

What's different about loans?  
An analysis of the risk structure of credit spreads

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**This Draft: October 2006**

**Abstract**

Understanding the determinants of credit spreads has always been an important objective of academic researchers, regulators and practitioners alike. While extensive research has been produced on bonds and loans separately, few empirical studies have analyzed those two classes of debt instruments jointly. The aim of this paper is twofold: first, we derive a simple structural model in which differences between bond and loan spreads are investigated, based on the different monitoring ability of bankers and bond-holders; second, we empirically analyse the determinants of bond and loan spreads and test the prescriptions of the model. The theoretical model is based on the introduction of a stochastic default barrier that accounts for informational noise met by lenders, while the empirical analysis is based on a sample of 7,926 Eurobonds and 5,469 syndicated loans originated between 1991 and 2003. The empirical results confirm the key finding of the model, that is, that while spreads increase as ratings worsen for both bonds and loans, the spread/rating link is quite steeper for the former.

JEL Classification Numbers: G15, G21, G28

EFM Classification Codes: 510, 550, 570

Keywords: Eurobonds, syndicated loans, credit ratings, spreads, default risk.

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## 1 Introduction

Understanding the determinants of credit spreads has always been an important objective for academic researchers, regulators and practitioners alike. Indeed, a proper understanding of the mechanism underlying credit pricing would allow researchers to estimate the impact on spreads of default risk changes, and to better calibrate credit risk models; also, it would enable practitioners to better forecast the evolution of bond markets and to assess the correct pricing of new debt issues; finally, it would help regulators to calibrate risk-based capital requirements and to use capital markets signals more effectively, as a market-discipline tool.

While extensive research has been produced on bonds and loans, separately, few studies have analyzed those two classes of debt instruments jointly. Comparing the mechanisms driving the cost of bonds and loans for borrowers of different quality may be of paramount importance to gain a full understanding of the advantages and disadvantages for bank-centered versus market-centered economies. Also, such a comparison may help to assess some of the likely consequences of the growing role played by capital markets in funding the industrial systems of many newly-developed countries. Finally, comparing bank loans and bonds might give us a better insight on the role played by financial institutions in local markets, where information asymmetries between borrower and bank lenders are more effectively overcome, due to the better knowledge of the socio-economic local environment.

The aim of this study is to draw a theoretical and empirical comparison between the determinants of bond and loan spreads, unifying two streams of research that have grown richer and livelier over the latest years. Accordingly, in the first part of this paper we derive a simple first passage structural model in which differences between bond and loan spreads are investigated, based on a stochastic default barrier that embeds the different monitoring ability of banks and bond-holders. In the second part, we empirically analyse the determinants of bond and loan spreads and test whether the prescriptions of the model are confirmed or rejected by market data.

Our empirical results tend to confirm the theoretical model's prescriptions, highlighting that the spread/rating relationship is quite steeper for bonds than for banks. In other words, by better controlling the informational noise surrounding the borrower's real

economic conditions, banks manage to keep the risk premium required on low-quality loans to much lower levels than the one demanded by private investors in the bond market.

The paper is structured as follows. Section 2 looks at previous related works on this subject and highlights the main objective and contribution of this empirical study. Section 3 presents our theoretical model, showing how it was derived from a standard *à la* Merton framework and made operational by means of discrete-time binomial tree. Section 4 presents our empirical test, describing our data sample (a wide dataset including more than 15,000 bond issues and syndicated loans), the model and variables used to analyze credit spreads and the estimation results (where more than 75% of the total variance in spreads is explained). Section 6 concludes.

## **2 Literature review**

This paragraph will briefly review the most relevant contributions on the three topics covered by this paper, namely: a) the “specialty” of bank loans versus public bonds; b) models that explain credit spreads by taking into account informational asymmetries; c) empirical studies on credit spreads, with a special focus on studies exploring the spread/rating relationship.

As far as the specialty of bank loans versus public bonds is concerned, the modern literature on financial intermediation has shown that bank loans attenuate information-related problems more effectively than is possible with direct financing transactions such as bonds, because of free-rider problems, coordination failures and duplicated monitoring costs dispersed among investors. According to this theoretical literature, the role of banks is to provide delegated monitoring and screening services on behalf of investors. The most important theoretical contributions of this type of research (e.g. Leland and Pyle, 1977; Diamond, 1984; and Ramakrishnan and Thakor, 1984) all focus on the problems generated by the informational asymmetry between lenders/investors and borrowers and rationalize the existence of financial intermediaries on these grounds. Banks are able to attenuate this information asymmetry through their superior *ex ante* screening ability and their *ex post* monitoring activity.

The assumption of a superior *ex ante* screening ability is consistent with the literature on “relationship lending” in commercial banking (Rajan, 1992; Petersen and Rajan,

1994; Berger and Udell, 1995; Boot and Thakor, 2000). Indeed, when the borrower has been a long-time customer of the bank, the bank might have some information not reflected in the borrower's public financial statements. This has also been shown more recently for syndicated loans by Dennis and Mullineaux (2000).

The findings of this theoretical literature on banks' monitoring activity also present implications on the role played by bank loans versus directly placed debt in the funding of differently rated borrowers. In an analysis focusing on the interaction of borrower reputation and monitoring, Diamond (1991) concludes that *"...borrowers with high credit ratings will borrow directly without monitoring, lower-rated borrowers will borrow from banks and monitoring will provide incentives, and still lower-rated borrowers (if monitoring costs are not too high) will borrow from banks and will be screened; some of these will be turned down for credit..."*.

More recent papers (e.g. Repullo and Suarez, 1998) have also addressed the issue of *how* banks monitor their borrowers. This process takes place through a credible threat of early liquidation on borrowers subject to moral hazard. This threat allows banks to indirectly participate in the borrower's decision making process and therefore reduce the problems of asymmetric information and the agency costs of outside finance under moral hazard.

As regards quantitative models explaining the "optimal" spreads on credit exposures, our review will focus mainly on the area where our contribution belongs: structural models with informational imperfections.

Structural models date back to the fundamental intuition of Merton (1974), where the value of the borrower's assets is described through a geometric Brownian motion and default occurs if, at maturity  $T$ , assets fall below the face value of debt ("the barrier"). Although it entails some rather simplistic assumptions (all debt is a zero coupon bond maturing at  $T$ , no covenants exist that may force default before the maturity date), Merton's model has the remarkable merit of showing how an investment in risky debt may be likened to a portfolio including a short put option, hence allowing option pricing models to be used in setting optimal credit spreads.

Black and Cox (1976) refined Merton's model by accounting for the possibility of a default before  $T$ , if the asset value falls below some safety threshold indicated in the debt covenants. The value of this barrier changes over time according to a deterministic law: e.g., it may accrue over time at some pre-determined instantaneous rate  $\gamma$ . Since default occurs as soon as the asset value crosses the barrier, Black and Cox's model is often referred to as a *first passage* model<sup>1</sup>.

Bielecki and Rutkowski (2001) generalise first passage models, by allowing the barrier (as well as the asset value) to follow a geometric Brownian motion. However, since in Brownian motions variance increases with time, the initial value of the barrier is thought to be known with certainty. The Authors also provide a more general framework in which stochastic interest rates are allowed, so interest rate risk can be accounted for, as well as its interaction with credit risk; within this framework falls the work by Saà-Requejo and Santa Clara (1999), where the barrier starts from a deterministic value and evolves stochastically, due to the same risk factors that drive asset values and interest rates.

Zhou (2001) introduces the possibility that the asset value  $V$  may undergo sudden "jumps", making its evolution totally unpredictable, even over a very small time interval. However, while being unpredictable,  $V$  is still assumed to be observable to investors.

Duffie and Lando (2001) introduce the case in which bond investors do not observe  $V$ , but rather receive imperfect information at discrete time intervals (that is, information on  $V$  is both delayed and noisy). The longer the delay and/or the noisier the information, the higher are default probabilities and credit spreads estimated by the market.

Finkelstein et al. (2002) propose a model in which asset values are observable but the current value of the barrier is not; however, for the sake of computational simplicity they choose to approximate the unobservable distance to default with an observable

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<sup>1</sup> A more sophisticated version of "first passage" models can be found in Brigo and Tarengi (2004), where the parameters of the processes for the asset value and the barrier (including interest rates and volatility) are time-dependent. This makes it possible to calibrate the model to market data with a substantially greater flexibility than in "classic" models like Merton or Black-Cox.

process (where the initial value of the barrier is known), while operating a shift in their time measure so that the present is portrayed as a sort of “future in the past”.

Giesecke and Goldberg (2004), too, assume that asset values are observable but the barrier is unknown to company outsiders. This implies that, even if default has technically occurred (that is, the asset value has fallen below the barrier), investors may not be fully certain about it; rather, they will assign a subjective probability to the fact that a default may already have occurred, and such a probability is going to translate into corporate bond prices. This kind of framework can correctly represent the case of “hidden defaults”, like Enron or Worldcom, that were fully disclosed to the market only some months (or years) after they had actually occurred.

The behaviour of credit spreads and the spread/rating relationship have been extensively researched over the latest years, with most analyses specializing either on loans or on bonds.

Spreads on syndicated loans were investigated, e.g., by Yi and Mullineaux (2002), where credit spreads are regressed on bank loan credit ratings and other factors reflecting information asymmetry and agency problems (the borrower's financial ratios, some characteristics of the loan facility and some market-environment variables)<sup>2</sup>. Coleman, Esho and Sharpe (2002) also focus on loan spreads, analyzing the impact on pricing of some features of the lending bank, as well as some borrower characteristics and other loan contract peculiarities. Harjoto, Mullineaux and Yi (2003) compare the loan pricing techniques of investment banks that originate syndicated loans to those of commercial bank arrangers; they find that, while investment banks appear to price loans differently on an unconditional basis, such differences are less robust when loan pricing is conditioned on some key explanatory variables (including the borrower's rating). Casolaro et al. (2002) analyze bank loan spreads trying to isolate the effect of the “certification” provided by the arranger of a syndicated credit facility (which is assumed

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<sup>2</sup> Yi and Mullineaux (2002) set up a comparison between loans and bonds concerning the factors underlying agency ratings (based on an ordered logit model). However, no comparative analysis is presented as far as credit spreads are concerned.

to be proportional to the share of the facility retained by the arranger); their results show that, after controlling for a large number of loan and borrower characteristics, syndicated facilities in which the arranger retains a larger share are charged lower spreads. Finally, Allen and Gottesman (2005) have studied the price formation process on the secondary market for subordinated loans, comparing it with the price changes experienced on the equity market.

Several works have analyzed bond spreads. E.g., Morgan and Stiroh (1999) compare bonds issued by banks and by non-financial companies, finding that, after controlling for a number of explanatory variables, the bond spread/rating relationship is the same for the two groups, especially among the investment grade issues. Elton et al. (2001) decompose the spreads observed on a sample of corporate bonds into three effects, representing expected default rates, tax factors and systematic risk: while state taxes are found to have a substantial effect on spreads, expected default rates seem to play a relatively minor role. John, Lynch and Puri (2002) study bond yields with a special focus on the effect of collateral, showing that collateralized debt pays *higher* yields, even after controlling for credit rating and several other characteristics of the issue and the issuers; such an effect is stronger for low credit ratings, non-mortgage assets and longer maturities. Esho, Kollo and Sharpe (2004) examine the determinants of underwriter spreads on straight/fixed rate Eurobonds; spreads are found to depend on the governing law (affecting renegotiation prospects), the distribution mechanism (public issues versus private placements), the underwriter reputation and the currency in which bonds are denominated.

The comparison of spread determinants for bonds and loans, that is the area covered by our paper, seems to have received relatively less attention. Empirical analyses were carried out, e. g., by Altman, Gande and Saunders (2004), who used secondary market data to show that the price decline near default is less marked for loans, since they are more carefully monitored and therefore are subject to a lower “surprise effect” than bonds. However, our paper differs from the Altman et al. one because empirical data (coming from a totally different dataset) are actually used to validate the implications of a structural credit-risk model.

### 3 A structural model of credit spreads with information asymmetries

#### 3.1 The process leading to a default

Following Merton (1974), we assume that the borrowing firm's assets follow a geometric brownian motion, that is:

$$dV = (\alpha V - C)dt + \sigma V dz \quad [1]$$

where  $V$  denotes the value of corporate assets,  $\alpha$  is the expected instantaneous return on these assets,  $C$  is the total euro payout by the firm per unit time to its shareholders and debtholders (to be defined in more detail below),  $\sigma$  is the instantaneous volatility of the return on the firm assets per unit time and  $dz$  denotes a standard Wiener process (with  $dz \equiv \varepsilon \sqrt{dt}$ , where  $\varepsilon$  follows a standard normal distribution).

Assuming that payouts  $C$  are proportionate to total assets, they will be given by

$$C \equiv (q + e)V \quad [2]$$

where  $q$  represents the instantaneous net outflow due to debt reimbursement and interest payments, net of new debt issues, and  $e$  indicates the instantaneous net outflow due to dividend payments and share buybacks, net of capital increases.

Based on this assumption, the model simplifies to

$$dV = (\alpha - q - e)Vdt + \sigma V dz = \mu V dt + \sigma V dz \quad [3]$$

where  $\mu$  denotes the expected *instantaneous net growth rate* of corporate assets. Accordingly, if  $V_0$  is the value of corporate assets at time  $t = 0$ , the future value for any  $t > 0$  is given by:

$$V_t = V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma \varepsilon \sqrt{t}} \quad [4]$$

Still following Merton (1974), we assume that default occurs if  $V_t$  falls below a certain threshold, which is given by the corporate debt of  $B_t$  maturing at time  $t$ .

The firm survives if and only if  $V_t > B_t$ , that is, if

$$\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma \varepsilon \sqrt{t} > \ln\left(\frac{B_t}{V_0}\right) \quad [5]$$

Furthermore, corporate debt evolves between time 0 and time  $t$  according to the following law:

$$B_t = B_0 e^{(r-q)t} \quad [6]$$

Where  $r$  is the continuously-compounded cost of debt and  $q$  (as defined above) indicates the instantaneous rate of decrease due to debt reimbursement and interest payments, net of any new debt issues. Note that  $q < 0$  if the value of new issues exceeds that of repayments, while the opposite holds if  $q > 0$ .

Equation [5] then becomes

$$\left( \mu - \frac{\sigma^2}{2} \right) t + \sigma \varepsilon \sqrt{t} > \ln \left( \frac{B_0}{V_0} \right) + (r - q)t \quad [7]$$

which, given the meaning of  $\mu$ , is equivalent to

$$\left( \alpha - e - \frac{\sigma^2}{2} \right) t + \sigma \varepsilon \sqrt{t} > \ln \left( \frac{B_0}{V_0} \right) + rt \quad [8]$$

Hence, the borrower survives if

$$\left( \alpha - r - e - \frac{\sigma^2}{2} \right) t + \sigma \varepsilon \sqrt{t} > \ln L \quad [9]$$

where  $L = \frac{B_0}{V_0}$  is a measure of the firm's current leverage.

### 3.2 The role of informational asymmetries

Now, suppose that  $L$  cannot be observed by the lender with certainty. Namely, the true value of  $L$  will differ from the one perceived by the lender ( $\hat{L}$ ) by a stochastic factor  $\eta$  which embodies the informational asymmetries, noise, uncertainty related to the true assessment of a company's creditworthiness. In symbols:

$$L = \hat{L} \eta \quad [10]$$

Note that the uncertainty surrounding  $L$  concerns both the actual current value of corporate debt (which could differ from the figure reported in the latest firm's financial accounts, which in turn could be subject to window-dressing or inaccuracies) and the

“fair” current value of corporate assets, which in turn depends on the discounted value of future gross earnings. Debt-holders can reduce this uncertainty by deploying adequate screening and monitoring techniques; however, there is always a positive amount of uncertainty surrounding  $L$ .

Since  $L$  is positive, one can reasonably assume that  $\eta$  follows a lognormal distribution. More precisely, since we want  $\hat{L}$  to be an unbiased estimate of  $L$ , we assume that  $\log \eta$  follows a normal distribution with mean  $-\frac{\nu^2}{2}$  and standard deviation  $\nu$ . Accordingly:

$$L = \hat{L} e^{\frac{\nu^2}{2} + \nu\psi} \quad [11]$$

where  $\psi$  denotes a standard normal variable.

By plugging [11] into the survival condition [9] one finds that the borrower will survive if

$$\left( \alpha - r - e - \frac{\sigma^2}{2} \right) t + \sigma \varepsilon \sqrt{t} > \ln \hat{L} - \frac{\nu^2}{2} + \nu\psi \quad [12]$$

That is, if

$$\sigma \varepsilon \sqrt{t} - \nu\psi > \ln \hat{L} - \frac{\nu^2}{2} - \left( \alpha - r - e - \frac{\sigma^2}{2} \right) t \quad [13]$$

Note that the left-hand side in [13] indicates risk and is given by two factors, the first one referring to the future evolution of corporate assets (and therefore becoming more and more uncertain as the risk horizon  $t$  increases), the second one originating from current informational asymmetries (hence, independent of the model's risk horizon); on the right-hand side, one can see that the position of the barrier depends on the company's estimated current leverage (a measure of financial risk), as well as on the net expected return on assets. More precisely, the barrier increases (and default becomes more likely) when the debt-holders estimate a higher current incidence of debt over total assets, when the firm has a higher cost of debt and/or a higher dividend rate.

### 3.3 The firm's leverage policy

In order to be able to estimate a company's future default risk, we need some assumptions on its leverage/dividend policy in the future. Since such a policy cannot be forecast by an outsider, we will assume, for simplicity, that the company's dividend and capital management policy will be set in such a way to keep the estimated leverage constant over time. This requires that the instantaneous net outflow  $e$  (due to dividend payments and share buybacks, net of capital increases) be set to

$$e = \alpha - r - \sigma^2 \quad [14]$$

To prove this, consider the expected leverage at time  $t$ :

$$E \left[ \frac{B_t}{V_t} \right] = E \left[ \frac{V_0 \hat{L} e^{-\frac{v^2}{2} + v\psi + (r-q)t}}{V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\varepsilon\sqrt{t}}} \right] = \hat{L} E \left[ e^{-\frac{v^2}{2} + v\psi + (r-q)t - \left(\mu - \frac{\sigma^2}{2}\right)t - \sigma\varepsilon\sqrt{t}} \right] = \hat{L} e^{(r-q-\mu)t + \sigma^2 t - v\sigma\sqrt{t} \text{cov}(\varepsilon, \psi)} \quad [15]$$

and assume that  $\psi$  and  $\varepsilon$  are uncorrelated, thereby getting:

$$E \left[ \frac{B_t}{V_t} \right] = \hat{L} e^{(r-\alpha+e)t + \sigma^2 t} \quad [16]$$

Now, if [14] holds, the expected leverage simply becomes:

$$E \left[ \frac{B_t}{V_t} \right] = \hat{L} \quad [17]$$

Based on [14], the survival condition [13] becomes

$$\sigma\varepsilon\sqrt{t} - v\psi > \ln \hat{L} - \frac{1}{2}(v^2 + \sigma^2 t) \quad [18]$$

### 3.4 The derivation of default probabilities

The next step in our model is to use the theoretical framework outlined above (and, namely, the survival condition in [18]) to derive an estimate of a company's default probability.

To keep complexity at a minimum, one might simply assume that all the company's debt is due at time  $t$ . This would enable us to derive the probability of default as

$$pr\left[\sigma\varepsilon\sqrt{t} - v\psi < \ln \hat{L} - \frac{1}{2}(v^2 + \sigma^2 t)\right] = \Phi\left[\frac{\ln \hat{L} - \frac{1}{2}(v^2 + \sigma^2 t)}{\sqrt{v^2 + \sigma^2 t}}\right] \quad [19]$$

(where  $pr[.]$  denotes probability).

However, this would be a very unrealistic scheme: since Black and Cox (1976) it is generally accepted that default may occur also before the final maturity date, e.g. because the borrower is not able to meet intermediate payments or covenants. We therefore have produced a discrete-time binomial approximation of our model, where default is modelled as a first-passage event. Details are provided in Appendix A.

Figure 1 shows an example of cumulative survival probabilities derived with our model, over a time horizon of 2 years (using a discrete time interval,  $\delta t$ , of one week).

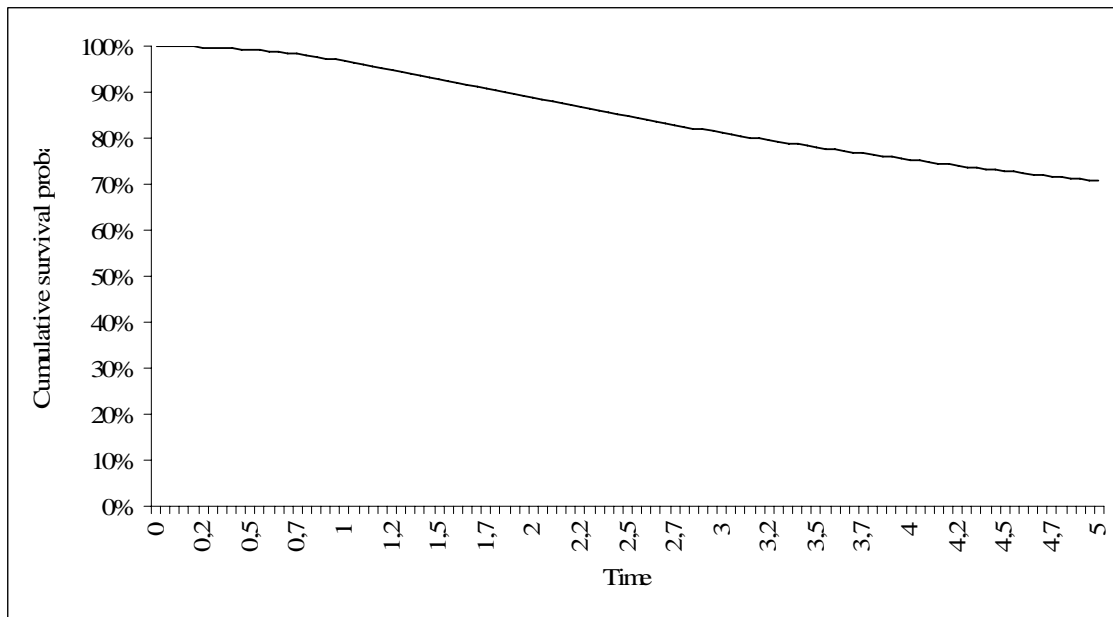


Figure 1: an example of survival probabilities derived with our model

(based on  $\mu = 6\%$ ,  $q = 3\%$ ,  $\sigma = 20\%$ ,  $\hat{L} = 50\%$ ,  $v = 20\%$ ,  $r = 5\%$ ,  $R = 50\%$ )

### 3.5 Fair credit spreads

Based on the survival probabilities derived in the previous section, we can now compute a measure of “fair” spread to be required against default risk. Since the credit spread on a risky exposure can be seen as the price of a credit risk swap, we use the latter as a conceptual framework.

Let  $c$  be the fair premium on a credit default swap maturing at time  $T\delta t$ . Assume the spread is paid at discrete time intervals,  $\delta t$ , which for the sake of simplicity are the same intervals used in the discrete-time model presented in Appendix A.

Expected costs, on a per-dollar basis, for the long party (holder of the CDS) will be<sup>3</sup>:

$$c \cdot \delta t \sum_{k=1}^T e^{-r \cdot k \cdot \delta t} s_{k\delta t} \quad [20]$$

(where  $s_{k\delta t}$  denotes the cumulated survival probability between time 0 and time  $T\delta t$ ) while expected per-dollar costs for the short party will be:

$$(1 - R) \sum_{k=1}^T e^{-r \cdot k \cdot \delta t} (1 - s'_{k\delta t}) \quad [21]$$

where  $s'_{k\delta t}$  denotes the marginal survival probability between time  $(k-1)\delta t$  and time  $k\delta t$  and  $R$  denotes the debt's recovery rate (which is assumed to be known, for the sake of simplicity).

By equating expected costs for the two parties one gets the fair spread as

$$c = \frac{(1 - R) \sum_{k=1}^T e^{-r \cdot k \cdot \delta t} (1 - s'_{k\delta t})}{\delta t \sum_{k=1}^T e^{-r \cdot k \cdot \delta t} s_{k\delta t}} \quad [22]$$

This formula will be used in the following paragraph to see how credit spreads behave in our model.

#### 4 Bank loans vs. bonds: results from our model

Based on the predictions of our model, we now investigate the behaviour of credit spreads for different categories of borrowers and lenders.

We consider borrowers belonging to seven rating classes, from AAA to CCC, as indicated in Table 1; for each class, the Table reports the mean and median asset volatility ( $\sigma$ ) and estimated leverage ( $\hat{L}$ ) at December 2005, based on a sample of 1,034

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<sup>3</sup> We assume that costs occur at the end of each period. Given that our period,  $\delta t$ , is very small and represents a discrete time approximation of continuous time, such an assumption is irrelevant.

rated non-financial companies. The sample was provided to us by Moody's KMV, a highly respected information provider on structural model-based credit risk analyses. As mean values are more subject to biases due to outliers, we decided to use the median values to feed our model.

*Table 1*  
**Borrower Characteristics for different rating classes**  
(December 2005)

<i>Rating Buckets</i>	<i>Leverage (<math>\hat{L}</math>)</i>		<i>Asset Volatility (<math>\sigma</math>)</i>		<i>Cases</i>
	<i>Mean</i>	<i>Median</i>	<i>Mean</i>	<i>Median</i>	
AAA	38%	32%	11%	12%	6
AA	42%	39%	13%	12%	25
A	44%	42%	14%	13%	161
BBB	42%	42%	15%	14%	302
BB	43%	41%	17%	17%	201
B	50%	48%	20%	19%	278
CCC or less	71%	60%	21%	19%	61
Total					1034

Source: Moody's KMV.

Note: M-KMV's "default point" is used as a proxy of debt in the computation of leverage; median values for leverage were smoothed (see Table 2) in order to make them strictly monotonic.

We also consider two types of lenders: banks and bond-holders. We assume that the former, thanks to their screening and monitoring activities, suffer from a lower degree of informational asymmetries, so that the stochastic disturbance  $\nu$  affecting their estimates of leverage is half as large as the one experienced by bond-holders. More specifically, while for bond-holders  $\nu$  will be assumed to be equal to  $\sigma$ , in the case of banks it will be set at  $\sigma/2^4$ . Note that this difference in  $\nu$  remains constant over time (in other words, informational disturbances of banks and bondholders do not converge), so that we are in fact assuming that banks not only benefit from better screening when originating the loan (as implied by the literature on relationship banking), but also preserve their informational advantage over bondholders through some superior monitoring abilities.

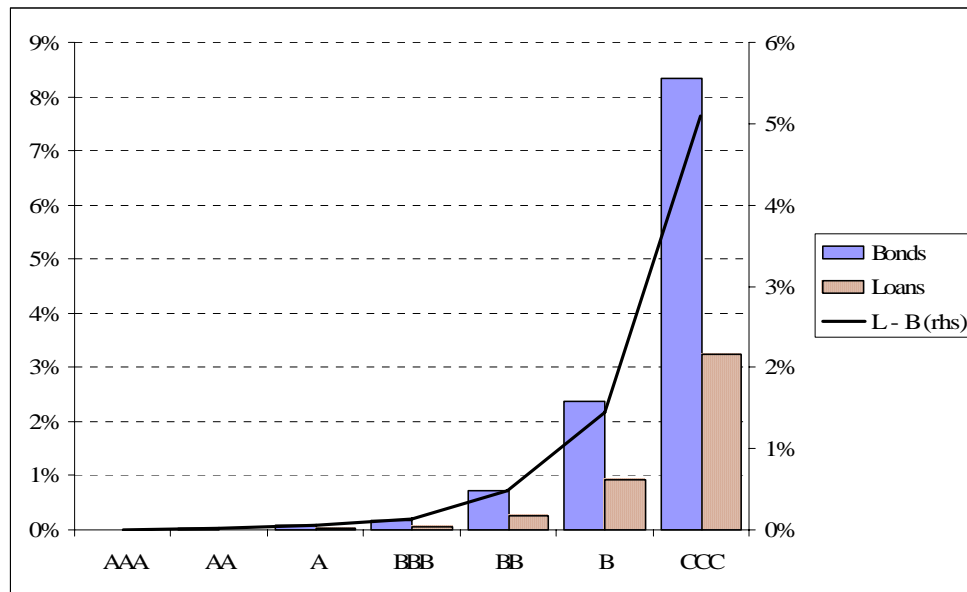
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<sup>4</sup> Experimenting with different values (provided that  $\nu$  is lower for bank loans) lead to similar results as those in Table 2.

In order to fully parametrise equations [A.1], [A.2] and [22], we also need to specify values for the cost of debt  $r$  (5%), the rate of debt outflow due to reimbursements and interest payments  $q$  (3%), the recovery rate  $R$  (50%) and the debt maturity  $t$  (5 years). Given these inputs, the model described in paragraph 3 leads to the results shown in Table 2 and Figure 2.

*Table 2*  
**Results from our model**  
*(December 2005)*

	AAA	AA	A	BBB	BB	B	CCC
<i>Borrower's characteristics</i>							
$\sigma$	12%	12%	13%	14%	17%	19%	20%
$\hat{L}$	32.0	39.0	40.0	41.0	42.0	48.0	60.0
<i>Credit spreads for a 5-year facility</i>							
Bank loans	0.0003%	0.006%	0.02%	0.05%	0.25%	0.93%	3.2%
Bonds	0.0018%	0.027%	0.08%	0.19%	0.73%	2.38%	8.3%
Loans - bonds	0.0016%	0.021%	0.06%	0.14%	0.48%	1.44%	5.1%



*Figure 2: spread curves for different rating categories*

It is important to stress that the results in the Table and Figure come from an “ideal” world where all debt characteristics stay constant as the borrower’s rating worsens; in real life, banks and bond-holders tend to reduce the risk of lending to low-rating firms by increasing the protection they get from, say, collateral (which increases the expected recovery rate  $R$ ), guarantees, or shorter maturities.

The results show that credit spreads tend to increase with higher leverage and asset volatility, and such an increase is considerably more dramatic in the case of bonds. In fact, bond-holders, who do not have the same screening and monitoring skills of banks, are faced with a larger amount of informational asymmetries. The effect of such informational noise becomes more and more substantial as the borrower's characteristics get worse, and this increasing uncertainty faced by bond-holders leads to a sharper rise in credit spreads.

This suggests that the competitive advantage of banks over securities markets becomes more sizeable in the funding of low-quality firms. In short, our model seems to indicate that, notwithstanding the growth of the market for speculative-grade corporate bonds and the increasing appetite of investors and asset managers for risky corporate debt to increase their portfolio "alphas", banks do enjoy a structural "leadership" when it comes to financing riskier firms.

The next paragraph investigates whether such a model-derived hypothesis is supported or rejected by the empirical analysis of a large sample of corporate debts, including both bank loans and bonds.

## **5 An empirical test of the model**

In order to empirically test whether the behaviour dictated by our model is consistent with real-life spreads, we now analyse a unique dataset comprising both bank loans and bonds, issued over a period of 12 years. By means of a multivariate model, we control for any variable that might affect spreads and to isolate the loan/bond gap for borrowers belonging to different rating classes.

In this paragraph, we first describe the sample used in our analysis; we then move to our multivariate model and, finally, to the empirical results.

### *5.1 Sample structure*

Our sample includes 7,926 Eurobonds and 5,469 syndicated loans<sup>5</sup> originated between 1991 and 2003. These are taken from the "Bondware" and "Loanware" database

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<sup>5</sup> One could argue that syndicated loans possess qualities of both public and private debt (Zhang, 2003), as they represent a "bundle" of loans to a group of institutional investors. However, what is most relevant for our analysis is that in the case of bank loans, an active screening and monitoring activity of the

maintained by Dealogic. Note that all bonds with special features (e.g. callable bonds, perpetual bonds, floating rate bonds) affecting their price have been discarded.

Table 3 shows a breakdown by year: the number of issues tends to increase over time (note that the last two bands include only two years), even though loans originated during the last months of 2003 are missing from our database.

*Table 3*

**Sample breakdown by time period and exposure type**

	<i>Bonds</i>	<i>Loans</i>	<i>Total</i>
1991-93	888	220	1,108
1994-96	1,494	1,169	2,663
1997-99	2,092	1,413	3,505
2000-01	1,662	1,682	3,344
2002-03	1,790	985	2,775
Total	7,926	5,469	13,395

The country of incorporation of the borrower is shown in Table 4: while bonds tend to be almost evenly distributed across the major countries, loan data are taken mainly from the US.

*Table 4*

**Sample breakdown by country and exposure type**

<i>Country</i>	<i>Bonds</i>	<i>Loans</i>	<i>Total</i>
Canada	325	153	478
France	788	36	824
Germany	1,018	14	1,032
Japan	761	36	797
The Neth	579	21	600
UK	969	186	1,155
US	1,767	3,934	5,701
Other	1,719	1,089	2,808
Total	7,926	5,469	13,395

---

lending banks is present. In the case of syndicated loans, this activity is partly delegated by the participating banks to the lead bank(s). However, given the informational asymmetry and moral hazard problems related to loan syndication (see Gorton and Pennacchi, 1995, and, more recently, Sufi, 2005), both the lead bank and the participating banks are motivated to use their superior information-processing abilities in order to avoid losses.

Finally, Figure 3 shows a frequency distribution by rating class, with loans spanning the rating spectrum much more evenly than bonds (for which we observe a stronger concentration in investment-grade classes, mainly AAA). This sounds as an indirect confirmation of the findings of our model, where banks enjoy a stronger comparative advantage for low-quality borrowers, and is consistent with the theoretical literature on the role played by bank loans versus directly placed debt in the funding of differently rated borrowers (Diamond, 1991).

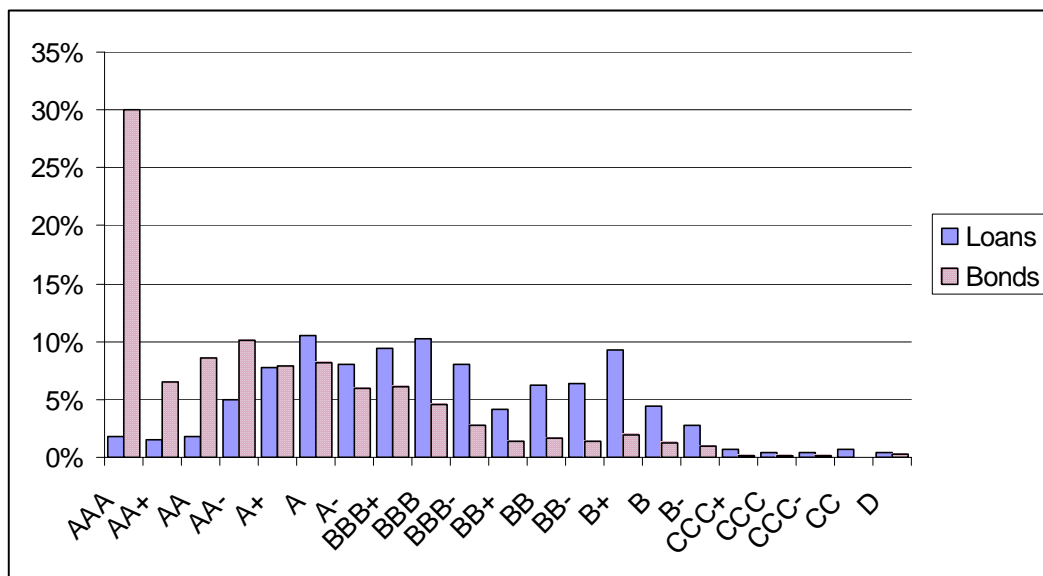


Figure 3: the rating mix of loans and bonds in our sample

## 5.2 Model specification and variables

The dependent variable in our model is given by the spread at issuance. The use of secondary-market spreads is avoided because of the relatively poor liquidity of this market for many smaller facilities.

In the case of bonds, spreads will be measured as “nearest-on-the-run” spreads (that is, as the difference between the yield to maturity at issuance of each individual Eurobond and the yield to maturity of the Treasury bond denominated in the same currency and with the nearest maturity). Spreads on loans are computed over Libor base rates<sup>6</sup>; they include the facility fee, when present.

<sup>6</sup> Loan spreads were computed as the spread over Libor, plus the facility fee. This does not include any upfront fee, such as underwriting and participation fees.

To explain spreads, we will make use of several groups of variables:

- a) variables that are common to both bonds and loans. These include a set of rating dummies<sup>7</sup> (see Table 5 below), the original maturity of the bond/loan, the total amount outstanding (expressed as the natural log of the amount in USD), two dummy variables for secured and subordinated exposures, the average share underwritten/retained by each financial institution participating in the bond issuance management group or the loan syndicate;
- b) bond-specific variables, namely the coupon rate and a dummy indicating whether the bonds are registered. These should proxy for the different expected tax treatment;
- c) loan-specific variables. These include dummies for multi-tranche/guaranteed/revolving facilities, as well as for loans with a sponsor (e.g., in project financing) and/or on which a utilization fee is required. Other variables relate to the purpose of the loan (including acquisitions, LBOs, refinancing, project finance, trade financing, working capital and “debtor-in-possession”) and to the amount of commitment fees charged on it;
- d) four sets of dummies associated with the quarter in which credit exposures were originated (to account for overall market conditions), the country of incorporation of the borrower (accounting for different regulatory and fiscal environments), the currency in which the bonds/loans are denominated and the industry in which the debtor operates. Country, currency and industry dummies are shown in more detail in Table 5<sup>8</sup>.

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<sup>7</sup> To avoid a severe reduction in our empirical sample, we had to content ourselves with issuer ratings in the case of syndicated loans, where facility ratings were missing for a high share of cases.

<sup>8</sup> The Table does not include coefficients for quarterly dummies (only an overall F-test is reported): however, a dummy was included in the model for each quarter between 1991/II and 2003/IV (1991/I was left out to avoid perfect collinearity). Also, some industry dummies were included that are not reported in the Table to save room, since they were not significantly different from zero. The complete list of industry dummies tested is as follows: Automobile, Building Societies, Banks, Chemicals, Computers, Constructions, Electronics and Electrics, Food and beverages, Financial cos. and holding cos., public entities other than governments and sovereigns, Health and pharmaceuticals, Hotels e Leisure, Industrials, Insurance, Oil and mines, Retail, Telecommunications, Trasportation, Energy and Utilities.

### 5.3 Main empirical results

Table 5 reports our estimates. Note that, as in Morgan and Stiroh (1999), some coefficients were estimated separately, by means of a set of multiplicative dummies, for the two subgroups in our sample (bonds and loans); the hypothesis that they are not statistically different is tested (and usually rejected) in the last column of the Table.

While we would not lay too much emphasis on the differences between the constant terms (given that spreads on loans and bonds are computed through different approaches, and may therefore not be fully comparable to each other), some results appear noteworthy.

Spreads increase as ratings worsen. This is true for both bonds and loans; however, the spread/rating link is quite steeper for the former, while the risk premium required by banks on low-quality loans appear much milder than the one demanded by private investors in the bond market. This looks fully consistent with the results of our simple structural model: we will return to this in the concluding paragraph of this paper.

These different spread/rating relationships are visualized in Figure 4, where the equations estimated in Table 5 are used to simulate spreads on bonds with different ratings and on the corresponding loans. As can be seen, the behaviour of empirical spreads on bank facilities and public debt is similar to the one dictated by our theoretical model (see Figure 2), in that they tend to diverge as default risk increases. This suggests that, notwithstanding the huge growth experienced by capital markets (and their increased ability to finance riskier/younger companies), banks still prove more efficient in deploying those screening and monitoring abilities which help them select and protect their credit exposures, funding riskier customers at more sustainable rates.

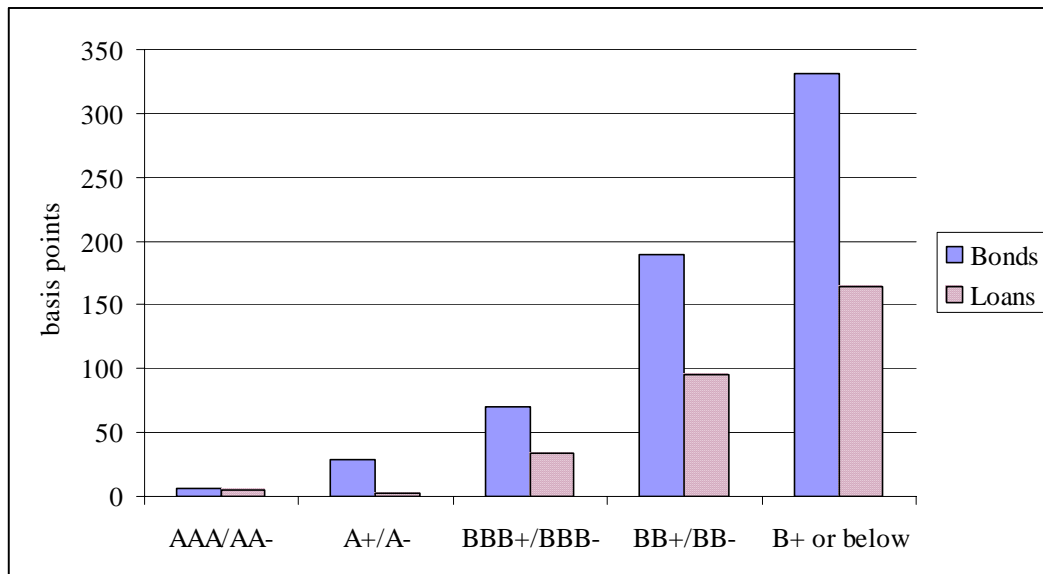


Figure 4: rating buckets and “typical” spreads

The maturity premium looks much larger for bank facilities than it is for bonds (where long maturities are more usual than in the syndicated loan market<sup>9</sup>). Note that, consistent with Fons (1994), we tested the hypotheses that the spread/maturity link be different for poorly-rated exposures: our data, however, seem to lend very little support to this assumption.

Regarding the amount, opposite results emerge for bonds and loans. The coefficient is significantly positive for the former, suggesting that a supply-side effect (due to the rigidity of the demand, larger issues are harder to place and have to pay a relatively higher spread) may prevail over liquidity issues. This sounds plausible for the Eurobond market, where many investors tend to hold securities until their final maturity, and are therefore, to some extent, indifferent to their secondary-market liquidity. As regards loans, spreads appear to decrease as size increases: besides liquidity issues, this may reflect the scale economies implied in information-gathering, screening and monitoring costs which are typically associated with lending.

<sup>9</sup> The average maturity for the bonds in our sample is 98 months, as opposed to 39 months for loans.

Table 5  
Multivariate results

Variable	Common		Bond-specific		Loan-specific		Test for $H_0$ : Bond = Loan
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	P-value
Constant			-190.75	0.0%	-17.89	6.8%	0.0%
Rating A+/A-			22.48	0.0%	-2.00	35.8%	0.0%
Rating BBB+/BBB-			63.31	0.0%	29.88	0.0%	0.0%
Rating BB+/BB-			176.16	0.0%	102.65	0.0%	0.0%
Rating B+ or below			316.87	0.0%	171.04	0.0%	0.0%
Maturity			0.02	6.7%	0.41	0.0%	0.0%
Maturity only if rating < BBB-			0.09	35.7%	-0.15	15.5%	9.4%
Total amount (log)			2.31	0.8%	-6.30	0.0%	0.0%
Subordinated exposures			13.25	0.0%	167.67	0.5%	1.0%
Average share			9.24	0.0%	24.46	0.0%	0.0%
Secured			4.98	25.5%	23.39	0.0%	0.6%
Coupon			25.55	0.0%			
Registered bonds			11.16	0.0%			
Maturity extension					36.79	1.6%	
Multitranche					71.93	0.0%	
Commitment fee					0.80	0.0%	
Utilization fee					-9.19	0.0%	
Guarantees					-10.07	10.5%	
Sponsor					48.84	0.0%	
Revolving					-29.64	0.0%	
Purpose: DIP					47.15	0.0%	
Purpose: acquisitions					23.08	0.0%	
Purpose: LBO					25.89	1.9%	
Purpose: refinancing					5.55	1.5%	
Purpose: project finance					-54.69	0.0%	
Purpose: trade financing					-59.07	0.0%	
Purpose: working capital					-18.77	0.0%	
Joint F-test					39.10	0.0%	
Quarterly dummies	-						
Joint F-test	53.53	0.0%					
Borrower country: US	8.62	0.0%					
Borrower country: UK	-8.26	0.0%					
Borrower country: Can	-1.65	60.2%					
Borrower country: Ger	1.96	30.5%					
Borrower country: Fra	-8.59	0.0%					
Borrower country: Jap	1.93	54.9%					
Borrower country: Net	21.01	30.3%					
Joint F-test	10.14	0.0%					
Currency: USD	5.06	6.3%					
Currency: DM	-10.71	0.0%					
Currency: EUR	15.31	0.0%					
Currency: FF	7.48	0.4%					
Currency: GBP	-10.55	0.1%					
Currency: Can \$	10.22	0.0%					
Joint F-test	12.45	0.0%					
Industry*: banks	-18.72	0.0%					
Industry*: chemicals	-15.86	0.0%					
Industry*: computers	-22.70	0.0%					
Industry*: constructions	-14.34	0.2%					
Industry*: public entities	-27.89	0.0%					
Industry*: pharmaceuticals	-9.38	4.7%					
Industry*: industrials	-11.16	0.6%					
Industry*: transportation	-24.61	0.0%					
Joint F-test	11.98	0.0%					
R-square	76.2%						
Adjusted R-square	76.0%						
Log-likelihood	-74371						
F-statistic	354.1	P-value	0.0%				

\* only industries for which dummies are 5%-significant have been reported. Coefficient p-values are based on t tests with White heteroskedasticity-corrected standard errors.

Subordinated exposures have to pay more than senior ones. This is especially true for loans, and may reflect the fact that senior syndicated loans enjoy higher recovery rates than bonds (Acharya et al., 2003), so that there is more scope for larger losses in the case of subordinated exposures.

The average share's coefficient is positive and statistically significant, suggesting that, on average, a low number of arrangers denotes higher spreads (as stronger difficulties in placing the loan/bond are probably being experienced). The effect is stronger for loans, as banks participating in the syndicate are likely to end up retaining the purchased exposure on their own balance sheets (while arrangers on the bond market only face a temporary underwriting risk). Larger syndicates may be associated with lower spreads also because (as pointed out by Coleman et al., 2002) they suffer from a decline in contractual flexibility.

Secured exposures appear to pay a higher spread than unsecured ones. While this may look a somewhat counterintuitive result, it is fully consistent with a wide stream of literature, including Casolaro et al. (2002), Yi and Mullineaux (2002), and dating back to Berger and Udell (1990)<sup>10</sup>. In fact, higher spreads and collateral tend to complement – rather than offset - each other in rewarding/limiting higher credit risk; riskier borrowers are therefore charged higher rates, while being asked to provide extra collateral. This is true even within our multivariate framework, which already controls for ratings; a similar result was found, for bonds only, by John, Lynch and Puri (2002).

As regards bond-specific variables, coupon rates and registered issues work as expected. Holders of registered bonds may find it more difficult to avoid being taxed, therefore ask for higher premia; moreover, since in most countries capital gains are taxed at the time of sale, bonds with lower coupons may be more valuable because some taxes are postponed.

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<sup>10</sup> Based on bank loans data, the paper shows that interest rates on secured loans are on average higher than those on unsecured exposures; this suggests guarantees are not enough, by themselves to offset a higher credit risk, and hence may be associated with higher spreads. See also Pozzolo (2001) and the references therein.

Turning to loan-specific features, maturity extension clauses increase the cost to the borrower (which looks correct, since they embed an option); also, multitranches facilities tend to pay higher rates (probably because they include one or more junior tranches facing speculative risks). Commitment fees tend to move together with spreads; this represents an expected result, as a higher level of risk (as well as a stronger bargaining power by the lending syndicate) is likely to affect the cost of both the drawn and the undrawn part of a loan. On the other hand, utilization fees (linked to the average level of utilisation during a specified period of time, and intended as a disincentive for the borrower to drawdown a back up line beyond a certain point) reduce spreads, as they help to reduce risk; personal guarantees are also found to have a (very weak) negative effect on spreads.

The presence of a sponsor (that is, a party wishing to develop the project being funded by the loan, often a public sector entity) is associated with higher spreads. This may be due to a mechanism like the one discussed for secured exposures: higher rates and the presence of a sponsor may be complementary devices used to reward/limit high risks.

Finally, revolving loans are found to be less expensive than term facilities (in line with Coleman, Esho and Sharpe, 2002), as they give the bank an extra monitoring tool (payments flowing in and out of the credit line) and make it easier for credit officers to trigger prompt recovery actions<sup>11</sup>.

## **6. Concluding remarks**

In this paper, a simple structural credit risk model was designed, to illustrate the effect on credit spreads of the different screening/monitoring abilities of banks and bondholders.

A wide empirical sample was then analysed, to see if the model's prescription were validated or rejected by real-life data.

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<sup>11</sup> Note that a dummy representing covenants was excluded from our model as it did not prove statistically significant (with a p-value of almost 40%). This may be due to the large variety of covenants included in loan contracts and their different effectiveness, as well as to their ambiguous effect on spreads: on one hand, they reduce risks, so may prompt lower rates; on the other hand, they are imposed on the most risky transactions, so, like collateral, they may be associated with higher spreads.

Our empirical estimates do confirm the key finding of the model, that is, that the spread/rating link is quite steeper for bonds than for banks. In other words, by better controlling the informational noise surrounding the borrower's real economic conditions, banks manage to keep the risk premium required on low-quality loans to much lower levels than the one demanded by private investors in the bond market.

The empirical model estimated in paragraph 5 also leads to the following other interesting results:

- the maturity premium looks much larger for bank facilities than for bonds (where long maturities are more usual than in the syndicated loan market); this is consistent with the fact that banks can express superior monitoring abilities only if the analysis of issuing companies is renewed over time, strengthening the bank/customer relationship by means of a repeated game;
- while larger bond issues tend to be associated with higher spreads, syndicated loans' spreads appear to decrease as size increases: besides liquidity issues and supply-side effects, this may reflect the scale economies implied in information-gathering, screening and monitoring costs, which are typically associated with bank lending;
- while subordinated exposures have to pay more than senior ones for both bonds and loans, this is especially true for the latter, and may reflect the fact that senior syndicated loans enjoy higher recovery rates than bonds, therefore have more to lose from an increase in the expected loss rate given default.

Overall, it seems to us that such findings lend a strong empirical support to the view that banks and capital markets are not perfect substitutes in funding non-financial companies, but rather specialize in different market niches. Bonds can be more effective in financing investment-grade companies in their long-term projects, while bank loans can bridge the gap separating riskier producers from private savings, using superior monitoring devices (including, possibly, shorter maturities) to reduce the risks incurred and the spreads charged to borrowers.

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### Appendix A: a discrete-time binomial first-passage model

Consider a small increase in time  $\delta t$  (e.g., one day). A discrete-time representation of [4] is to assume that, over each  $\delta t$ , the asset value can take one of the following two values:

$$V_{t+\delta t} = \begin{cases} uV_t & \text{with prob } p = \frac{e^{\mu\delta t} - d}{u - d} \\ dV_t & \text{with prob } 1 - p \end{cases} \quad [\text{A.1}]$$

where:

$$u = e^{\sigma\sqrt{\delta t}}$$

$$d = e^{-\sigma\sqrt{\delta t}}$$

As shown by Cox et al. (1979), this will ensure that the standard deviation of the distribution for  $\log V_t$  be  $\sigma$ , as expected.

Equation [A.1] gives rise to the binomial tree for  $V$  depicted in Figure A.1.

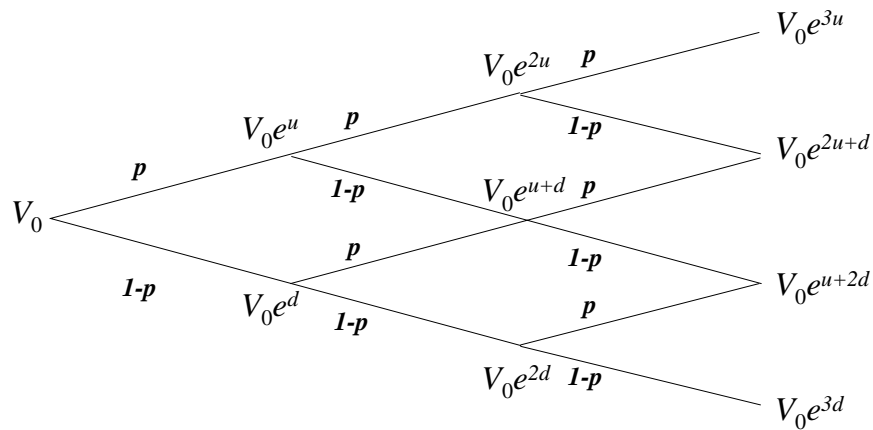


Figure A.1: a representation of our discrete-time binomial tree

As shown in the Figure, we have one value for  $V_0$ , two values for  $V_{\delta t}$ , three values for  $V_{2\delta t}$ ,  $k+1$  values for  $V_{k\delta t}$  and so on.

Any possible value for  $V_{k\delta t}$  is compared to the distribution of possible values for corporate debt. The latter is derived by recalling that, from [6], [11] and the definition of  $L$ , it follows that:

$$B_0 = V_0 \hat{L} e^{-\frac{v^2}{2} + v\psi} \quad [\text{A.2}]$$

which can be transposed into our discrete-time world as follows:

$$B_{k\delta t} = B_0 e^{(r-q)k\delta t} = V_0 \hat{L} e^{-\frac{v^2}{2} + v\psi + (r-q)k\delta t} \quad [\text{A.3}]$$

Consider any of the  $k+1$  values for  $V_{k\delta t}$  and call it  $V_{k\delta t}^*$ . The survival probability ( $s'$ ) conditional on this  $V_{k\delta t}^*$  is simply

$$\begin{aligned} s' \Big|_{V_{k\delta t}^*} &= pr \left[ V_{k\delta t}^* > V_0 \hat{L} e^{-\frac{v^2}{2} + v\psi + (r-q)k\delta t} \right] = pr \left[ \log \frac{V_{k\delta t}^*}{V_0 \hat{L} e^{-\frac{v^2}{2} + (r-q)k\delta t}} > v\psi \right] = \\ &= pr \left[ \frac{1}{v} \log \frac{V_{k\delta t}^*}{V_0 \hat{L} e^{-\frac{v^2}{2} + (r-q)k\delta t}} > \psi \right] = \Phi \left[ \frac{1}{v} \log \frac{V_{k\delta t}^*}{V_0 \hat{L} e^{-\frac{v^2}{2} + (r-q)k\delta t}} \right] \end{aligned} \quad [\text{A.4}]$$

Note that  $s'$  is a *marginal* survival probability (that is, the probability associated with survival at time  $k\delta t$  only). In fact, by assuming that the asset value is  $V_{k\delta t}^*$ , we are implicitly assuming that the borrower has already survived until time  $(k-1)\delta t$ .

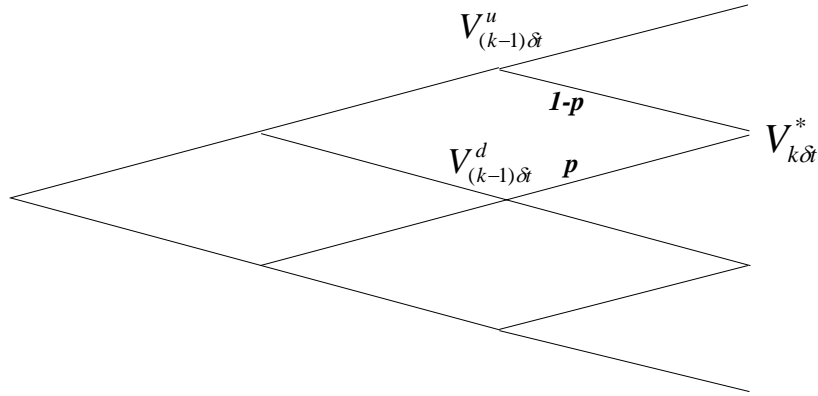


Figure A.2: an example of values leading to a given  $V_{k\delta t}^*$

Now, consider the two values for  $V_{(k-1)\delta t}$  that might lead to  $V_{k\delta t}^*$ , that is (see an example in Figure 2):

$$V_{(k-1)\delta t}^u \Big|_{V_{k\delta t}^*} = V_{(k-1)\delta t}^u d$$

$$V_{(k-1)\delta t}^d \Big|_{V_{k\delta t}^*} = V_{(k-1)\delta t}^d u$$

and denote with  $s \Big|_{V_{(k-1)\delta t}^u}$  and  $s \Big|_{V_{(k-1)\delta t}^d}$  the cumulative survival probabilities associated with those two values.

In other words, let  $s \Big|_{V_{(k-1)\delta t}^u}$  denote the probability that the borrower has survived between time zero and time  $(k-1)\delta t$  and that, at time  $(k-1)\delta t$ , the value of her assets be  $V_{(k-1)\delta t}^u$ .

The same meaning holds for  $s \Big|_{V_{(k-1)\delta t}^d}$ <sup>12</sup>.

It is easy now to observe that the cumulative survival probability associated with the value  $V_{k\delta t}^*$  will simply be

$$s \Big|_{V_{k\delta t}^*} = \left[ p \cdot s \Big|_{V_{(k-1)\delta t}^d} + (1-p) s \Big|_{V_{(k-1)\delta t}^u} \right] s' \Big|_{V_{k\delta t}^*} \quad [\text{A.5}]$$

This simple relationship, together with the fact that  $s \Big|_{V_0^d} = s \Big|_{V_0^u} = 1$ , will enable us to compute (by backward recursion) the cumulative survival probability associated with any  $V_{k\delta t}^*$ .

Finally, to learn the cumulative survival probability associated with time  $k\delta t$ , we will simply have to add up all the conditional survival probabilities associated with the different values that  $V^*$  can take at time  $k\delta t$ :

$$s_{k\delta t} = \sum_{V_{k\delta t}^*} s \Big|_{V_{k\delta t}^*} \quad [\text{A.6}]$$

These are the survival probabilities shown in Figure 1 in the paper and used to compute “fair” spreads.

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<sup>12</sup> Note that, if  $V_{k\delta t}^*$  is an “extreme” value,  $V_{(k-1)\delta t}^u$  or  $V_{(k-1)\delta t}^d$  might as well not exist. For example, if  $V_{k\delta t}^*$  is the highest value that  $V$  can take at time  $k\delta t$ , no  $V_{(k-1)\delta t}^d$  will exist, such that  $V_{(k-1)\delta t}^d e^u = V_{k\delta t}^*$ . In such cases, the associated survival probability  $s \Big|_{V_{(k-1)\delta t}^d}$  will, by definition, be zero.