Equity Issuance and Expected Returns: Theory and New Evidence

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Abstract

This paper examines an economy in which firm managers seek to maximize proceeds from equity issuance. The share issues are accommodated by competitive risk-averse investors. All agents are fully rational. Yet, the model produces what looks like market-timing behavior by firm managers. That is, using standard statistical tests, post equity issuance there appears to exist negative return predictability. We calibrate the model numerically and test its implications empirically. The data supports the prediction that, among others, stocks with higher volatilities experience lower subsequent returns controlling for the level of equity issuance.
Do managers time the market? That is, do they sell shares when their stock is overvalued and buy them back when they are undervalued? There is certainly empirical support for this conjecture. Ritter (1991) finds strong underperformance following initial public offerings. Loughran and Ritter (1995) come to similar conclusions for seasoned equity offerings. Conversely, Ikenberry, Lakonishok, and Vermaelen (1995) document positive abnormal returns after share repurchase announcements. Results related to these event studies are confirmed in the cross-sectional regression studies of Daniel and Titman (2004), and Pontiff and Woodgate (2006). It is also possible to construct calendar time strategies that exploit the negative return predictability of equity issuance, a point important from an asset pricing perspective (Fama and French (2006, 2007)).

Despite abundant empirical evidence, however, there is little consensus as to why equity issuance predicts future returns. Are managers taking advantage of systematic mispricing induced by irrational investors, or are they reacting to other economic forces that then lead to the observed correlations? The analysis presented here attempts to shed light on this issue from a rational perspective and to bring out its additional implications.

This paper examines a model in which firms seek to maximize proceeds from net equity issuance. These proceeds are then invested in real assets which change the composition of the risks faced by the model’s population. Overlapping generations of competitive risk-averse investors not only purchase and sell corporate securities but their labor endowment as well. Their labor is also converted into real assets by firms. This creates two channels via which capital creation can occur: labor and investment. It is the interaction between them that then leads the model to generate predictions in line with existing empirical studies.

Consider stock prices per dollar of real capital. If real capital in an industry is in relatively short supply, its

\footnote{For international evidence, see Ikenberry, Lakonishok, and Vermaelen (2000) and McLean, Pontiff, and Watanabe (2007).}
share price per dollar of real capital will be relatively high. This induces managers to issue
more shares in order to raise funds to create additional real assets in their firm. The result is
a sequence of events that looks like market-timing. It is, however, fully rational as opposed to
the typical context in which that term is used.

Second, the apparent market timing behavior by firms, coupled with the exogenous
labour capital supply shocks, leads to the return predictability of equity issuance. Consider a
positive labour created capital shock. When this occurs risk-averse investors require a price
discount on the stock that now owns the additional capital since they have to accommodate a
larger supply of the real underlying assets. However, this discount then induces the firm to sell
real assets (i.e. repurchase its own shares) to reduce its capital stock. The offsetting capital
supply forces of labor and investment make each firm’s total capital supply mean-reverting.
Thus, negative shocks to the prices induce share repurchases (negative net equity issuance),
followed by an upward drift to the mean price levels on average.

Third, the model quantifies the negative relation between net equity issuance and subse-
quent returns. Since this relation derives from a risk-return trade-off, the expected return is
proportional to the market price of the risk, which is further proportional to the variance of
investors’ future wealth. A cross-sectional implication, therefore, is that a unit equity issuance
leads to lower subsequent returns for more volatile stocks relative to their less volatile counter-
parts. Conversely, controlling for the level of volatility, stocks with larger equity issuance
should experience stronger underperformance subsequently.

Before taking the model’s implications to the data, we demonstrate its empirical relevance
via calibration. The model is fit to the data by matching it to its two key quantities: net equity
issuance and return variance. From there additional empirical implications are brought out
that allow it to be tested via market data using as yet unconduted tests.

Finally, the paper presents an empirical analysis of the model’s predictions. We form portfolios by two dimensional sorting on net equity issuance and return volatility. Our equity issuance measure is adjusted for nominal share changes such as stock dividends and splits and therefore captures only real share changes. Consistent with the model’s predictions, stocks with larger equity issuance experience lower subsequent returns controlling for volatility. Similarly, stocks with higher return volatility earn substantially lower returns controlling for the level of equity issuance. The returns on the zero investment portfolio that goes long the least volatile stocks and short the most volatile stocks is 1.00% ($t = 3.32$) monthly. The performance of this long-short portfolio is persistent as it earns a monthly excess return of 0.57% ($t = 2.03$) even if it is held for the next six months. A concern, however, is that these excess returns may represent reward for bearing known risk. To examine this point, we compute risk-adjusted returns by regressing portfolio returns on the market, size, value, and momentum factors. The risk-adjusted return, whose means equal the alphas from the four factor model by construction, on the long-short volatility portfolio is significant at all volatility levels and reaches 1.07% ($t = 5.89$) in the highest equity issuance group. We further conduct panel regressions to test, and confirm, the cross-sectional restriction that the model imposes.

Our paper makes three contributions to the existing literature. First, the theory produces the market-timing behavior of firm managers and the negative return predictability of equity issuance in an economy populated by only rational agents who optimally execute their transactions. Second, the model numerically demonstrates these points using returns rather than prices. This is useful since empirical work is conducted on returns and not the price changes typically generated within the negative exponential utility-normal payoff framework. Finally,
the empirical analysis confirms a novel prediction that, controlling for the equity issuance, more volatile stocks earn lower future returns.

The rest of the paper is organized as follows. The next section sets up the model, solves for an equilibrium, calibrates the parameters, and derives empirical implications. Section 2 presents empirical evidence. Section 3 provides extensions and discussions. The final section concludes.

1 A Model of Endogenous Equity Issuance

1.1 Set-up

The economy contains $K$ firms. Each firm lives forever. As in most overlapping generations models people live for two periods. Those born in period $t$ come endowed with labor capital that they sell to firms which convert it to corporate capital. In period $t+1$ the old generation sells its holdings of stocks and bonds, consumes its claim to the economy’s single consumption good and then dies.

At the beginning of period $t$, capital units pay a vector of random dividends of the consumption good,

$$D_t = D_{t-1} + \delta_t,$$

where $\delta_t$ is a $K$ vector of dividend shocks. The $\delta_t$ vector is distributed multivariate normally with mean zero and variance-covariance matrix $\Sigma_\delta$.

The size of the firms’ production base, $N_t$ varies over time. For example, firms purchase new machinery and equipment during expansions and scale back during contractions. This portion of changes in the production base, as a result of the firm managers’ optimal choices, is denoted
by $y_t$. Expansions are financed with equity sales and contractions with equity repurchases.

In the model real assets $N_t$ also vary randomly with the new generation’s (random) endowment of labor capital. In each period the new generation arrives with labor capital that can be sold to firms which will then convert it into $\eta_t$ units of corporate capital. The model assumes all agents and firms act competitively which implies that labor capital is sold to firms at that period’s equilibrium market price for the corporate capital it will produce. With these assumptions, the production base vector, $N_t$, evolves through the following dynamics:

$$N_t = N_{t-1} + \eta_t + y_t.$$  \hfill (1)

Stocks are claims to these production technologies. For analytical tractability it is useful to set the number of shares in a firm equal to its capital stock. Therefore, $N_t$ also represents number of shares outstanding. Thus, $y_t$ equals net equity issuance to investors, and $\eta_t$ the payment in shares to labor in exchange for selling their supply shock to the firm. All these variables are $K$ vectors. The model also assumes that $\eta_t$ is distributed multivariate normally with mean zero and variance-covariance matrix $\Sigma_{\eta}$.

The riskless bond is in perfectly elastic supply and pays $r > 0$ units of the consumption good as interest at the beginning of each period. It serves as numeraire for the economy and thus always sells for the price of unity. The gross interest rate is denoted by $R = 1 + r$.

In each period, a new generation is born. They have a mass of unity and as described above come endowed with a personal share of labor supply and units of the bond. People derive utility from consumption of the single good. They possess a negative exponential utility with a constant absolute-risk aversion parameter $\theta$. After the stocks and the bond pay their owners
at the beginning of period $t$, trading takes place. All agents form rational expectations about the market prices of stocks.

Each firm manager seeks to maximize proceeds from the issuance or repurchase of his own firm’s shares. Investors provide liquidity to the market by absorbing the shares issued. All agents observe current prices, dividends, supply levels, shares issued, and the whole history of these quantities.

### 1.2 Equilibrium

The model’s solution is found by conjecturing a price process, solving for its unknown parameters and verifying that the result is a Nash equilibrium. Market participants conjecture that prices $P_t$ follow,

$$P_t = AN_t + BD_t,$$

where $A$ and $B$ are $K$ by $K$ matrices to be determined. This paper only examines stationary equilibria in which these coefficient matrices are time invariant. Given the supply process of real assets in (1), this price function implies that there is a price impact associated with new capital purchases or sales $Ay_t$. As will be seen below this comes about because risk averse investors require a premium for holding real assets that are in relatively abundant supply.

Firm $k$’s manager decides the net issuance, $y_{kt}$, of his own firm’s shares to maximize the proceeds,

$$\max_{y_{kt}} y_{kt}p_{kt} = y_{kt}(a'_k N_t + b'_k D_t),$$

where $p_{kt}$ is his firm’s share price and $a'_k$ and $b'_k$ are the $k$’th row of the $A$ and $B$ matrices, respectively. Note that the vector products inside the parentheses in Equation (3) are scalars.
The investors maximize their expected utility. Let $X_t$ be their stock position. The investors’ future wealth, $W_{t+1}$, is

$$W_{t+1} = X_t'Q_{t+1} + RW_t,$$

$$Q_{t+1} = P_{t+1} + D_{t+1} - RP_t,$$

where $W_t$ is their exogenously given endowment. $Q_{t+1}$ represents the vector of payoffs from the zero-investment portfolio that purchases one share of each stock financed by a short position in the bond. We call $Q_{t+1}$ “excess payoffs.” By the property of the negative exponential utility, the investors’ utility maximization problem, $\max_{X_t} E_t[-\exp(-\theta W_{t+1})]$, amounts to maximizing the certainty equivalent of future wealth,

$$\max_{X_t} E_t[W_{t+1}] - \frac{\theta}{2} Var_t(W_{t+1}).$$

The first order condition is given by

$$X_t = \frac{1}{\theta} Var_t^{-1}(Q_{t+1})E_t[Q_{t+1}].$$

The second order condition is met if $Var_t(Q_{t+1})$ is positive definite. Because investors absorb the shares issued in each period, they collectively hold the total shares outstanding at any time $t$,

$$X_t = N_t.$$

This is the usual equilibrium condition that prices equilibrate per capita demand and supply.
The following theorem characterizes the equilibrium prices and the managers’ equity-issuance strategy.

**Theorem 1 (Equilibrium)** There exists an equilibrium if the coefficient matrices in the price function (2) are a real-valued solution to the following system of nonlinear matrix equations:

\[
0 = AF^{-1}A + rA + \theta D_A F^{-1} A \Sigma \eta A' F' \Sigma A A' D_A \\
+ \theta (I + D_A F^{-1} B) \Sigma \delta (I + D_A F^{-1} B)',
\]

**(9)**

\[
B = (AF^{-1} + rI)^{-1},
\]

**(10)**

\[
F = A + D_A,
\]

**(11)**

where \(D_A\) is a negative definite diagonal matrix containing the principal diagonal of \(A\). The vector of optimal equity issuances is given by

\[
y_t = -D_A^{-1} P_t.
\]

**(12)**

**Proof.** All proofs are contained in the Appendix. ■

Note that, by substituting equations (10) and (11) into (9), the “system” of nonlinear matrix equations presented in this theorem reduces merely to a single nonlinear matrix equation for \(A\). Since all diagonal elements of \(D_A\) are negative in equilibrium, Equation (12) immediately implies the following result regarding the market-timing behavior of equity-issuing managers:

**Proposition 1 (Equity issuance)** A firm’s manager issues more (fewer) shares as the market price of his firm’s stock rises (falls). Equivalently, the firm’s addition of real capital is positively related to its share price.

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1.3 A Special Case with Analytic Solutions

When \( A \) is a diagonal matrix or a scalar, Equations (11) and (10) reduce to \( F = 2A \) and \( B = \frac{2}{1+2r}I \), respectively. This can occur when there is a single security \( (K = 1) \) or when there are multiple but independent securities whose supply and dividend shocks are cross-sectionally uncorrelated (i.e., \( \Sigma_{\delta} \) and \( \Sigma_{\eta} \) are diagonal). Substituting the simplified expressions for \( F \) and \( B \) into Equation (9) yields the following result.

**Corollary 1** (Equilibrium with independent securities) When there is a single security \( (K = 1) \) or multiple securities with cross-sectionally uncorrelated supply and dividend shocks \( (\Sigma_{\delta} \) and \( \Sigma_{\eta} \) are diagonal), the equilibrium is characterized by the price function

\[
P_t = AN_t + \frac{2}{1+2r}D_t,
\]

where \( A \) is a diagonal matrix satisfying the following quadratic matrix equation:

\[
A\Sigma_{\eta}A + \frac{2}{\theta}(1 + 2r)A + 4[1 + (1 + 2r)^{-1}]^2\Sigma_{\delta} = 0.
\]

The \( A \) matrix is always negative definite if it is a solution to this equation. The vector of optimal equity issuances is given by

\[
y_t = -A^{-1}P_t.
\]

Since all the diagonal elements of \( A \) are negative in equilibrium, Equation (15) again implies that a firm’s manager issues more shares as the market price of his firm’s stock rises and vice versa (Proposition 1). Additionally, explicit expressions allow us to examine the relation
between the state variables of the economy and equity issuance. First, substituting the price function (13) into Equation (15), we see that equity issuance and hence the supply of shares to marginal investors depends on the fundamentals (dividends) inherent in prices. Thus, the level of equity issuance can vary with the business cycle and rise in periods of good economic conditions with strong output. This is another distinction of our model from Spiegel (1998) and Watanabe (2008), in which supply of shares is exogenously given and is independent of dividends.

Second, equity issuance (real capital investment) tracks stock price changes. The risk neutral value equals the $D_t$ term in Equation (13). Adding the “discount” $AN_t$ (recall that $A$ is negative definite) yields the market prices on the left hand side. A smaller discount implies a higher market price which induces the firm to create real assets by raising money through an equity issuance. Importantly, this suggests that equity issuance predicts future returns, because higher current prices imply lower expected returns. The next section examines this possibility in detail.

1.4 Return Predictability of Equity Issuance

So far the analysis has examined prices. But for empirical work the paper’s results need to be reworked in terms of returns. Since investors are mean-variance optimizers (see Equation (6)), the relationship between expected returns and risk is completely characterized by the first two moments of the expected excess payoffs. The following corollary calculates these moments.

**Corollary 2 (Expected excess payoffs)** The expected excess payoffs are given by

$$E_t[Q_{t+1}] = -\theta V A^{-1} D_y t + D_t,$$

(16)
where $V \equiv \text{Var}_t(Q_{t+1})$. In particular, when there is a single or multiple but independent securities (denoted by superscript “ind”), the expressions simplifies as

$$
\mathbf{E}_t[Q_{t+1}^{\text{ind}}] = -\theta V y_t + D_t.
$$

Equation (17) for the expected excess payoffs in independent markets is particularly insightful. Note that $V$ is diagonal in this case and all of its diagonal elements are positive. Thus, ceteris paribus, larger net equity issuance (real capital investment) predicts lower future returns. Furthermore, the magnitude of this effect is stronger for high volatility stocks.

With this in mind, we derive empirical hypotheses using simulated returns from the general model. Define the excess return vector, $r_{t+1}$, and the relative net share change vector, $z_t$, as

$$
\begin{align*}
\begin{cases}
r_{t+1} & \equiv Q_{t+1} \otimes P_t = \mathcal{D}_{P,t}^{-1}Q_{t+1}, \\
z_t & \equiv y_t \otimes N_{t-1} = \mathcal{D}_{N,t-1}^{-1}y_t, 
\end{cases}
\end{align*}
$$

where $\otimes$ is the elementwise division operator and $\mathcal{D}_{P,t}^{-1}$ and $\mathcal{D}_{N,t-1}^{-1}$ are the diagonal matrices containing $P_t$ and $N_{t-1}$, respectively, in their main diagonal. Let the $k$’th element of $r_{t+1}$ and $z_t$ be $r_{k,t+1}$ and $z_{k,t}$, respectively. A regression analogue of Equations (16) and (17) using the return is

$$
\begin{align*}
\begin{cases}
r_{k,t+1} & = \gamma_{k,t-1}z_{k,t} + \varepsilon_{k,t+1}, \\
\gamma_{k,t-1} & = \frac{\text{Cov}_{t-1}(r_{k,t+1}, z_{k,t})}{\text{Var}_{t-1}(z_{k,t})}.
\end{cases}
\end{align*}
$$
The $\gamma$ coefficient in Equation (19) involves a ratio of normals and can be computed most conveniently by simulation.\footnote{The univariate distribution of a ratio of normals is known as the Fieller distribution (Fieller (1932)). We need to go further and consider the joint distribution between the ratio of normals, $r_{k,t+1}$, and a normal variable, $y_{k,t}$ (scaled by $1/N_{k,t-1}$), given the time $t-1$ information set. Yatchew (1986) discusses how to obtain a joint density function involving ratios of multivariate normals. Even if the density function has a closed form, moments may not; see Yatchew (1985, 1986).} In doing so, we carefully choose the parameter values of the model to match the key quantities: the conditional variance of individual stock returns, $\sigma^2_{r,k,t-1} \equiv Var_{t-1}(r_{k,t})$, and the mean normalized share change, $\bar{z}_{k,t-1} \equiv E_{t-1}[z_{k,t}]$. These are the sorting keys that we will use in the empirical analysis below. We choose parameter values so that simulated $\sigma^2_{r,k,t-1} = 0.00112$ and $\bar{z}_{k,t-1} = 0.00549$ in the high volatility equilibrium to be identified below. These are the monthly means over the 30 industry portfolios defined on Kenneth French’s web site (the aggregate figures without first collapsing stocks to industry portfolios are 0.00119 and 0.00579, respectively). We simulate 100,000 draws at each set of parameter values with moment matching and a variance reduction technique known as antithetic sampling. An important consideration in a negative exponential utility-normal payoff framework is to ensure positive prices. The probability of such an event, however, can be made arbitrarily small with proper parameter choices. We ensure that prices, dividends, and supply at time $t-1$, $t$, and $t+1$ are almost positive at any point in the simulation.\footnote{In our simulation, negative values of these quantities occur only a few out of 100,000 draws.}

Two caveats follow before proceeding further. First, Equation (18) is a univariate simple regression rather than a multivariate multiple regression that Equation (16) implies when $A$ is nondiagonal. Second, in Equation (16) $y_t$ is correlated with $D_t$ (see Equation (A2) in the Appendix), whose omission will generally bias the $\gamma$ coefficient in Equation (18). These assumptions are made for numerical tractability in the simulation and for empirical ease when confronted with the data. In any case, our goal here is not to prove a proposition, but to derive...
refutable empirical hypotheses.

Figure 1 plots the elements of $A = \begin{pmatrix} 1 & \rho_A \\ \rho_A & 1 \end{pmatrix}$. The parameter values are shown in the caption. We numerically search for the range $-0.99 \leq \rho_A \leq 0.99$ and find two equilibria.\(^4\) These equilibria exhibit contrasting levels of the common diagonal element, $a$ (Panel A). While it is negative in both equilibria, the equilibrium marked by stars yields a much larger absolute value of $a$. Panel B indicates that the off-diagonal ratio, $\rho_A$, is zero in these equilibria.

Figure 2 plots simulated equilibrium quantities. Panels A and B present the common variance ($\sigma^2_{r,k,t-1}$) and the cross-sectional correlation ($\rho_{r,t-1}$) of returns, respectively, where

$$Var_{t-1}(r_t) = \sigma^2_{r,k,t-1} \begin{pmatrix} 1 & \rho_{r,t-1} \\ \rho_{r,t-1} & 1 \end{pmatrix}.$$  

These panels tell us the following properties of the two equilibria:

- Stars: low volatility,
- Circles: high volatility.

While the return variance is almost invariant with the supply shock volatility in the low volatility equilibrium, it decreases in the high volatility equilibrium (Panel A). At the leftmost point in the high volatility equilibrium, the return variance takes the fitted value of $0.00112$ (at that point, $\tau_{k,t-1}$ also takes the fitted value of $0.00549$, which is not depicted). Panel B indicates that cross-sectional return correlation is zero in both equilibria. This confirms the findings of Spiegel (1998) and Watanabe (2008) using returns rather than the prices that they work with.

A new implication is presented in Panel C, which depicts the $\gamma$ coefficient in Equation (19).

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\(^4\)The number of equilibria is hard to pin down due to the lack of analytic solutions in the general case. Spiegel (1998) and Watanabe (2008) find $2^K$ equilibria in a $K$-security model. Trial computation at other parameter values indicates that there can be four equilibria, two of which exhibit positive and negative return correlation even if all underlying shocks are cross-sectionally uncorrelated. Such correlated equilibria are discussed in Watanabe (2008).
The coefficient is negative in both equilibria, and in the case of high volatility equilibrium, the graph is roughly the mirror image of the return variance in Panel A. Therefore, the implication from Equation (17) appears to hold in return space as well. We summarize these results as the following empirical hypothesis, which will be examined in the next section:

**Hypothesis 1** *(Equity issuance and expected return)*

A. *Controlling for return variance, stocks with higher normalized share changes (investment) have higher subsequent returns.*

B. *Controlling for normalized share changes (investment), stocks with a higher return variance have higher subsequent returns.*

### 2 Empirical Evidence

Daniel and Titman (2006), Fama and French (2006, 2007), and Pontiff and Woodgate (2006) find that stocks with larger net equity issuances have lower subsequent returns in cross-sectional regressions and calendar-time portfolio strategies. Hypothesis 1 offers a way to take a closer look at this equity issuance effect. We now examine the hypothesis empirically.

#### 2.1 Methodology

We form portfolios of stocks sorted on normalized share change and volatility. Following Pontiff and Woodgate (2006), we compute one-year percentage changes in the number of shares outstanding (*SHRCHG*) for a period between 18 and 6 months ago. The six-month lag accounts for possible delay in the dissemination of accounting information. This is in accordance with the usual practice of lagging accounting measures such as the book-to-market ratio. The
issuance measure captures only real share changes and is adjusted for nominal share changes such as stock dividends and splits. We compute return volatility ($VOL$) as the standard deviation of daily returns in the previous month. We calculate these measures monthly for each stock on NYSE, AMEX, and NASDAQ included in the CRSP dataset. The sample period is from January 1963 through December 2006. To be consistent with Equation (19), we employ monthly rebalancing and construct overlapping portfolios a la Jegadeesh and Titman (1993). This also increases the power of the test. Each month, stocks are sorted independently by $SHRCHG$ and $VOL$ into triplets. Nine portfolios are formed as the cross section of these triplets. The NYSE breakpoints are used for $SHRCHG$. The portfolios are held for the next $M$ months, $M = 1, 6$, and value-weighted returns are calculated. Thus, there are $M$ cohorts of value-weighted portfolios at any given month. We compute the equally-weighted return of these $M$ cohorts.

2.2 Result

Table 1 shows the characteristics of the constructed portfolios. The top left panel indicates that, as expected, volatile stocks tend to have small market capitalization ($SIZE$). The relation between $SHRCHG$ and $SIZE$, however, is not monotone. Within the least volatile stocks, firms that issue more shares tend to be smaller in size, but the reverse is true for the most volatile stocks. The top right panel indicates that growth stocks issue more shares. This implies that we should rigorously control for the value effect. The next two panels demonstrate that the sorting procedure adequately controls for the level of post-ranking volatility, but there remains some dispersion in share changes within a $SHRCHG$ triplet. The bottom two panels show trade and liquidity variables. While share turnover ($TOV$) increases monotonically with
equity issuance controlling for volatility, Amihud’s illiquidity ratio (ILLIQ) has a U-shaped relation with equity issuance.

Table 2 reports the portfolio excess returns. Consistent with Part A of Hypothesis 1, returns are monotonically decreasing in SHRCHG at all levels of VOL across all the holding periods. The returns on the zero-investment portfolios that go long the lowest SHRCHG stocks and short the highest SHRCHG stocks (denoted by ‘1 − 3’ in the bottom row of each panel) are always positive. They are also significant at the 1% level except only for two cases, (M, VOL) = (1, 1) and (6, 1) (the latter is still significant at 5%). Thus, we find a very strong support for Part A of Hypothesis 1. Moreover, the zero-investment portfolio returns in the bottom row are monotonically increasing with VOL at both horizons. At the highest VOL level, they range from 0.82% (t = 5.12) for M = 6 to 0.99% (t = 4.99) for M = 1. Furthermore, returns of highest SHRCHG stocks are strongly decreasing in VOL. Return spread between the lowest and the highest VOL stocks within that triplet is 1.00% (t = 3.32) for M = 1 and 0.57% (t = 2.03) for M = 6 (denoted by ‘1 − 3’ in the rightmost column of each panel). However, the return spread within other triplets is insignificant. Thus, there is only limited support for Part B of Hypothesis 1. It is possible that highest volatility stocks load on known risk factors and therefore earn extra risk premia. We now control for this possibility.

We regress the excess returns of each portfolio \( r_{k,t}^e \) on the excess market return (MKTRF) and the size (SMB), value (HML), and momentum (MOM) factors:

\[
r_{k,t}^e = \alpha_k + \beta_k^{MKTRF} MKTRF_t + \beta_k^{SMB} SMB_t + \beta_k^{HML} HML_t + \beta_k^{MOM} MOM_t + \varepsilon_{k,t}.
\]

The sum of the intercept and the residual from this regression, \( \alpha_k + \varepsilon_{k,t} \), is a measure of
risk-adjusted return. By construction, its mean equals the alpha from the four factor model. This will also be used as a risk-adjusted return in the panel regressions below to test further restrictions. Table 3 reports the means and the significance of these risk-adjusted returns. Generally, the risk-adjusted adjusted returns are much lower than excess returns in Table 2. More importantly, the risk adjustment is larger for more volatile stocks. Consequently, the zero investment portfolios that go long least volatile stocks and short most volatile stocks earn significantly positive risk-adjusted returns. These are shown in the rightmost column labeled ‘1 − 3,’ and range from 0.36% ($t = 1.97$) to 1.07% ($t = 5.89$) for $M = 1$ and from 0.24% ($t = 1.87$) to 0.64% ($t = 4.04$) for $M = 6$. All these numbers are significant at the 5% level except for one case, which is still significant at 10%.

Finally, we examine the model’s implication in a regression framework. Recall Panel C of Figure 2, which visually indicates that the $\gamma$ coefficient in Equation (19) is roughly proportional to the return variance in Panel A. In prices, it has also been demonstrated analytically using Equation (17). Thus, consider a restriction

$$\gamma_{k,t-1} = \phi \sigma^2_{r,k,t-1}$$

in the regression in Equation (18). Note that $\phi$ is constant across assets, which makes the equation amenable to a panel regression of the form:

$$r_{k,t+1} = \alpha + \phi_1 z_{k,t} + \phi_2 \sigma^2_{r,k,t-1} z_{k,t} + \varepsilon_{k,t+1} + \epsilon_{t+1},$$

where we have included the term $\phi_1 z_{k,t}$ as a control and an additional time-series error term,
That is, we estimate this equation as a two-way random effects model. We expect that
\[ \phi_2 < 0. \]

While \( SHRCHG \) is already lagged for six months, to be sure, we further take one-month lag and use it as a proxy for \( z_{k,t} \). For \( \sigma^2_{k,t-1} \), we simply use lagged squared \( VOL \), denoted as \( VAR \).

Table 4 presents the result of the panel regression. Panel A uses excess returns as \( r_{k,t+1} \), and Panel B the risk-adjusted returns. In each panel, Columns 1 and 2 restrict \( \phi_2 = 0 \) and \( \phi_1 = 0 \), respectively, in Equation (20). Column 3 is the full model. We see that \( \phi_2 \), the coefficient on \( VAR \times SHRCHG \), is negative and significant at 1% in all specifications. This is consistent with our model’s implications.

3 Discussion

3.1 Correlated Shocks

The numerical analysis in Section 1.4 assumes independent markets, i.e., the supply and dividend shocks are cross-sectionally uncorrelated. In reality, we have reason to believe that dividend shocks may be correlated; for example, in the expansion stage of a business cycle, a rise in the productivity of one industry may positively affect that of another industry. Our result is robust to such correlated shocks. Figures 3 and 4 vary the dividend shock correlation, \( \rho_\delta \), holding other parameters at our base values. Our key result remains valid; the \( \gamma \) coefficient in Panel C of Figure 4 is negative in the high volatility equilibrium, and its graph is the mirror images of the return variance in Panel A. We note that the \( \gamma \) coefficient in the low volatility
equilibrium (stars) does become positive when $\rho_\delta$ is negative, as indicated by the truncated graph. However, its magnitude is negligibly small; note that the graphs are plotted in log scale. Moreover, both Spiegel (1998) and Watanabe (2008) argue that high volatility equilibria are empirically more relevant than a low volatility equilibrium; calibrating their models with the estimates in Shiller (1981), they conclude that the price volatility observed in the data is consistent with high volatility equilibria rather than a low volatility equilibrium.

Interestingly, Pontiff and Woodgate (2006) report that the return predictability of equity issuance is insignificant or even becomes positive and significant at some horizon in the pre-1970 period. Does this suggest the possibility that the economy was in a low volatility equilibrium before 1970? We will return to this question in the conclusion section.

3.2 Risk or Misparking?

This subsection discusses implications of our model further. Some researchers hold a view that firm managers issue shares when they see their firms' shares “overpriced” in the market. To understand the sense in which they may be apparently “overpriced” in our model, it is illuminating to compare the equilibrium price function to those of Spiegel (1998) and Watanabe (2008). These authors show that in a fully rational model with overlapping generations of competitive agents, there exists an equilibrium in which supply shocks are amplified to produce “excessive volatility.”

If there is no share issuance in the current model, the resulting price function will be identical to Spiegel (1998):

$$P_t = AN_t + \frac{1}{r}D_t,$$
where the $A$ matrix satisfies

$$A \Sigma \eta A + \frac{r}{\theta} A + \frac{R^2}{\tau^2} \Sigma \delta = 0.$$  \hfill (21)

In the full-information model of Watanabe (2008), investors know one-period-ahead dividends, $D_{t+1}$. The timing of the dividends then leads to the following price function:

$$P_t = AN_t + \frac{1}{r} D_{t+1},$$

where the $A$ matrix satisfies

$$A \Sigma \eta A + \frac{r}{ \theta} A + \frac{1}{r^2} \Sigma \delta = 0.$$  \hfill (22)

In these no-share-issuance models, prices are given by the perpetuity of known dividends ($\frac{1}{r} D_t$ in Spiegel (1998) and $\frac{1}{r} D_{t+1}$ in Watanabe (2008)) plus a discount due to supply pressure, $AN_t$. This is a “discount” because $A$ is negative definite in both models. The intercept term in Equation (22) is smaller by a factor of $1/R^2$ than that in Equation (21), because the dividend uncertainty is put forward by one period.\footnote{See Watanabe (2008) for the analysis with intermediate levels of information between these two extreme cases.} Since the $A$ matrix is determined by a quadratic equation, there are multiple equilibria in both cases. The authors show that the prices can be “excessively volatile” relative to the dividend-shock volatility. For ease of exposition, assume that there is a single security ($K = 1$) and hence all matrices are scalars. The excessively volatile equilibrium corresponds to the smaller (negative and larger in magnitude) root of $A$. In such an equilibrium, prices are sensitive to the supply shocks, producing apparent volatility in excess of the fundamentals’ (dividend’s) variability.

The same intuition carries over to our model, with $A$ now measuring not only stock price
variance, but also the degree of negative returns subsequent to unit equity issuance. For simplicity, consider the case of independent securities. By analogy, the $D_t$ in the price function (13) is the fundamental value of the stocks. It is altered from the standard perpetuity formula due to the price impact of equity issuance. Added to this is a discount $AN_t$. Because $A$ is the solution to the quadratic matrix equation (14), there are multiple equilibria. The smaller (negative and larger in magnitude) root of $A$ corresponds to a highly volatile equilibrium, in which prices tend to deviate from the fundamental value $\frac{2}{1+2r}D_t$ by a larger spread, $AN_t$. While firm managers will issue fewer shares in a high-volatility equilibrium than in a low-volatility equilibrium (notice the coefficient $-A^{-1}$ in Equation (15)), its predictability is disproportionately large; Equation (A8) in the Appendix implies that $V = -\frac{1}{2\theta}(1 + 2r)A$ and therefore Equation (17) becomes

$$E_t[Q_{t+1}^{ind}] = \frac{1}{2}(1 + 2r)Ay_t + D_t.$$  

It is in this sense that firm managers behave as if they are issuing more shares when the prices appear to be “overpriced,” in that they are followed by lower returns. And yet all agents are rational and optimally execute their transactions.

As stated above, in addition to the negative returns following equity issuance, our model can reconcile another major empirical regularity, the excess volatility of stock returns. Calibrating their models using empirically estimated parameter values, both Spiegel (1998) and Watanabe (2008) demonstrate that only very small supply shocks are necessary to produce the levels of return volatility observed in the data. Moreover, Watanabe (2008) shows that a high-volatility equilibrium can exhibit strong or weak correlations between stock returns. Our model adds asset-pricing implications, namely, implications on the first moment of stock returns, to those
on the second moments studied by these authors.

4 Conclusions

This paper presents and tests a model of endogenous equity issuance. Competitive risk-averse investors accommodate shares issued by proceeds-maximizing firm managers. The managers exhibit market timing behavior because they issue more shares as their firm’s stock price rises. Since the endogenous equity issuance offsets the exogenous supply shocks, both the issuance and stock prices are mean reverting. Consequently, higher equity issuance accompanying higher current prices are followed by lower subsequent returns and vice versa. This negative return predictability of equity issuance, along with the managers’ market timing behavior, results from the interaction of fully rational agents. We calibrate the model to the data using returns and derive a primary implication that, holding the level of equity issuance constant, stocks with higher return volatility should earn lower subsequent returns. The empirical analysis supports this prediction among others.

Our analysis leaves some unresolved issues. First, our result is not to be interpreted as precluding irrational stories for the return predictability of net equity issuance. Second, it is puzzling that, as reported in Pontiff and Woodgate (2006), the return predictability of equity issuance is insignificant before 1970. In fact, they find that equity issuance positively predicts future returns at some horizon in the pre-1970 period. Does this suggest the possibility that the stock market was in a different equilibrium before 1970? In our terminology, the economy in the early years might have been in a low volatility equilibrium, which exhibits weak or even positive return predictability of equity issuance (Recall Panel C of Figure 4). However, it is still hard for
our low volatility equilibrium to explain the pre-1970 result, since it comes with counter-factual properties such as negative dividend shock correlation and little excess volatility.
A Appendix

A.1 Proof of Theorem 1

Rewrite Equation (3) as

$$\max_{y_{kt}} p_{kt} = y_{kt}[a'_{k}(N_{t-1} + \eta_t + y_t) + b'_k D_t].$$

Noting that $y_t$ contains $y_{kt}$ in its $k$’th position, the first order condition is given by

$$a'_{k}(N_{t-1} + \eta_t + \overline{y}_{kt}) + 2a_{kk}y_{kt} + b'_k D_t = 0,$$

where $\overline{y}_{kt}$ is the vector created by replacing the $k$’th element of $y_k$ by zero and $a_{kk}$ is the $k$’th element of $a'_k$ (the $k$’th diagonal element of $A$). This can further be rewritten as

$$a_{kk}y_{kt} = -a'_{k}(N_{t-1} + \eta_t + y_{kt}) - b'_k D_t, \forall k.$$

Observe that we can stack this equation in a matrix form,

$$\mathcal{D}_A y_t = -AN_t - BD_t = -P_t, \quad (A1)$$

where $\mathcal{D}_A$ is again the diagonal matrix containing the principal diagonal elements of $A$. This immediately yields Equation (12) in the theorem. Substitute Equation (1) and solve for $y_t$ to
write it with only exogenous shocks:

\[ y_t = -F^{-1}[A(N_{t-1} + \eta_t) + BD_t], \]  

\[ F \equiv A + D_A. \]

Using this, we can write the prices in (2) with exogenous shocks only:

\[ P_t = A(N_{t-1} + \eta_t + y_t) + BD_t \]

\[ = (I - AF^{-1})[A(N_{t-1} + \eta_t) + BD_t]. \]

This indicates that the price vector follows a vector autoregressive process

\[ P_{t+1} = (I - AF^{-1})[P_t + A\eta_{t+1} + B\delta_{t+1}], \]

provided that the norm of \( I - AF^{-1} \) is sufficiently small. Substitute this expression into Equation (5) and rearrange to write

\[ Q_{t+1} = (I - AF^{-1})[P_t + A\eta_{t+1} + B\delta_{t+1}] + D_{t+1} - RP_t \]

\[ = (I - AF^{-1})A\eta_{t+1} + [(I - AF^{-1})B + I]\delta_{t+1} \]

\[ + D_t - (AF^{-1} + rI)P_t. \]
From this, compute the first two conditional moments of future wealth:

\[
E_t[Q_{t+1}] = D_t - (AF^{-1} + rI)P_t \tag{A3}
\]

\[
= -(AF^{-1} + rI)AN_t + (I - AF^{-1}B - rB)D_t.
\]

\[
Var_t(Q_{t+1}) = (I - AF^{-1})A\Sigma(\eta A'(I - AF^{-1})' + [(I - AF^{-1})B + I]\Sigma[I - (I - AF^{-1})B + I]' \tag{A4}
\]

Substitute the first order condition (7) into the market clearing condition (8) and rearrange to obtain

\[
\theta V N_t = E_t[Q_{t+1}] = -(AF^{-1} + rI)AN_t + \{I - (AF^{-1} + rI)B\}D_t. \tag{A5}
\]

Again, use Equation (A2) to write the supply by only exogenous shocks:

\[
N_t = N_{t-1} + \eta_t + y_t = (I - F^{-1}A)(N_{t-1} + \eta_t) - F^{-1}BD_t.
\]

Substitute this equation into (A5) and rearrange:

\[
0 = -\{\theta V + (AF^{-1} + rI)A\}(I - F^{-1}A)(N_{t-1} + \eta_t)
\]

\[
+ [\{\theta V + (AF^{-1} + rI)A\}F^{-1}B + \{I - (AF^{-1} + rI)B\}]D_t. \tag{A6}
\]

For this equation to hold almost surely, the coefficients on \(N_{t-1} + \eta_t\) and \(D_t\) must be zero.

Assuming that

\[
I - F^{-1}A = F^{-1}(F - A) = F^{-1}DA \tag{A7}
\]
or $F$ is nonsingular, this implies that the curly brackets in Equation (A6) are all zero:

$$0 = \theta V + (AF^{-1} + rI)A, \quad (A8)$$

$$0 = I - (AF^{-1} + rI)B.$$

The second equation immediately gives Equation (10) in the theorem. Substituting Equation (A4) for $V$ in the first equation and using the relation $I - AF^{-1} = D_AF^{-1}$ gives Equation (9) in the theorem. ■

A.2 Proof of Corollary 2

Rewrite Equation (A3) as

$$E_t[Q_{t+1}] = D_t - B^{-1}P_t \text{ by (10)}$$

$$= D_t + B^{-1}D_A y_t \text{ by (A1)}.$$ 

$$= -\theta V A^{-1}D_A y_t + D_t,$$

since $B^{-1} = -\theta V A^{-1}$ by Equation (A8). ■
References


Table 1: Characteristics of portfolios sorted on changes in shares outstanding and volatility

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Characteristics of portfolios sorted on changes in shares outstanding and volatility. SHRCHG is the one-year percentage change in the number of shares outstanding for a period between 18 and 6 months ago. VOL is the standard deviation of daily returns in the previous month. These measures are computed monthly for each stock on NYSE, AMEX, and NASDAQ included in the CRSP dataset. The sample period is from January 1963 through December 2006. Each month, stocks are sorted independently by SHRCHG and VOL into triplets. Nine portfolios are formed as the cross section of these triplets. The NYSE breakpoints are used for SHRCHG. The portfolios are held and value-weighted returns are calculated for the next month. The strategy holds the equally-weighted portfolio of these M cohorts. Reported are the post-ranking characteristics of the nine portfolios. SIZE is the market capitalization in millions of dollars. BM is the book-to-market ratio. TOV is the share turnover in units of one thousandths. ILLIQ is Amihud’s (2002) illiquidity ratio.
Table 2: Excess returns of portfolios sorted on changes in shares outstanding and volatility

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<td>( M = 1 )</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>( SHRCHG )</td>
<td>1</td>
<td>0.0061*** (3.65)</td>
<td>0.0091*** (3.89)</td>
<td>0.0046 (1.44)</td>
<td>0.0015 (0.58)</td>
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<td>2</td>
<td>0.0050*** (2.97)</td>
<td>0.0068*** (2.90)</td>
<td>0.0034 (0.98)</td>
<td>0.0016 (0.57)</td>
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<td>3</td>
<td>0.0047*** (2.75)</td>
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<td>0.0014* (1.73)</td>
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<tr>
<td>( SHRCHG )</td>
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<td>0.0060*** (3.60)</td>
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<td>0.0063*** (2.74)</td>
<td>0.0032 (0.97)</td>
<td>0.0018 (0.73)</td>
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<td>3</td>
<td>0.0042** (2.46)</td>
<td>0.0039 (1.46)</td>
<td>-0.0015 (-0.40)</td>
<td>0.0057** (2.03)</td>
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<td>1-3</td>
<td>0.0018** (2.48)</td>
<td>0.0042*** (4.20)</td>
<td>0.0082*** (5.12)</td>
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Excess returns of portfolios sorted on changes in shares outstanding and volatility. \( SHRCHG \) is the one-year percentage change in the number of shares outstanding for a period between 18 and 6 months ago. \( VOL \) is the standard deviation of daily returns in the previous month. These measures are computed monthly for each stock on NYSE, AMEX, and NASDAQ included in the CRSP dataset. The sample period is from January 1963 through December 2006. Each month, stocks are sorted independently by \( SHRCHG \) and \( VOL \) into triplets. Nine portfolios are formed as the cross section of these triplets. The NYSE breakpoints are used for \( SHRCHG \). The portfolios are held and value-weighted returns are calculated for the next \( M \) months. The strategy holds the equally-weighted portfolio of these \( M \) cohorts. Reported are the returns and t-statistics in parentheses. *, **, *** represent significance at 10, 5, and 1%, respectively.
Table 3: Risk-adjusted returns of portfolios sorted on changes in shares outstanding and volatility

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<td>$SHRCHG$</td>
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<tr>
<td>1</td>
<td>0.0010* (1.74)</td>
<td>0.0035*** (4.90)</td>
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<td>0.0001 (0.16)</td>
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<td>-0.0002 (-0.30)</td>
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<td>1 − 3</td>
<td>0.0012 (1.48)</td>
<td>0.0047*** (4.31)</td>
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Risk-adjusted returns of portfolios sorted on changes in shares outstanding and volatility. $SHRCHG$ is the one-year percentage change in the number of shares outstanding for a period between 18 and 6 months ago. $VOL$ is the standard deviation of daily returns in the previous month. These measures are computed monthly for each stock on NYSE, AMEX, and NASDAQ included in the CRSP dataset. The sample period is from January 1963 through December 2006. Each month, stocks are sorted independently by $SHRCHG$ and $VOL$ into triplets. Nine portfolios are formed as the cross section of these triplets. The NYSE breakpoints are used for $SHRCHG$. The portfolios are held and value-weighted returns are calculated for the next $M$ months. The strategy holds the equally-weighted portfolio of these $M$ cohorts. We regress excess return of each portfolio on a constant, the excess market return ($MKTRF$), the size ($SMB$), book-to-market ($HML$), and momentum ($MOM$) factors. The Risk-adjusted return is computed as the excess portfolio return less the sum of the four factors times their respective betas. Reported are the alphas and t-statistics in parentheses. *, **, *** represent significance at 10, 5, and 1%, respectively.
### Table 4: Panel regressions

#### Panel A: Excess returns

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<td>$VAR \times SHRCHG$</td>
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<td>-3.83***(-2.70)</td>
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#### Panel B: Risk-adjusted returns

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<tr>
<td>Const</td>
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<tr>
<td>$VAR \times SHRCHG$</td>
<td>-4.27***(-5.04)</td>
<td>-4.52***(-4.24)</td>
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This table shows the estimated parameters of the panel regressions. Dependent variables are excess returns (Panel A) or the four-factor risk-adjusted returns (Panel B) of the nine portfolios sorted on changes in shares outstanding and volatility. See the caption to Table 1 for portfolio formation. The independent variables are as follows: $SHRCHG$ is the one-year percentage change in the number of shares outstanding for a period between 18 and 6 months ago. $VAR$ is the squared standard deviation of daily returns in the previous month. $VAR \times SHRCHG$ is their product.
Figure 1: Elements of matrix $A$ representing price sensitivity to supply shocks. Panel A: The common diagonal elements, $a$; Panel B: The off-diagonal ratio, $\rho_a$, such that $A = a \begin{pmatrix} 1 & \rho_a \\ \rho_a & 1 \end{pmatrix}$. The panels show these quantities in the symmetric equilibria of the two security model. Parameter values: the dividend shock covariance matrix $\Sigma_\delta = 0.464^2 I$, the supply shock covariance matrix $\Sigma_\eta = \sigma_\eta^2 I$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D_{t-1} = 4.64 \times 10^4$, $N_{t-1} = 40.9$, interest rate $r = 0.05/12$, coefficient of absolute risk aversion $\theta = 1$. 
Figure 2: Equilibrium quantities in the symmetric equilibria of the two security model. Panel A: The common return variance, $\sigma^2_{r,k,t-1}$. Panel B: Return correlation, $\rho_{r,t-1}$. Panel C: Slope coefficients, $\gamma_{k,t-1}$. Parameter values: the dividend shock covariance matrix $\Sigma_\delta = 0.464^2 I$, the supply shock covariance matrix $\Sigma_\eta = \sigma^2_\eta I$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D_{t-1} = 4.64 \times 10^4$, $N_{t-1} = 40.9$, interest rate $r = 0.05/12$, coefficient of absolute risk aversion $\theta = 1$. 
Figure 3: Elements of matrix $A$ representing price sensitivity to supply shocks. Panel A: The common diagonal elements, $a$; Panel B: The off-diagonal ratio, $\rho_a$, such that $A = a \begin{pmatrix} 1 & \rho_a \\ \rho_a & 1 \end{pmatrix}$. The panels show these quantities in the symmetric equilibria of the two security model. Parameter values: the dividend shock covariance matrix $\Sigma_\delta = 0.464^2 \begin{pmatrix} 1 & \rho_\delta \\ \rho_\delta & 1 \end{pmatrix}$, the supply shock covariance matrix $\Sigma_\eta = 0.0301^2 I$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D_{t-1} = 4.64 \times 10^4$, $N_{t-1} = 40.9$, interest rate $r = 0.05/12$, coefficient of absolute risk aversion $\theta = 1$. 
Figure 4: Equilibrium quantities in the symmetric equilibria of the two security model. Panel A: The common return variance, $\sigma^2_{r,k,t-1}$. Panel B: Return correlation, $\rho_{r,t-1}$. Panel C: Slope coefficients, $\gamma_{k,t-1}$. Parameter values: the dividend shock covariance matrix $\Sigma_\delta = 0.464^2 \begin{pmatrix} 1 & \rho_\delta \\ \rho_\delta & 1 \end{pmatrix}$, the supply shock covariance matrix $\Sigma_\eta = 0.0301^2 I$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D_{t-1} = 4.64 \times 10^4$, $N_{t-1} = 40.9$, interest rate $r = 0.05/12$, coefficient of absolute risk aversion $\theta = 1$. 