The Effect of Trading Commissions on Analysts’ Forecast Bias

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Motivation

- Empirical studies on analysts’ forecasts
  - forecast bias
  - analysts’ incentives

- Lack of theoretical models

- Trading commissions are a “tangible” incentive – natural place to start
  - “Brokerage firms usually reward their research analysts using a single measure of performance: trading volume in the stocks that they cover.” Cowen/Groysberg/Healy 2006
Objective

- Study the interaction between a sell-side analyst and a risk-averse investor

- Focus: Given his private information what forecast should the analyst issue?

- Takes other aspects of the analyst’s forecasting decision as exogenous
  - Timing (Guttman 2007)
  - Information gathering (Hayes 1998)
Main Findings

- Unique fully separating equilibrium
- Analyst’s bias is an increasing, bounded function of his private signal
  - “High” signal → upward bias
  - “Low” signal → downward bias
- Equilibrium is robust to
  - short-sale restrictions
  - additional information asymmetry (cost function)
**Literature**

- Analysts’ incentives to bias their forecasts:
  - Jackson (2005), trade off between upwards bias and accuracy.
  - Cowen, Groysberg and Healy (2006), analysts’ optimism is driven by trading incentives.
  - Underwriting business; Career concerns; Access to management; Selection bias:

- Analysts care about the accuracy of their forecasts.
- Analysts’ weighting of private and public information.
Setup

- The firm’s earnings, $\tilde{x} \sim N\left(\mu_x, \tau_x = \frac{1}{\text{Var}(\tilde{x})}\right)$.

- Analyst privately obtains $\psi = x + \epsilon$
  where $\tilde{\epsilon} \sim N\left(0, \tau_\epsilon\right)$ and independent.

- Based on $\psi$, the analyst provides a forecast, $x^R$, to his client investor.

- The informed investor initially holds $D_0$ shares and updates his holding to $D_1(x^R)$ after obtaining the forecast.

- The informed investor’s trade, $|D_1(x^R) - D_0|$, determines the analyst’s commission.
A continuum of investors with CARA utility function

\[ u(W_1) = -e^{-\rho W_1} \]

Initial demand/holding of the representative investor

\[ D = \frac{\mu_x - P_0}{\rho} \tau_x \]

\( P_0 \) is the equilibrium stock price

\[ P_0 = \mu_x - \frac{\rho S}{\tau_x} \]
Setup (Cont.) - Informed Investor

- The informed investor is assumed to be a price-taker.

- After privately obtaining the analyst’s forecast, the informed investor trades $|D_1 (x^R) - D_0|$ s.t.

$$\max_{D_1} \mathbb{E} \left[ u^I (D_1) \mid x^R \right] = - \int_{-\infty}^{\infty} e^{-\rho(W_0 + D_1(\tilde{x} - P_0) - c_I|D_1 - D_0|)} f \left( \tilde{x} \mid x^R \right) d\tilde{x}$$

where $c_I$ is the investor’s per share trading costs.

- Anticipating arrival of future forecast and $D_1 (x^R)$, the investor chooses $D_0$ to maximize his expected utility.

- In a fully separating equilibrium

$$D_0 = D$$
Setup (Cont.) - Analyst’s payoff function

- When determining his forecast, the analyst trades off trading commission against expected cost from forecast errors.

- Analyst’s payoff function:

\[ u^A (x^R, \psi) = c_A |D_1 (x^R) - D_0| - E \left[ g \left( x^R - \tilde{x} \right) \right] \psi \]

- \( c_A \) – analyst’s per share benefit from trading commissions.

- \( g (\cdot) \) is twice-differentiable, convex and steep at the tails.

- Zero bias minimizes the expected cost from forecast error.
Zero marginal trading cost, \(c_I = 0\)

In a fully separating equilibrium, where the informed investor perfectly infers \(\psi\), his demand is

\[
D_1 (\psi) = \frac{E[\bar{x}|\psi] - P_0}{\rho \text{Var}(\bar{x}|\psi)}
\]

The unique realization of \(\psi\), for which the investor does not trade, \(\psi^*\), is

\[
D_1 (\psi^*) = D_0
\]

That is

\[
\psi^* = P_0
\]
Zero marginal trading cost, $c_I = 0$

Proposition

There exists a unique fully separating equilibrium where

(i) The analyst’s equilibrium forecasting strategy is

$$x^R(\psi) = E[\tilde{x}|\psi] + b(\psi)$$

where the bias function $b(\psi)$ is increasing, continuous, convex for $\psi < \psi^*$, concave for $\psi > \psi^*$, bounded from above and below and $b(\psi^*) = 0$;

(ii) The investor’s demand is given by

$$D_1(x^R) = \frac{x^R - b(x^R) - P_0}{\rho \text{Var}(\tilde{x}|\psi)}$$

where $b(x^R(\psi)) = b(\psi)$ is the investor’s belief about the analyst’s bias given $x^R$. 
Zero marginal trading cost (Cont.)

The analyst’s bias as a function of his private signal

The analyst’s forecast as a function of his private signal

Figure: The analyst’s equilibrium bias and forecasting strategy.
Zero marginal trading cost (Cont.)

Investor’s trade as a function of the analyst’s forecast $x^R$

$D_1(x^R) - D_0$

Figure: The investor’s equilibrium trade as a function of the forecast.
Investor’s trading decision with positive trading costs

- For $c_I = 0$ the investor buys shares only if $E[\tilde{x}|\psi] > x^* (= E[\tilde{x}|\psi^*])$

- For $c_I > 0$ the investor buys shares only if $E[\tilde{x}|\psi] > x^* + c_I$

- Equivalently for investor’s decision to sell shares

- “No-Trading Zone” for

$$x^* - c_I \leq E[\tilde{x}|\psi] \leq x^* + c_I$$
Positive marginal trading cost

The analyst’s bias as a function of his private signal (per share trading costs of 1)

Investor’s trade as a function of the analyst’s forecast $x^R$ if the investor incurs a trading cost per share of $c_I = 1$

**Figure:** The analyst’s bias and investor’s demand for $c_I > 0$. 

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Analysts’ Forecast Bias
Short-sale constraints

- $s$ – maximum number of shares an investor can short-sell

$$D_1 \geq -s$$

- Maximum number of shares he can sell is

$$s + D_0$$

- For $\psi < \psi$ the unconstrained demand $D_1(\psi) < -s$

$\implies$ short sale constraint is binding

$\implies$ the analyst’s incentive to bias his forecast diminishes.
Short-sale constraints

Investor’s trade as a function of the analyst’s forecast $x^R$, in the presence of short sale constraint

$$D_1(x^R) - D_0$$

Figure: Investor’s demand in the presence of short-sale constraint.
Short-sale constraints

The analyst’s bias as a function of his private signal, in the presence of short sale constraint

The analyst’s forecast in the presence of short sale constraint

Figure: The analyst’s bias and forecast – short-sale constraint.
Unknown analyst’s objective function

- Additional information asymmetry regarding the analyst’s cost function

\[ u^A \left( x^R, \psi, \nu \right) = c_A \left| D_1 \left( x^R \right) - D_0 \right| - E \left[ g \left( x^R - \tilde{x} - \nu \right) \right] \psi, \nu \]

\[ \tilde{\nu} \sim N \], observed by the analyst but not by the investor

- Before – investor learned analyst’s expectation \( E \left[ \tilde{x} | \psi \right] \)

- Now – investor only learns \( E \left[ \tilde{x} | \psi \right] + \nu \)

- Other than that the equilibrium is robust
Forecast Bias

- Higher likelihood of positive (optimistic) bias
- Positive expected bias, $E \left[ b \left( \tilde{\psi} \right) \right] > 0$, if the cost function $g \left( \cdot \right)$ is symmetric
Analyst’s trading commission, $c_A$

- Higher per share trading commission, $c_A$, increases the analyst’s incentive to bias his forecast.

- For any $\psi \in \Psi_{NT}$, the absolute value of $b(\tilde{\psi})$ is increasing in $c_A$ and $\lim_{c_A \to 0} b(\psi) = 0$.

- $E[FE^2]$ increases in $c_A$; and

- If the cost function is symmetric, $E[b(\tilde{\psi})]$ increases in $c_A$;

- Chen and Jiang (2006)
"Weighting" of private information

For all $\psi \notin [P_0, \mu_x]$ the analyst issues a forecast as if he overweights his private information $\psi$.

- $\exists w \geq \frac{\tau_{\epsilon}}{\tau_x + \tau_{\epsilon}}$ such that $x^R(\psi) = w\psi + (1 - w)\mu_x$
  - For $\psi > \mu_x$: overweights by more
  - For $\psi < P_0$: overweights by less

For all $\psi \in (P_0, \mu_x)$ the analyst issues a forecast as if he underweights his private information $\psi$.

- $\exists w \leq \frac{\tau_{\epsilon}}{\tau_x + \tau_{\epsilon}}$ such that $x^R(\psi) = w\psi + (1 - w)\mu_x$

The inequalities hold strictly for $\psi \notin \Psi_{NT}$.

Effect of precision of private information on the bias

For any $\psi \neq \Psi_{NT}$, if $g'(x^R - x)$ is weakly convex in $x^R$ then

$$\frac{\partial |b(\psi)|}{\partial \tau_\varepsilon} > 0$$

Higher precision of the private information:

- Increases the sensitivity of the informed investor’s demand to the information conveyed in the forecast.
- Provides the analyst with stronger incentives to bias his forecast (similar to higher trading commission).
Effect of precision on the squared forecast error

\[ E \left[ FE^2 \right] = \text{Var} (\hat{x} | \psi) + E \left[ b (\psi)^2 \right] \]
Additional empirical predictions

- Informed investor’s trading costs $c_I$
  - $E \left[ FE^2 \right]$ is decreasing in $c_I$;
  - for symmetric cost functions $E \left[ b (\tilde{\psi}) \right]$ is decreasing in $c_I$.

- A less binding short-sale constraint induces
  - higher $E \left[ FE^2 \right]$;
  - lower $E \left[ b (\tilde{\psi}) \right]$. 
Summary

- Unique fully separating equilibrium
- Analyst’s bias is an increasing function of his private signal
- Robustness of the equilibrium

Empirical predictions

- Forecast is on average optimistic
- Ambiguous effect of the precision on the expected squared forecast error
- Weighting of private information