Discussion:
The more we know, the less we agree:
Public announcements, higher-order expectations
and rational inattention

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Discussion by

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Paper’s contribution

• ‘Canonical’ information structure used in incomplete info models: public, private signals about common state

• Certain implications of canonical information structure are special (i.e. not generally valid)

• In particular: public information can increase dispersion, disagreement among traders, in particular in higher-order beliefs

• In noisy REE model: Potential explanation for increase in trading activity around public announcement dates.

• Assumptions driving polarization may naturally arise from specialization in information gathering.
Two parts of the contribution:

- Clarifying special nature of informational assumptions: nice and important theoretical insight
- But what about plausibility of implications for trading activity...?

Discussion:

- Step 1: Discuss role of key assumptions for polarization in a toy model
- Base analysis on extension of ‘beauty contest’ game: static, simpler to handle, fewer ‘GE complications’ than noisy REE model...
- Step 2: Discuss how key assumptions arise in dynamic trading model, and ask whether the required restrictions are plausible.
Toy model: Beauty contest

• State variable:
  \[ \theta = \alpha \theta_1 + (1 - \alpha) \theta_2, \text{ where } \theta_1, \theta_2 \sim \mathcal{N}(0, \tau_{\theta}^{-1}) \]

• Continuum of traders, observe signal \( x_i \sim \mathcal{N}(\theta_1, \tau_x^{-1}) \)

• Public signal:
  \[ z = \beta \theta_1 + (1 - \beta) \theta_2 + \nu, \text{ where } \nu \sim \mathcal{N}(0, \tau_{\nu}^{-1}) \]

• Idea: use comparative statics w.r.t. \( \tau_{\nu} \) to assess impact of public signal.

• Beauty contest: Optimal actions set according to
  \[ a_i = (1 - r)E^i(\theta) + rE^i(a) \]
Role of parameters (Nested Benchmark cases):

- First- vs. Higher-order expectations: $r \in (0, 1)$.
  $\iff$ If $r = 0$, there is no role for higher-order expectations.

- Canonical information structure: $\alpha = 1, \beta < 1$.
  $\iff$ $\alpha < 1$ key for departure from canonical information structure

- Target vs. signal: if $\alpha = \beta$, then signal is public signal of target.
  $\iff$ When $\alpha \neq \beta$, signal more informative about one factor than the other.

Proposition:

- Contrarianism and Polarization in Beliefs and Actions result only when $\alpha < \beta < 1$. (In particular when $\alpha$ is low).
Polarization in First-order expectations:

\[ \beta E^i(\theta_1|x,z) = \frac{\tau_x \hat{\theta}_x + \beta^{-1} \hat{\tau}_x \hat{\theta}_z}{\tau_x + \tau_{\theta} + \hat{\tau}_z}, \]

where \( \hat{\tau}_z = \left( \frac{(1 - \beta)^2 \tau_{\theta}^{-1} + \tau_v^{-1}}{\beta^2} \right)^{-1} = \beta^2 \Lambda \tau_{\theta}, \)

and \( \Lambda = \frac{\tau_v}{(1 - \beta)^2 \tau_{\theta} + \tau_v} \)

- \( \Lambda \) measures informativeness of public signal

- Expectations about \( \theta_2 \):

\[ (1 - \beta)E^i(\theta_2|x,z) = \Lambda E^i((1 - \beta) \theta_2 + \nu|x,z) = \Lambda (z - \beta E^i(\theta_1|x,z)) \]

- Therefore, with public signal \( (\Lambda > 0) \), expectations about \( \theta_2 \) are decreasing in \( x \), even though expectations about \( \theta_1 \) are increasing in \( x \).

- Now, need to combine the two expressions to see when effect of \( \theta_2 \) dominates.
Polarization in First-order expectations (ctd):

\[ \alpha E^i(\theta_1|x,z) + (1 - \alpha) E^i(\theta_2|x,z) = \alpha E^i(\theta_1|x,z) + \frac{1 - \alpha}{1 - \beta} \Lambda(z - \beta E^i(\theta_1|x,z)) \]

\[ = \frac{1 - \alpha}{1 - \beta} \Lambda z + \left( \frac{\alpha}{\beta} - \frac{1 - \alpha}{1 - \beta} \Lambda \right) \beta E^i(\theta_1|x,z) \]

where \( E^i(\theta_1|x,z) = \frac{\tau_x x + \beta \Lambda \tau_\theta z}{\tau_x + \tau_\theta + \beta^2 \Lambda \tau_\theta} \)

- Key: signing the bracket...

- ‘Canonical information structure’: \( \alpha = 1 \), only \( E^i(\theta_1|x,z) \) matters, effect of \( \Lambda \) through weights in expectations (standard channel).

- Two factors, but \( \alpha = \beta \):

  \[ E^i(\theta|x,z) = \Lambda z + (1 - \Lambda) \beta E^i(\theta_1|x,z) \]

  Additional shift towards public signal, still no contrarianism (bracket is positive)
Polarization in First-order expectations (ctd):

\[ E^i(\theta|x,z) = \frac{1 - \alpha}{1 - \beta} \Lambda z + \left( \frac{\alpha}{\beta} - \frac{1 - \alpha}{1 - \beta} \Lambda \right) \beta E^i(\theta_1|x,z) \]

- \( \alpha < \beta < 1 \): public signal may flip sign of bracket (contrarianism), increase magnitude of response to private signal (if \( \alpha \) small, \( \beta, \Lambda \) large)

- Contrarianism: \( \beta > \alpha \): Signal puts more weight on \( \theta_2 \) than target - needed so that effect on \( E^i(\theta_1|x,z) \) (positive) is outweighed by negative effect from \( E^i(\theta_2|x,z) \).

- \( \Lambda \) sufficiently high: public signal needs to convey enough information to make it worthwhile to disentangle \( \theta_1 \).

- \( \alpha \) small: agents have signals about \( \theta_1 \), but would really like to know \( \theta_2 \)... implies that on their own private signals are fairly uninformative about \( \theta \)
General beauty contest case with \( r > 0 \):

- Guess a linear equilibrium strategy, then verify the guess

- Equilibrium displays typical change in weights on private signals from higher-order uncertainty:

- Solution replaces

\[
E^i(\theta_1|x,z) = \frac{\tau_xx + \beta\Lambda_\theta z}{\tau_x + \tau_\theta + \beta^2\Lambda_\theta} \text{ with } \frac{(1-r)\tau_xx + \beta\Lambda_\theta z}{(1-r)\tau_x + \tau_\theta + \beta^2\Lambda_\theta}
\]

- Recap: Contrarianism due to discrepancy between target and signal in factor weights: private signal serves mainly to back out \( \theta_1 \) from public signal.

- No specific role for higher-order beliefs, correlation in signals across agents, etc.
From example to dynamic CARA trading model

- Multiple trading rounds: 1, 2, 3. Asset dividend is $\theta_1 + \theta_2$.

- Announcement $z = \alpha\theta_1 + (1 - \alpha)\theta_2 + \nu$ in period 2. Dividend realized after round 3.

- Traders: short-lived (OLG), receive private signals about $\theta_1$ in period 1 and 2, $\theta_2$ in period 3.

- Normal Signals, CARA preferences, normal (mean 0) supply. Demand:
  
  $$a_t^i = \rho(E_t^i(\pi_{t+1}) - p_t)/V_t^i(\pi_{t+1}).$$

- Assume $V_t^i(\pi_{t+1}) = V_t$, and impose market clearing:
  
  $$p_t = \bar{E}_t(\pi_{t+1}) - s_t \frac{V_t}{\rho}$$

- Price is average expectation of tomorrow’s return (plus noise).
Segmentation across time and agents:

- Short-lived OLG traders: $\pi_2 = p_2$, $\pi_3 = p_3$, $\pi_4 = \alpha \theta_1 + (1 - \alpha)\theta_2$

- Through OLG assumption, it’s higher-order expectations that matter...

- Informational segmentation: early traders learn about $\theta_1$, late traders about $\theta_2$.

- Without announcement: no more information about $\theta_1$ priced in after period 2. But $p_3$ responds mainly to period 3 information - informational segmentation limits value of signal about $\theta_1$, unless...

- ...Public signal allows for disentangling motive (and contrarianism).

- **Key assumptions:*** no buy-and-hold possibility, no possibility to learn about $\theta_2$ at date 1.
More on informational segmentation

- OLG assumption motivated by appeal to short-horizon performance motives
- In practice, traders are long-lived (even if incentives have short-horizon).
- Informational assumptions as initial set-up of analyst activities.
- OLG assumption implies ‘forgetfulness’ - private information gained in round 1 cannot be used again in subsequent periods.

Endogenous information choice to generate segmentation:

- Key technical restriction: no learning about $\theta_2$ possible until date 3.
- Important: early traders would really like to learn about $\theta_2$ if they could.
- But $\theta_2$ is already included in public signal, why not in anyone’s private signal?
Conclusion

- Nice theoretical contribution: public information can actually increase dispersion of private beliefs

- Contrarianism and public announcements increasing dispersion in beliefs and trading activity.

- Reminds us of the danger of extrapolating from examples without discussing which insights are due to (restrictive) modeling assumptions, and which ones are general.

- Empirical robustness: Do we believe the assumptions required to sustain the necessary informational segmentation (across time and agents)?

- Contrarianism: Do traders speculate against their own private info because they believe that the announcement will push the market in the other direction?