 7 suggest that the capital structure arbitrage monthly returns are not sensitive to the assumptions of the basic strategy. In particular, rebalancing the hedging position daily does not improve the risk and return trade-off even in the absence of equity market trading costs. This is perhaps due to the fact that the equity hedge is already imperfect in the first place. In addition, raising the CDS market bid/ask spread to 10% does not eliminate the monthly excess returns. It does, however, reduce the mean monthly excess returns by about 10 to 20bp.
<table>
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<tr>
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<th>Mean</th>
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<th>Corr</th>
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<td>-3.20</td>
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<td>0.21</td>
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<td>0.72</td>
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<td></td>
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<td>46</td>
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<td>0.05</td>
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<td>1.50</td>
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<td>0.14</td>
<td>0.38</td>
<td>0.87</td>
</tr>
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<td>0.40</td>
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</tr>
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<td>0.08</td>
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<td>1.40</td>
<td>0.16</td>
<td>0.42</td>
<td>0.47</td>
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</table>

Table 7: Summary statistics of monthly excess returns for variations to the basic strategy. The return statistics are in percentages.

8 Conclusion

This paper presents the most comprehensive study to date of capital structure arbitrage, a popular trading strategy in recent years in which the arbitrageur takes advantage of the temporary divergence between CDS market spreads and predicted spreads from a structural credit risk model. My simple implementation computes CDS spreads by calibrating the industry standard CreditGrades model using the first ten daily observations of CDS market spreads, then compares the time-series of market and model spreads. A CDS position is entered into whenever the two series diverge from each other by a fixed threshold level, with an accompanying equity position taken out as a hedge.

I study this type of trading strategies using daily spreads of 5-year CDS on 261 North American industrial obligors from 2001 to 2004. By varying the trading threshold level, the initial capital invested, and the maximum holding period, I find that the individual trades can be quite risky. In particular, the arbitrageur risks large losses should the CDS market spread rise rapidly, culminating in some type of credit event. The equity hedge can be completely ineffective, offering minimal resistance during such periods of “crisis.” However, I also find that when the individual trades are aggregated into monthly capital structure arbitrage portfolio returns, the strategy appears to offer attractive Sharpe ratios that are in line with other types of fixed income arbitrage strategies and hedge fund industry bench-
marks. In addition, I find that the monthly excess returns cannot be explained by several well known equity and bond market risk factors. These results are robust to CDS market transaction costs and variations to the implementation of the trading strategy.

A CreditGrades Model

This appendix contains a summary of the CreditGrades model, including its basic assumptions and the requisite formulas for implementing the trading strategy described in the main text. Further details about the model can be found in the CreditGrades Technical Document (2002).

The CreditGrades model assumes that under the pricing measure the firm’s value per equity share is given by

\[ \frac{dV_t}{V_t} = \sigma \, dW_t, \]  

(9)

where \( W_t \) is a standard Brownian motion and \( \sigma \) is the asset volatility. The firm’s debt per share is a constant \( D \) and the default threshold is

\[ LD = \overline{L} De^{\lambda Z - \lambda^2/2}, \]  

(10)

where \( L \) is the (random) recovery rate given default, \( \overline{L} = \mathbb{E}(L) \), \( Z \) is a standard normal random variable, and \( \lambda^2 = \text{var}(\ln L) \). The variable \( L \) represents the global recovery on all liabilities of the firm, while \( R \) defined in the main text is the recovery on a specific debt issue which constitutes the underlying asset for the credit default swap. Note also that the firm value process is assumed to have zero drift. This assumption is consistent with the observation of stationary leverage ratios and the model of Collin-Dufresne and Goldstein (2001).

Default is defined as the first passage of \( V_t \) to the default threshold \( LD \). The density of the default time can be obtained by integrating the first passage time density of a geometric Brownian motion to a fixed boundary over the distribution of \( L \). However, CreditGrades provides an approximate solution to the survival
probability using a time-shifted Brownian motion, yielding the following result:\footnote{The approximation assumes that \( W_t \) starts not at \( t = 0 \), but from an earlier time. In essence, the uncertainty in the default threshold is shifted to the starting value of the Brownian motion. As a result, even for very small \( t \) (or even \( t = 0 \), for that matter) the default probability is not equal to 0. While this may be a practical way to increase short-term default probability, a theoretically more appealing approach is given by Duffie and Lando (2001).}

\[
q(t) = \Phi \left( -\frac{A_t}{2} + \ln d \right) - d \cdot \Phi \left( -\frac{A_t}{2} - \ln d \right),
\]

(11)

where \( \Phi(\cdot) \) is the cumulative normal distribution function,

\[
d = \frac{V_0}{LD} e^{\lambda^2},
A_t^2 = \sigma^2 t + \lambda^2.
\]

Substituting \( q(t) \) into Eq. (5) and assuming constant interest rate \( r \), the CDS spread for maturity \( T \) is given by

\[
c(0, T) = r (1 - R) \frac{1 - q(0) + H(T)}{q(0) - q(T) e^{-rT} - H(T)},
\]

(12)

where

\[
H(T) = e^{\xi} (G(T + \xi) - G(\xi)),
G(T) = d^{\xi + 1/2} \Phi \left( -\frac{\ln d}{\sigma \sqrt{T}} - z \sigma \sqrt{T} \right) + d^{-\xi + 1/2} \Phi \left( -\frac{\ln d}{\sigma \sqrt{T}} + z \sigma \sqrt{T} \right),
\]

\[
\xi = \frac{\lambda^2}{\sigma^2},
z = \sqrt{1/4 + 2r/\sigma^2}.
\]

Normally, the equity value \( S \) as a function of firm value \( V \) is needed to relate asset volatility \( \sigma \) to a more easily measurable equity volatility \( \sigma_S \). Instead of using the full formula for equity value, CreditGrades uses a linear approximation \( V = S + LD \) to arrive at

\[
\sigma = \sigma_S \frac{S}{S + LD}.
\]

(13)

To find the value of an existing contract using Eq. (6), we need an expression for \( q_t(s) \), the survival probability through \( s \) at time \( t \). In structural models with uncertain but fixed default barriers, the hazard rate of default is zero unless the
firm value is at its running minimum. Therefore, although the uncertain recovery rate assumption in the CreditGrades model may help increase short-term default probabilities at one point in time, it cannot do so consistently through time. To circumvent this problem, I assume that $t$ is small compared to the maturity of the contract $T$. The value of the contract is then approximated by

$$
\pi (0, T) = (c (0, T) - c) \int_0^T e^{-rs} q (s) \, ds
$$

$$
= \frac{c (0, T) - c}{r} (q (0) - q (T) e^{-rT} - e^{r \xi} (G (T + \xi) - G (T))) . \tag{14}
$$

In this expression, $c$ is the CDS spread of the contract when it was first initiated, and $c (0, T)$ is a function of the equity price $S$ as shown in Eq. (12). The proper way to understand Eq. (14) is that it represents the value of a contract which was entered into one instant ago at spread $c$ but now has a quoted spread of $c (0, T)$ due to changes in the equity price.

By Eqs. (7) and (14), the hedge ratio is given by

$$
\delta (0, T) = \frac{1}{r} \frac{\partial c (0, T)}{\partial S} (q (0) - q (T) e^{-rT} - e^{r \xi} (G (T + \xi) - G (T))) , \tag{15}
$$

because by definition $c$ is numerically equal to $c (0, T)$, which corresponds to an equity price of $S$. I then differentiate $c (0, T)$ numerically with respect to $S$ to complete the evaluation of $\delta$.

**References**


