THE EFFICIENCY & WELFARE FOUNDATIONS OF FREEZEOUT LAWS IN TAKEOVERS

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Abstract

We provide an economic basis for permitting freezeouts of non-tendering shareholders following successful takeovers. We describe a specific freezeout mechanism that is based on easily verifiable information, making it simple to implement in practice. We show that this mechanism induces desirable efficiency and welfare properties in models of both corporations with widely-dispersed shareholdings (as in Grossman and Hart, 1980) and corporations with large pivotal shareholders (as in Bagnoli and Lipman, 1988), and that it strictly dominates previous proposals along some important dimensions. The mechanism we describe is very closely related to the practice of takeover law in the US.
1 Introduction

In a well-known paper, Grossman and Hart (1980) note that free-riding considerations could thwart takeovers of widely-held corporations where each shareholder is negligibly small. Briefly put, target shareholders can reasonably anticipate that the post-takeover value of the raider-run firm will be higher than the tender price (otherwise the raider loses). Thus, they are better off holding on to their shares and free-riding on the raider’s improvement in firm value than tendering them. As a result, no shareholder will tender and raids cannot succeed even if they are potentially value-increasing. These observations indicate that to enable takeovers to succeed an exclusionary device is needed that deters free-riding by “punishing” non-tendering shareholders.

However, any exclusionary device that has an impact on free-riding ipso facto also carries other important economic implications: by influencing the likelihood of successful takeovers, it affects the incentives of raiders to mount takeovers and incumbent managers to generate firm value, and, more generally, is a key determinant of overall welfare levels. Efficiency considerations further demand that a proper exclusionary device encourage only value-enhancing takeovers (those where the value of the raider-run firm exceeds the value under current management) and deter value-decreasing ones. Thus, the central issue is not just identifying an exclusionary mechanism that deters free-riding, but one that also satisfies other desirable properties. This question was the focal point of the Grossman–Hart analysis but oddly has received next to no attention in the subsequent literature (see Section 2 below). It forms the subject matter of our paper.

We propose and analyze an exclusionary mechanism that relies on a particular freezeout of non-tendering shareholders.\(^1\) Specifically, we assume that in the event of a successful takeover, non-tendering shareholders receive only the maximum of two values: (a) the target company’s share price immediately prior to the tender offer, and (b) the tender offer itself. This simple rule faces no informational or legal obstacles to its implementation. It is evidently informationally undemanding: the two values required for its implementation are trivially both observable and verifiable. It is also legally permissible; indeed, as we discuss further in Section 8, it is consistent with the structure of actual tender offers in the US. In this paper, we show that it also induces powerful efficiency and welfare properties in equilibrium.

In the first step of our analysis, we examine the impact of this mechanism in the context of a firm with widely-dispersed shareholdings as in Grossman and Hart (1980). We adopt a framework that is basically similar to Grossman and Hart’s: the only exogenous variable in the model is the ex-ante distribution of values of a raider-run firm. All other quantities such as the value of the firm under incumbent management, the ex-ante probability of a successful raid, and the firm’s share price prior to a raid being mounted, are determined endogenously in equilibrium as a function

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\(^1\)In a freezeout, the shares of non-tendering shareholders can be bought out by a successful raider at a price determined by pre-specified rules.
of the exclusionary mechanism in place. Grossman and Hart propose an exclusionary mechanism based on dilution in which a successful raider is allowed to transfer to himself a fixed dollar amount from the post-takeover value of the firm. If this dilution lowers the residual value of the firm below the tender price, free-riding is inhibited. Grossman and Hart identify and characterize the ex-ante optimal (social-welfare maximizing) level of dilution in their model.

We show that our mechanism has all the desirable efficiency and welfare properties of the optimal device proposed by Grossman and Hart, but without the latter’s important shortcomings. Specifically, we show that our proposed freezeout rule induces a unique equilibrium. Ex-post efficiency is assured in this equilibrium: takeovers succeed when, and only when, they are value-enhancing. This equilibrium also has welfare and incentive implications that, in a literal sense, coincide with those under the optimal dilution level of Grossman–Hart. In particular, both devices offer the same incentives to incumbent managers for value generation; and they lead to identical levels of aggregate social welfare.

On the other hand, the Grossman–Hart mechanism has three serious shortcomings that are not present in our model. First, to ensure that shareholders do not tender at any price, Grossman and Hart impose an ad-hoc lower bound on acceptable tender offers. While this bound eliminates inefficient outcomes by assumption, it depends centrally on information that is unverifiable, and arguably even unobservable, by a third party such as a court of law. Second, this constraint, while arbitrary, is critical. We show that if it is dropped, new equilibria arise that could fail to be efficient in two ways: value-increasing takeovers could fail, while value-decreasing ones could succeed. This multiplicity of equilibria moreover creates another problem: it is no longer apparent how to choose a single optimal level of dilution, since dilution levels that are optimal conditional on one equilibrium are no longer so if a different equilibrium is played. Third, the Grossman–Hart solution to the free-rider problems—their dilution device—is in violation of US law, making it unimplementable from a legal standpoint. This indicates why exclusionary devices along these lines are never observed in practice.

Our rule is notably more “benevolent” to target firm shareholders than the dilution mechanism of Grossman and Hart. The Grossman-Hart mechanism resorts to the use of a threat of a severe penalty to non-tendering shareholders of (possibly total) dilution of their post-takeover wealth. Our rule does not penalize the non-tendering shareholders compared to those who tender; rather, it eliminates free-riding by limiting the gain of the non-tendering shareholders. It is therefore remarkable that without a threat of penalty and without our rule being dependent on the unverifiable information that is essential for the Grossman-Hart model to work, our rule achieves at least the same objective as they do.

In the second step of our analysis, we show that our proposed solution also achieves dominant outcomes in the case of a corporation with “large” shareholders in which each shareholder is potentially pivotal. Bagnoli and Lipman (1988), who introduced this case, show that exclusion is
not required to eliminate free-riding in this case.\textsuperscript{2} In particular, they show that there are always Nash equilibria of the game without exclusion in which takeovers succeed. However, some of these equilibria may involve the use of mixed-strategies, creating inefficient outcomes in which value-increasing takeovers fail to occur with positive probability. On the other hand, they show that introducing a Grossman–Hart style exclusionary mechanism into this world may create another inefficiency by permitting value-reducing takeovers to succeed.

We show that both kinds of inefficiencies are eliminated by our freezeout mechanism. Value-reducing takeovers cannot succeed under our mechanism, thus eliminating the inefficiencies of the sort created by the Grossman–Hart dilution mechanism. Second, value-improving takeovers always succeed, so the inefficiencies associated with having no exclusionary mechanism also do not arise.

Since our solution is consistent with US legal practice, our analysis may be viewed as providing an economic foundation for the current takeover law in the US. Our analysis also raises a question with respect to the robustness of a number of theoretical studies in the literature that have offered explanations of various documented empirical regularities seen in takeovers. The models in these studies enable free-riding by non-tendering shareholders and, indeed, free-riding is often central to sustaining candidate equilibria. Since freezeouts that are admissible by law and that eliminate free riding are not modelled, the question arises: to what extent do the arguments in these studies retain force given that freezeouts are permitted? We do not analyze this question in detail, but a brief reading suggests that its impact may be substantial.

The remainder of this paper is organized as follows. Section 2 indicates the related literature and relates our paper to some of the earlier work. Section 3 presents the model that forms the subject of our analysis. Section 4 describes and characterizes our proposed efficient resolution of the free-rider problem. Section 5 presents and evaluates the Grossman–Hart solution and compares our solution to theirs. Section 6 examines the implications of the freezeout rule in a Bagnoli-Lipman (1988) world of corporations with large, pivotal shareholders. Section 7 makes a comment on the irrelevance for our solution of perfect information regarding the value of the raider-run firm. Section 8 provides a description of US takeover law and relates our freezeout rule to the law. Section 9 concludes.

2 Literature Review

In this section we briefly discuss studies that relate to Grossman and Hart’s (1980) free-rider problem in takeovers. (The Grossman–Hart analysis is itself is described in Section 5.) A central

\textsuperscript{2}That the pattern of large shareholding makes a difference in takeovers has also been empirically documented. See, e.g., Brickley, Lease, and Smith (1988).
objective of the Grossman–Hart paper, as also the present paper, is to identify and characterize the proposed resolution of the free-rider problem from an efficiency/welfare perspective. In contrast, much of the literature following the Grossman–Hart analysis has centered attention on the free-rider problem itself, examining factors that could mitigate its impact. We discuss some of these papers below.

As mentioned above, Bagnoli and Lipman (1988) show that if the target firm has a finite number of shareholders (so any shareholder is potentially pivotal), takeovers can succeed without a dilution mechanism (see also Bebchuk, 1989). Some of the Nash equilibria involve the use of mixed-strategies, so outcomes in the absence of an exclusionary mechanism may be stochastic. Bagnoli and Lipman also show that the introduction of Grossman–Hart’s exclusion mechanism makes inefficient (specifically, value-reducing) takeovers feasible. The Bagnoli-Lipman setting and the impact of our rule in this model are discussed in Section 6.

Shleifer and Vishny (1986) show that takeovers can succeed without exclusion in some cases if the raider has accumulated a fraction of the target firm’s shares prior to the takeover attempt. They assume that the actual improvement in firm value from a successful takeover is private knowledge and known only to the raider. Other shareholders rationally forecast a distribution of possible values, and use this to determine their responses to a tender offer. In the absence of an exclusionary mechanism, free-riding by shareholders implies that the raider cannot realize a profit (on average) on the shares purchased in a successful takeover attempt, but even in this case, he can profit from the increase in firm value on his original shareholding. Thus, if the latter quantity is large enough, takeover attempts can succeed even without exclusion.

In the Shleifer–Vishny model, only bids that are sure to succeed are made. Hirshleifer and Titman (1990) extend the Shleifer–Vishny model to one where tender offers are made that may sometimes fail owing to either the use of randomized strategies by shareholders or information asymmetries that prevent the raider from knowing the reservation prices of the shareholders. Ravid and Spiegel (1999) look at allowing the bidder to strategically determine the size of its toehold in the target firm prior to the takeover, assuming that its acquisition affects the market price of the stock and that there is possible competition from other bidders. Toehold provides insurance should entry of rival bidders occur and increase the price the rival has to pay. They derive the optimal toehold acquisition strategy of the bidder given the likelihood of entry of rival bidders and their valuation of the target.

Cornelli and Li (2001) and Gomes (2001) look at the role of risk-arbitrageurs in takeovers. In their models, risk-arbitrageurs play the role of large shareholders who can potentially hold out on tender offers; the remaining shareholders are negligible. The number of arbitrageurs who enter in equilibrium and the number of shares they buy are both endogenously determined. Cornelli and Li show that the presence of risk-arbitrageurs can resolve the free-riding problem. They derive several implications of their resolution of the free-rider problem including the size of the takeover
premium. The Cornelli–Li framework, however, does not incorporate any exclusionary mechanism. If freezeouts of the sort permitted by the law were allowed, there is a superior equilibrium strategy for the raider which by-passes the risk-arbitrageurs altogether. Gomes focuses on the effect of the arbitrageurs on the size of the takeover premium; he shows, among other things, that it rises in the quantity of shares that they hold prior to the takeover announcement as well as of the illiquidity of the target stock.

Ravid and Spiegel (1999) and Gomes (2001) are, to our knowledge, the only papers that incorporate freezeouts in their analyses. However, their objectives are different from ours, and in a sense, our analyses are complementary. Ravid/Spiegel and Gomes take the freezeout rule as given and employ it in deriving their results—the optimal toehold acquisition strategy (Ravid/Spiegel) or the strategy game between raider and arbitrageurs and the effect on the takeover premium (Gomes). They are not concerned with the welfare/efficiency implications of the rule; in particular, both papers take the value of the firm under incumbent management as a given constant. Both provide a role for large shareholders in the presence of freezeouts. Gomes considers the effect of the majority that is needed to affect a freezeout on the bargaining position of the arbitrageurs and the takeover premium. In our paper, we study the optimality of freezeouts as a mechanism to overcome the free-riding problem. We characterize their ex-ante effects—the incentives they provide to managers for increasing the firm's value and the level of social welfare they generate—and we examine their ex-post implications for efficiency in takeovers.

3 The Basic Model

This section presents the basic model that we use to study takeovers, which is similar to Grossman and Hart’s (1980) model. Section 3.1 describes the basic features of the model, and Section 3.2 identifies the free-rider problem that forms the basis for the discussion in the rest of the paper.

3.1 The Basic Features

The model considers a firm with a large number of shareholders. Each shareholder is individually negligible; thus, in determining their response to the raider’s tender offer, shareholders ignore the impact of their own actions on overall outcomes.

The firm is potentially a takeover target. The cost to the raider of mounting the raid is denoted $c$. In the event of a successful takeover, the value of the raider-run firm will be $v$ (all values are in per-share terms). Ex-ante, $v$ and $c$ are random variables with exogenously specified

\footnote{The implicit mathematical model is one with an atomless measure space of shareholdings. This mathematical formalism plays no role in our analysis, so we do not pursue it.}
distributions. However, by the time of the takeover bid, their realizations are known and are common knowledge.\footnote{The assumption that \( v \) and \( c \) are common knowledge is made here for expositional simplicity. As we make clear in Section 7, the properties of our exclusionary mechanism are independent of whether these values are private information to the raider or are public information.}

The value of the firm under current management is denoted \( q \). This value is determined before the raider’s tender offer is made and is a function of the actions taken by the firm’s manager. These actions are, in turn, influenced in part by the probability of a successful takeover. In particular, the manager’s actions are chosen with knowledge of the distribution of \( v \) and \( c \), but before their realizations are observed.

Specifically, there is a set of actions \( A \) available to the manager, and that the action \( a \in A \) leads to the value \( q(a) \) for the firm and the utility level \( u(a) \) for the manager. We rewrite the manager’s utility directly as a function of \( q \) rather than \( a \) by defining\footnote{This formulation of the manager’s role and preferences is borrowed from Grossman–Hart (1980). We refer the reader to that paper for further motivation and details.}

\[
U(q) = \max\{u(a) \mid q(a) = q\},
\]

and presume that \( U \) is continuous as a function of \( q \). In the event of a takeover, the manager suffers a utility loss. His utility in this case is denoted \( U_0 \); without loss of generality, we take \( U_0 = 0 \). Thus, if \( \pi \) denotes the probability of a successful takeover (\( \pi \) is endogenously determined in equilibrium), the manager’s expected utility is given by

\[
EU(q) = (1 - \pi) U(q).
\] (3.1)

The manager sets \( q \) to maximize his expected utility. This exercise is made non-trivial by the fact that the probability \( \pi \) itself depends on \( q \): ceteris paribus, a higher value of the firm under current management reduces the likelihood of a successful takeover. In addition, \( \pi \) also depends on (a) the distribution of the variables \( v \) and \( c \); and (b) the exclusionary mechanism in place that determines payoffs to non-tendering shareholders in the event of a successful takeover (see below). The manager takes these dependencies into account in determining his action \( q \).

**Takeover Offers and Exclusion**

The tender price offered by the raider is denoted \( p \). In this paper, we focus on “any-and-all” offers in which the raider agrees to buy the shares tendered by any shareholder at the offered tender price regardless of how many others tender.\footnote{There is no particular advantage in our model to the raider in using conditional offers rather than unconditional ones: homogeneity of shareholder beliefs and behavior implies that either all shareholders tender in response to a given tender offer or no shareholder tenders.} Thus, all tendering shareholders receive a price of \( p \) for the shares tendered.
Consider the outcome for a shareholder who does not tender. If the takeover attempt fails, the current management remains in charge, and a non-tendering shareholder receives $q$. If the attempt succeeds, the payoff to a non-tendering shareholder depends on the exclusionary mechanism in place. Absent any exclusionary mechanism, a non-tendering shareholder participates fully in the post-takeover value of the firm and receives $v$. At the other extreme, if the exclusionary mechanism allows complete dilution of non-tendering shareholders, the latter receive nothing. At this point, we make no specific assumptions regarding exclusion.

Given a particular exclusionary mechanism, shareholders compare the value of tendering their shares to not tendering and choose the action with the higher value. For definiteness, we will assume that if shareholders are indifferent between tendering and not tendering, they will choose to tender. This eliminates the trivial “openness” problem in identifying the lowest winning bid.\footnote{The openness problem is the following. Suppose non-tendering shareholders will receive some amount $z$. Then, shareholders will strictly prefer to tender given an offer of $z + \varepsilon$ for any $\varepsilon > 0$, but unless shareholders will also accept the offer $z$, there is no lowest winning bid defined.}

**Equilibrium**

All participants have rational expectations about the outcomes of bids and base their behavior on these expectations. Our assumption that shareholders will tender their shares in the event of indifference ensures, as in Grossman and Hart [8], that all equilibria involve only certainty outcomes, i.e., takeovers succeed or fail with probability one. Analytically, this implies that the raider will make a bid if and only if he anticipates it will succeed, so the only bids that are made in equilibrium are successful ones.

**Pre-Takeover Share Prices**

Given a choice of $q$, the pre-takeover market price of the firm’s shares, denoted $r$, is determined as a function of the value of the firm $q$ under current management, the probability of a successful takeover $\pi$, and the payoff received by the shareholders in the event of a successful takeover (which, of course, depends on the exclusionary mechanism). We assume that the market is risk-neutral with respect to the activities of the firm; thus, $r$ is given by

$$r = (1 - \pi) q + \pi E[\text{payoff received by shareholders} | | \text{takeover is successful}].$$

\footnote{This assumption also eliminates another tricky problem: that of aggregating continua of random variables which would arise if indifferent shareholders chose mixed strategies. The nature of these problems may be illustrated with a simple example. Suppose shareholdings are uniformly distributed on $[0, 1]$, and each shareholder accepts the tender offer with probability $p \in (0, 1)$ and rejects it with probability $1 - p$. Intuition suggests that with i.i.d. shareholder choices, the aggregate should be deterministic with a measure $p$ tendering and a measure $1 - p$ not tendering. However, Feldman and Gilles (1985) show that this is not true, and that aggregate outcomes are stochastic in a very complex way, making describing mixed-strategy equilibria impossible. See also Judd (1985).}
Note that $r$ is share price of the firm before it is known whether a takeover will occur. Note also that $r$ is an observable and verifiable quantity, since it is a market price.

Social Welfare

As in Grossman and Hart [8], we assume that society is risk-neutral with respect to the activities of the firm. The aggregate social payoff is thus given by $q$ if there is no takeover, and by $v - c$ (the post-takeover value of the firm minus the resources consumed in the takeover process) if the takeover is successful.\footnote{This ignores distributional considerations—how much the shareholders get vis-a-vis the raider—and equates the social and private costs of the raid.} Thus, social welfare is measured by

$$R = (1 - \pi)q + \pi E[v - c | \text{takeover is successful}]$$  \hspace{1cm} (3.3)

Efficiency

An exclusionary mechanism is efficient if all value-enhancing takeovers succeed, while all value-consuming takeovers fail, i.e., if it is such that

$$\text{Takeover attempts are made and succeed if and only if } v - c \geq q.$$  \hspace{1cm} (3.4)

It is essential to note that efficiency depends on two variables, $v$ and $q$, that are unverifiable by a third party such as a court of law. Note the important point that a welfare-maximizing mechanism need not be efficient and vice-versa, since the former is an ex-ante construct, while the latter is ex-post. In particular, some efficient mechanisms may provide poor ex-ante incentives to managers to generate high $q$ values.

Objective

A central objective of this paper is the identification of an efficient exclusionary mechanism. To ensure implementability of the rule that we identify, we will require it to be based solely on verifiable information. This makes the problem non-trivial; it means, in particular, that while the mechanism may depend on the market price $r$ of the firm's shares, it cannot depend directly on $q$ or $v$. We begin our study by first examining outcomes when there is no exclusionary mechanism employed.
3.2 The Free-Rider Problem

If there is no exclusionary device in place to “punish” non-tendering shareholders in the event of a successful takeover, then a typical shareholder faces the following choice in responding to the raider’s tender offer. If he tenders his shares, he receives \( p \). If he does not, he receives

\[
\begin{align*}
&v, &\text{if the raid is successful} \\
&q, &\text{if the raid fails}
\end{align*}
\]

The well-known result of Grossman–Hart follows immediately from these payoffs:

**Proposition 3.1 (The Free-Riding Problem)** For any \( c > 0 \), there is no rational expectations equilibrium in which takeover attempts are made and succeed.

**Proof** Suppose \( p < v \). If a shareholder believed the raid would be successful, he is better off not tendering his shares, since tendering will fetch \( p \), but retaining the shares will result in a value of \( v > p \). But this means no shareholder will tender, so the raid fails. On the other hand, an offer \( p \geq v \) will induce all shareholders to tender and result in a successful raid; however, any such bid will lead to a loss for the raider, so such an offer will not be made either. Since raids cannot simultaneously succeed and be profitable for the raider, no raids will be made in equilibrium. \( \square \)

Note that outcomes are inefficient, since raids fail with certainty even if they are potentially value-improving (i.e., \( v - c > q \)). Nor are outcomes welfare-maximizing; in particular, there is no threat of takeovers to induce the firm’s incumbent manager to generate a high value of \( q \).

In summary, the problem is that non-tendering shareholders try to free-ride on the improvements in firm value resulting from the raider, and so do not tender their shares if they anticipate a raid will succeed. To overcome this obstacle, an exclusionary device must be added to the model. In the following section, we discuss our proposed solution.

4 A Freezeout-Based Resolution

We propose the following rule: in the event of a successful bid, non-tendering shareholders receive

\[
\max\{r, p\},
\]

(4.1)
where \( p \) is the tender offer price and \( r \) the share-price of the firm prior to the takeover offer. This rule is based entirely on observable and verifiable information, since it only depends on market prices (actually, on a single market price and the tender offer). It is very closely related to the freezeout rule posited by Gomes (2001), but the two rules are not quite identical. In Section 8, we argue that (4.1) may be viewed as the rule that reflects the payoffs which non-tendering shareholders effectively receive in the U.S.

In this section and the next, our focus is on the theoretical properties of using (4.1) as an exclusionary device. We will show that it induces a number of desirable properties. In particular:

**Proposition 4.1** Under the freezeout rule (4.1), outcomes are efficient: takeovers succeed if, and only if, \( v - c \geq q \).

**Proof** To prove Proposition 4.1, we must identify the equilibrium values of several variables under (4.1), including the firm-value \( q \) under incumbent management; the probability \( \pi \) of a successful takeover; the share price \( r \) prior to the takeover; and the optimal tender offer \( p \). We derive these equilibrium implications by working backwards, identifying first the response of shareholders to a tender offer given (4.1), then the optimal bid of the raider for any realization of \((v, c)\), and finally, the resulting equilibrium values.

To this end, note first that under (4.1), a typical shareholder will elect to tender if and only if

\[
p \geq \max\{p, r\},
\]

i.e., if, and only if, \( p \geq r \). In equilibrium, this means that a profit-maximizing raider will bid \( p = r \). On the other hand, a bid of \( p = r \) will be profitable for the raider only if \( v - c \geq r \). Therefore:

- If \( v - c \geq r \), the raider will make a bid of \( r \), and the bid succeeds.
- If \( v - c < r \), no takeover bid is mounted.

Thus, the ex-ante probability of a successful bid is

\[
\pi(r) = \text{Prob}\{(v, c) \mid v - c \geq r\}.
\]

Now, given that shareholders receive \( q \) if there is no takeover and \( r \) if there is a takeover, the share price \( r \) is itself determined as

\[
r = (1 - \pi(r))q + \pi(r)r.
\]
which implies \( r = q \). Thus, we have \( p = r = q \).

Now, as mentioned above, the raider in our model mounts a bid if and only if \( v - c \geq p \), but whenever the raider bids, the takeover attempt is successful. Since \( p = q \), this means a successful takeover attempt is made when, and only when, \( v - c \geq q \), which is exactly the criterion for efficiency. \(\square\)

What are the equilibrium levels of social welfare \( R^* \) and ex-ante firm value \( q^* \) implied by our exclusionary rule (4.1)? From the proof of Proposition 4.1, the equilibrium probability of a successful takeover under this rule is given by (4.3). Since \( r = q \) in equilibrium, the incumbent manager of the firm picks \( q \) to solve

\[
\max_{z \in \mathbb{R}} (1 - \pi(z)) U(z). \tag{4.4}
\]

The solution \( q^* \) to (4.4) is the ex-ante value of the firm, i.e., the value generated by incumbent management. From \( q^* \) and (4.3), the equilibrium probability \( \pi^* \) of a successful takeover is determined as

\[
\pi^* = \text{Prob}\{(v, c) \mid v - c \geq q^*\}.
\]

This determines the level of social welfare \( R^* \) in our equilibrium as

\[
R^* = [1 - \pi(q^*)] q^* + E[v - c \mid v - c \geq q^*] \tag{4.5}
\]

In the next section, we compare the firm value \( q^* \) and social welfare \( R^* \) under our rule to those under the optimal (social-welfare maximizing) dilution level of Grossman–Hart (1980).

5 The Grossman–Hart Dilution Mechanism

Grossman and Hart [8] suggest that in the event of a successful takeover, the raider be allowed to transfer an amount \( \phi \) from the firm to himself. Given this value transfer, the post-takeover value of the firm to a non-tendering shareholder is now \( \max\{v - \phi, 0\} \). Thus, if a shareholder anticipates that the raid will succeed, he will tender his shares if and only if

\[
p \geq \max\{v - \phi, 0\}. \tag{5.1}
\]

Since a profit-maximizing raider will make the lowest possible winning bid, Grossman and Hart impose an important constraint, which is exogenous to the model, that any bid must satisfy:

\[
p \geq q. \tag{5.2}
\]
Thus, a successful bid must satisfy
\[ p \geq \max\{v - \phi, q\}, \] (5.3)
which implies that a profit-maximizing raider would bid \( p = \max\{v - \phi, q\} \). At this offer, all shareholders tender, and each shareholder receives \( \max\{v - \phi, q\} \). Thus, the raider’s net profit is
\[ v - \max\{v - \phi, q\} - c = \min\{\phi, v - q\} - c. \] (5.4)
We now have

**Proposition 5.1** If \( \min\{\phi, v - q\} \geq c \), there is a unique rational expectations equilibrium in which the raider bids \( \max\{v - \phi, q\} \) and all shareholders tender their shares.

**Proof** Straightforward.

The choice of \( \phi \) affects equilibrium social welfare in many ways, in particular, through its impact on the probability \( \pi \) of a successful raid and its influence on the choice of firm value \( q \) under current management. From (5.4), the probability of a successful raid given \( \phi \) and \( q \), is
\[ \pi(\phi, q) = \operatorname{Prob}\{v, c) | \min\{\phi, v - q\} \geq c}\]. (5.5)

The value \( q \) of the firm under the incumbent manager is now determined by the solution to the manager’s utility maximization problem:
\[ q(\phi) = \arg \max_q (1 - \pi(\phi, q))U(q). \] (5.6)

In turn, this means that the equilibrium level of social welfare resolves as
\[ R(\phi) = [1 - \pi(\phi, q(\phi))] q(\phi) + \pi E[v - c | \min\{\phi, v - q(\phi)\} \geq c]. \] (5.7)

Under some innocuous conditions, Grossman and Hart (1980) establish the following result:

**Proposition 5.2** \( q \) and \( R \) are both non-decreasing in \( \phi \). In particular, both are maximized at \( \phi = +\infty \).

In words, Proposition 5.2 states that optimality is achieved when the raider can divert the entire value of the target firm to himself after the takeover. This solution to the free-rider problem is inconsistent with observed behavior, and would violate the law. The following result is of particular interest in this context. It shows that our solution produces welfare and efficiency properties identical to Grossman–Hart, and, indeed, that, in a literal sense, the solutions coincide. This coincidence is striking because our rule depends solely on verifiable price information, while the Grossman-Hart mechanism utilizes information that is both more primary and unverifiable.

Proposition 5.3 Let \( q(\infty) \) and \( R(\infty) \) denote the welfare-maximizing equilibrium values in the Grossman–Hart framework. Then:

1. The Grossman–Hart solution is efficient: raids take place and are successful if and only if \( v - c \geq q \).
2. \( q(\infty) = q^* \) and \( R(\infty) = R^* \), where \( q^* \) and \( R^* \) are the values identified in (4.4) and (4.5), respectively, under our solution.

Proof  Suppose \( \phi = +\infty \). Then, from (5.3), the raider bids \( p = q \) in any takeover attempt, and the attempt succeeds with certainty. Thus, to prove Part 1 of the result, it suffices to show that the raider will find it profitable to make a tender offer if and only if \( v - c \geq q \). But this is immediate, since \( \min(\phi, v - q) = v - q \) in this case, so from (5.4), the raider will find the takeover profitable if and only if \( v - c \geq q \).

To see Part 2, observe that with \( \phi = +\infty \), the probability of a successful takeover given \( q \) in the Grossman–Hart model reduces to

\[
\pi(\infty, q) = \text{Prob} \{(v, c) \mid v - q \geq c\}. \tag{5.8}
\]

Thus, the optimization problem of the Grossman–Hart manager is

\[
\max_q (1 - \pi(\infty, q))U(q). \tag{5.9}
\]

It is immediate from the definition of \( \pi(\infty, q) \) that this optimization problem (5.9) is identical to the problem (4.4) faced by the manager in our solution. Thus, the equilibrium values of \( q \) coincide in the two models. The conditions determining successful takeovers also evidently coincide. Thus, finally, it is immediate that the social welfare levels are identical.

An important weakness of the Grossman–Hart model is that it imposes some difficult requirements from an informational standpoint. Admissible bids in the model must meet the condition
(5.2), which depends on the variable \( q \). Winning bids must also meet (5.1), which depends further on \( r \) and \( \phi \). Of these variables, it is doubtful if \( r \) and \( q \) may be regarded as even observable values to an outside entity such as a court of law. Certainly, a convincing case cannot be made that these variables are verifiable, that is, that an outside entity could ascertain these values with any degree of certitude. Thus, implementation of the Grossman–Hart solution is problematic.

More troubling, perhaps, is that (5.2) is an ad-hoc and exogenous condition imposed on the model that has no obvious justification. Yet, this latter condition is central to the Grossman–Hart model. Indeed, we will show that if this condition is dropped, then efficiency of the Grossman–Hart solution is also sacrificed. As a first step, we show the presence of other equilibria that arise if this condition is dropped:

**Lemma 5.4** Suppose (5.2) is not required of admissible bids. Pick any dilution level \( \phi \). If \( \min\{\phi, v\} \geq c \), there is a rational expectations equilibrium in which the raider makes a bid of \( p = \max\{v - \phi, 0\} \) and the bid succeeds with certainty.\(^\text{11}\)

**Proof** If a bid \( p \geq \max\{v - \phi, 0\} \) is submitted, a shareholder who believes it will succeed is faced with the choice of tendering and receiving \( p \), or retaining his shares and receiving \( \max\{v - \phi, 0\} \); thus, tendering is optimal for the shareholder, and the bid succeeds with certainty. Moreover, the raider will find it profitable to submit such a bid since his profit from doing so is \( v - \max\{v - \phi, 0\} - c = \min\{\phi, v\} - c \), which is positive by hypothesis.

Lemma 5.4 implies, for example, that for sufficiently large dilution \( \phi \geq v \), the raider’s offer price will be zero, and shareholders will still tender under the threat of receiving \( v - \phi \) if they do not, which means they too end up receiving zero. We can now show that this second equilibrium induces inefficiency in outcomes:

**Proposition 5.5** If (5.2) is not required in the Grossman–Hart solution, then outcomes may be inefficient in either direction: value-reducing bids can succeed and value-enhancing bids may fail.

**Proof** Pick any dilution level \( \phi \). Efficiency demands that a tender offer succeed if and only if \( v - c \geq q \), or equivalently,

\[
v - q \geq c.
\]

(5.10)

On the other hand, Lemma 5.4 indicates that there is a rational expectations equilibrium in which a tender offer will succeed whenever

\[
\min\{\phi, v\} \geq c.
\]

(5.11)

\(^{11}\)That this second rational expectations equilibrium exists when (5.2) is dropped is recognized in Grossman–Hart (1980); see their footnote 8 on p.47.
Expressions (5.10) and (5.11) imply the same thing only if \( v - q = \min\{\phi, v\} \), which is impossible to ensure for any a priori choice of \( \phi \). In particular, for any fixed choice of \( \phi \), it is always possible that

\[
\min\{\phi, v\} \geq c > v - q,
\]

so value-reducing bids may succeed. If \( \phi \) is finite, then it is also possible that

\[
v - q \geq c > \min\{\phi, v\},
\]

so value-enhancing bids could fail.

6 Large Shareholdings: The Bagnoli-Lipman Setting

This section analyzes the case of a firm whose shareholdings are all non-trivial, as opposed to the model above where each individual shareholding was negligible. Bagnoli and Lipman (1988) study such a model. In this setting, every shareholder is potentially pivotal, i.e., his action could determine whether a takeover succeeds or not. As a consequence, competitive behavior (ignoring the effect of one's own action on aggregate outcomes) is no longer tenable. Bagnoli and Lipman examine Nash equilibria of the resulting game and establish the following results:

1. Even without any exclusionary mechanisms, there are always Nash equilibria in which takeovers succeed.

2. Some of these equilibria involve mixed-strategies, so outcomes are stochastic, implying that some value-increasing takeovers will fail to take place.

3. The introduction of the Grossman–Hart mechanism into the game can induce value-reducing takeovers.

Thus, while free-riding is mitigated in this framework, the second property identified above implies that the solution may be socially inefficient, while introducing the Grossman-Hart dilution mechanism may introduce another kind of inefficiency.

In this section, we examine the impact of the freezeout rule (4.1) in the Bagnoli–Lipman world. We show that using (4.1) guarantees the success of all value-increasing takeovers and only of those takeovers. This eliminates inefficiencies created when value-increasing takeovers do not take place. At the same time, (4.1) also guarantees that value-decreasing takeovers will not occur, as is the case with the Grossman-Hart solution.
We first illustrate the Bagnoli–Lipman results using a simple example to highlight the different inefficiencies that could arise. Consider a model in which there are three shareholders, each holding one share. As above, let the per-share value of the firm under incumbent management be \( q \), the per-share value of the raider-run firm be \( v \), and the tender price be \( p \). Bagnoli and Lipman are not concerned with the impact that different exclusionary mechanisms may have on the choice of \( q \); thus, they take \( v \) and \( q \) as fixed known quantities with \( v > q \). They also do not consider tendering costs \( c \); to retain consistency with their framework, we set \( c \) to zero in this section.

Now suppose that a raider must obtain at least two shares for a successful takeover. It is easy to see that a tender offer \( p < q \) will fail with certainty. We will show that there is a tender offer \( p^* > q \) at which there is a mixed strategy equilibrium. Pick any \( p \geq q \). Suppose shareholders 1 and 2 use mixed strategies in which each tenders with probability \( a \) and does not tender with probability \( 1 - a \). Then, shareholder 3 receives \( p \) from tendering and an expected payoff of \( a^2v + (1 - a^2)q \) from not tendering, so is indifferent between the alternatives if \( a = [(p - q)/(v - q)]^{1/2} \). By symmetry, it follows that each player randomizing with this probability \( a \) is a Nash equilibrium of the tendering game. Given this behavior, the expected profit of the raider from a tender offer \( p > q \) is given by \( [a^3 + 2a^2(1 - a)](v - p) \). From the form of \( a \), this profit is zero if \( p = q \); it is evidently also zero if \( p = v \). Since \( p \in (q, v) \) leads to a strictly positive profit, it follows that the optimal tender offer must be some \( p^* \in (q, v) \). At such a tender offer, the probability of a successful takeover is strictly less than one, even though \( v > q \). Thus, in the absence of an exclusionary mechanism, we can have inefficient equilibria in which value-improving takeovers fail to occur with positive probability.

Suppose the Grossman–Hart exclusionary mechanism with \( \phi = +\infty \) is introduced into this setting. Consider a partial tender offer of \( p = q \) for two shares; and suppose that if more than two shareholders tender, pro-rating occurs. Since non-tendering shareholders now receive zero, it is immediate that the strategy of everyone tendering is a Nash equilibrium. The payoff to the raider in this Nash equilibrium is \( v - 2q/3 \) which can be positive even if \( v < q \). Thus, inefficient value-reducing takeovers become possible.

In the remainder of this section, we examine the impact of introducing the freezeout rule (4.1) into this finite shareholder model. We make only a very weak assumption: that no shareholder uses a weakly-dominated strategy. Under this condition, we show that there is an essentially unique game-perfect equilibrium under (4.1) in which takeovers succeed with probability one when and only when they are value-increasing, so both kinds of inefficiencies identified above are avoided. Unlike the usual Nash equilibrium constructions involving best-response mappings, our derivation of this equilibrium utilizes primarily the non-use of dominated strategies.

**Proposition 6.1** Suppose no shareholder uses a weakly dominated strategy. Then, the finite shareholder game under the exclusionary mechanism (4.1) involves a unique equilibrium in which tender offers are made and succeed with certainty when and only when \( v \geq q \).
Proof We first show that under (4.1) we must have \( r = q \) in equilibrium. To this end, note first that the raider will never make a tender offer \( p < q \); accepting such an offer results in the shareholder receiving \( p \), but rejecting it dominates this since the shareholder receives either \( q > p \) (if the takeover fails) or \( \max\{r, p\} \geq p \) (if it succeeds). Thus, we can partition the set of values \( v \) of the raider-run firm into two sets: \( V_0 \), the set of \( v \) for which no tender offer is made, and \( V_1 \), the set of \( v \) for which an offer \( p(v) \geq q \) is made. Thus, for any \( v \), shareholders receive either \( q \) or \( p(v) \geq q \), implying the equilibrium share price \( r \) must satisfy \( r \geq q \).

Now observe that any bid \( p > r \) will succeed with certainty. Accepting such an offer means receiving \( p \) for certain, while rejecting it results in a payoff of either \( \max\{r, p\} = p \) (if the takeover succeeds) or a payoff of \( q \leq r < p \) (if it fails), so rejection is dominated by acceptance. No such tender offer can be part of an equilibrium for any \( v \). For any \( p > r \), there is \( p' \) such that \( r < p' \leq p \), and making the tender offer \( p' \) would result in strictly larger profits for the raider. Thus, for any \( v \in V_1 \), we must have \( p(v) \leq r \).

Now, combining these observations and letting \( \pi_0 \) denote the probability of the set \( V_0 \), the equilibrium share price \( r \) must satisfy

\[
\begin{align*}
    r &= \pi_0 q + (1 - \pi_0) E[\text{Shareholders’ receipts} \mid v \in V_1] \\
    &\leq \pi_0 q + (1 - \pi_0) r
\end{align*}
\]

so we must have \( r = q \), as claimed at the outset. Indeed, we must have \( p = r = q \).

Now, if \( v \geq q \), it is always an equilibrium for the raider to bid \( p = q \) and for (a relevant majority of) shareholders to accept, and from the arguments presented above, this is the only equilibrium in this case. If \( v < q \), the raider will not make a tender offer, since an offer \( p < q \) cannot succeed, and an offer \( p \geq q \) will lead to a loss for him. Thus, outcomes are guaranteed to be efficient. \( \square \)

7 A Comment on Common Knowledge

In the analysis above, we have made the assumption that at the time of the raid, the raider’s valuation is known and is common knowledge to the raider and the shareholders. It is easy to see that this assumption is irrelevant under our freezeout rule (4.1). Under (4.1), tendering shareholders receive \( p \) while non-tendering shareholders receive either \( \max\{r, p\} \) (if the raid succeeds) or \( q \) (if it fails). These payoffs indicate that in no contingency do shareholders’ payoffs depend on their knowledge or estimate of \( v \); consequently, neither will their acceptance or rejection decisions. In turn, this implies that the raider’s valuation of the post-takeover firm may be private knowledge without affecting our equilibrium or its efficiency and welfare levels in any way.
A similar point is also true of the Grossman–Hart mechanism provided $\phi = +\infty$. In this case, shareholders' payoffs are independent of $v$, since tendering shareholders receive $p$, and non-tendering shareholders receive either $q$ if the raid fails or $\max\{v - \phi, q\} = q$ if it succeeds. However, for the general form of the Grossman–Hart mechanism, this independence is false: if $\phi$ is finite, then non-tendering shareholders receive $\max\{v - \phi, q\}$ if the raid succeeds, which depends on $v$ for all sufficiently large $v$. Hence, shareholders' knowledge of $v$ matters, and since the raider's bid conveys some information about this valuation, the solution to the model becomes significantly more complicated.\(^{12}\)

8 The Legal Environment

Our proposed resolution of the free riding problem is consistent with the legal regulation of the treatment of non-tendering shares in the US. After acquiring majority control of a company, a raider can acquire any non-tendering shareholders, without their approval, through a so-called freeze-out merger between the target and another company controlled by the raider. The securities laws require that the intention to effect a freeze-out merger, together with the terms of the merger, are announced by the raider when making the tender offer. Non-tendering shareholders usually receive in the freeze-out merger the same consideration that is paid to tendering shareholders in the tender offer. Importantly, if the non-tendering shareholders sue the raider for higher consideration in an appraisal proceeding, the court assesses the fair value of the company exclusive of any element of value arising from the accomplishment or expectation of the merger.\(^{13}\) This excludes free riding.

In practice, therefore, raiders employ the very mechanism we have described to overcome the free-rider problem: they buy out non-tendering shareholders for a value that does not exceed the tender price. This mechanism has been sanctioned by law. From a practical perspective, non-tendering shareholders have no effective legal recourse against the raider. Below we describe the legal rights available to non-tendering shareholders – appraisal rights and suits for breaches of fiduciary duties – focusing on the law of Delaware, where well over half of public companies are incorporated. The law of other states does not differ significantly from Delaware law.

8.1 Appraisal Rights

Appraisal rights are available in Delaware to non-tendering shareholders either where these shareholders receive cash in the freeze-out merger or where the freeze-out merger takes the form of a

\(^{12}\)As mentioned in Section 2, Shleifer and Vishny (1986) consider such a model.

\(^{13}\)Delaware General Corporation Law, 262(h).
short-form merger. To effect a short-form merger, the raider must own at least 90% of the target’s stock after the tender offer. In an appraisal action, a shareholder receives a fair value of her shares as assessed by the court instead of the consideration offered in the freeze-out merger. If the court-assessed value is below the consideration offered in the freeze-out merger, the shareholder thus suffers losses from having sought an appraisal.

Appraisal rights do not provide effective recourse against a raider that pursues a freeze-out merger above the pre-tender offer share price. Principally, the law\textsuperscript{14} provides that the fair value that the dissenting shareholders can receive should not include the improvement in value that arises from the merger. Though the court may consider factors other than the pre-tender offer share price, there is no a priori reason to expect that the court will award a value in excess of that price (let alone in excess of the tender offer price). In particular, the raider’s proposed strategy for running the company does not affect the value awarded in the appraisal unless the raider starts to implement that strategy prior to the freeze-out merger.\textsuperscript{15}

Appraisal rights are very rarely pursued because this entails substantial costs to individual shareholders. To exercise appraisal rights, a shareholder must inform the company of her intention to do so prior to the merger and petition the court after the merger. Failing to take these actions, a shareholder does not receive appraisal rights. Thus, appraisal rights cannot be brought as a class action, though the court can consolidate the appraisal proceeding of the shareholders that sought appraisal rights. Ordinarily, shareholders are not entitled to be compensated for the legal and professional fees that they incur in the course of the appraisal proceeding. The costs of the appraisal proceeding are thus borne by those shareholders who exercise these rights. These costs are likely to make it irrational to exercise appraisal rights for any small shareholder, unless the shareholder expects many other shareholders to exercise appraisal rights as well. That is, appraisal rights are least effective in circumstances when shareholders are small and coordination is difficult—the very situation where the free-riding problem arises.

\section*{8.2 Suits for Breaches of Fiduciary Duty}

To bring a suit for breach of fiduciary duty, a shareholder must show that the raider breached its fiduciary duties by effecting the freeze-out merger. In principle, such suits can be brought to challenge a self-interested transaction, such as freeze-out merger between the target and a company controlled by the raider. In practice, however, the raider can and does insulate the merger against such challenges.

First, if the raider acquires at least 90% of the target’s stock, it can effect a short-from

\textsuperscript{14}Delaware General Corporation Law, 262(h).
\textsuperscript{15}Cede v. Technicolor.
merger. Short-form mergers cannot be challenged for breaches of fiduciary duty.  

Second, in a friendly tender offer, which happens in the great majority of takeovers (see Schwert, 2000) the raider can negotiate the terms of the freeze-out merger, and sign a merger agreement, with the target’s board prior to the consummation of the tender offer. Since the raider did not control the target when the merger agreement was signed, the agreement cannot be challenged as a self-dealing transaction even though the raider will control the company by the time shareholders will vote on the merger.  

Even takeovers that start as hostile often end up being negotiated and the raider reaches a merger agreement with the target’s board, which can insulate it from being sued by non-tendering shareholders.

Third, even if no merger agreement is signed prior to the consummation of the tender offer, it is difficult to challenge the merger if the merger consideration is equivalent to the consideration paid in the tender offer. A court in such circumstances is likely to give substantial weight to the fact that a majority of target shareholders tendered their shares and thereby indicated their approval of the transaction.

The law thus affords non-tendering shareholders no effective recourse against a raider who freezes them out at a price between the pre-offer share price and the tender price. To our knowledge, there has been no instance in which a raider has been held to have violated its fiduciary duties by effecting a freeze-out merger in the wake of, and on the terms announced in, such a tender offer.

While our proposed solution to the free riding problem is consistent with the law and therefore feasible, GH’s solution is not. The legal regulation of controlling shareholders has not sanctioned the dilution suggested by Grossman–Hart, by which the raider would be allowed to transfer an amount \( \phi \) from the firm to itself. Such a transfer would constitute a breach of the raider’s duty of loyalty and would subject the raider to legal sanctions.

### 9 Concluding Remarks

In this paper, we have offered an economic basis for permitting freezeouts in takeovers. We have shown that, suitably defined, freezeouts can guarantee desirable normative properties in models of both widely-held firms (as in Grossman and Hart, 1980) and firms with large, pivotal

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17Shareholders could still bring a claim for breach of the duty of care against the target board. See, e.g., Cede & Co. v. Technicolor, Inc., 634 A.2d 345 (Del. 1993). However, monetary damages for breaches of the duty of care are usually not available. See Delaware General Corporation Law, 102(b)(7). Even when available in principle, the conduct of the members of the target board who negotiated with the raider, rather than the raider’s conduct, would determine whether there was a breach of the duty of care, and damages would have to be paid by these target board members, rather than by the raider.
shareholders (as in Bagnoli and Lipman, 1988). In particular, our freezeout mechanism provides the same desirable properties as the optimal Grossman–Hart mechanism without the problems that the latter faces.

Two avenues for future research are indicated by our analysis. The first is a continuation of the normative analysis in this paper. We have focussed on the case when there is a single raider. While the case of multiple competing raiders presents considerable additional complexity, intuition suggests that the ultimate impact of competition between raiders for the firm will be to generate a premium, since a portion of the surplus from the takeover will be transferred from the raiders to the target shareholders. This implies that the the takeover price $p$ will exceed the share price $r$, and, as an immediate consequence, the equilibrium share price $r$ will also strictly exceed the value $q$ of the firm under current management. This suggests that under freezeouts some value-improving takeovers may fail to occur, and when combined with the analysis in this paper, this raises the question: how much competition is actually a good thing from a social standpoint?

A second area for future investigation concerns the positive aspects of freezeout-based equilibria. Many existing theoretical analyses of takeovers in the literature (e.g., Shleifer and Vishny (1986) or Cornelli and Lee (2001)) examine models in which no exclusionary mechanisms are employed to eliminate free-riding. While these analyses have provided valuable theoretical insights into the limitations of the free-riding problem, their empirical implications must be judged with caution. In most cases, the elimination of free-riding possibilities unwinds the equilibria that are shown to exist. Since freezeouts are, in fact, permitted by law, these models cannot be viewed as completely satisfactory explanations of the various empirical phenomena associated with takeovers. Rather, there is a need to build models that explicitly incorporate freezeout possibilities (as Gomes (2001) and Ravid and Spiegel (2001) have done), and use these to reconcile theory with observation.
References


