On the Conditions under which Audit Risk Increases with Information *

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Abstract

It has been reported in the literature on strategic auditing that audit risk (the probability of audit failure) may increase when the auditor obtains information, whereas conditions for such cases have not been identified as yet. This paper provides simple models to analyze the general tendencies of exogenous parameters for such cases. The analysis shows that audit risk increases with more information if the auditee has a sufficiently strong incentive to commit fraud. If the auditee is penalized by auditor rejection even when he does not commit fraud, the detection risk increases with more information. In this case, if the auditor has a sufficiently strong incentive to avoid false rejection, audit risk increases with more information.

Key Words:	strategic audits; audit risk; audit evidence; acceptance sampling
JEL Classification:	C72, D81, D82, M42

1 Introduction

The purpose of this study is to provide two simple audit games for analysis of the conditions under which audit risk (the probability of audit failure) increases with more information. Strategic auditing research shows that audit risk does not always decrease with more information, whereas the exact conditions for the anomalous increase in audit risk remain to be identified.

Early contributions to the literature on strategic auditing mainly focus on the difference between decision theory and game theory. Traditionally, stylized audit models have been decision theoretic or, equivalently, formulated as games against nature (a mechanical device playing chance moves). However, the decision theoretic models fail to explain certain audit phenomena such as randomized audit strategies. In the first study in the strategic auditing literature, Fellingham and Newman (1985) introduce a strategic auditee into their analysis and formulate an audit as a multi-stage game as a combination of a simple simultaneous game and a simple sequential game with perfect information. On analyzing the model, Fellingham and Newman (1985) report that audit risk assessment in a game theoretic setting can be different from assessment in a decision theoretic setting.

The difference between decision theory and game theory and subsequent effects on audit risk continue to be the focus of research on subsequent strategic auditing studies, such as those by Newman and Noel (1989), Fellingham, Newman and Patterson (1989) and Shibano (1990), among others. Newman and Noel (1989) develop a defalcation game between an auditee and an auditor and compare the model with a decision theoretic model. They also derive comparative statics on various risks with respect to the players' payoff parameters, and find that type I and type II audit risks may either increase or decrease with the auditee's payoff parameters, depending on the other parameters.

Fellingham, Newman and Patterson (1989) investigate the strategic incentives for audit information acquisition by analyzing a game with costless perfect information, a game with costly perfect information, and a game with costly imperfect information. In particular, they find a counterintuitive phenomenon whereby the auditor is worse off with the availability of costly perfect information and establish the conditions for this case.

Shibano (1990) introduces a misreporting game that he terms *strategic testing of a report of hidden information*, in addition to a defalcation game that he terms *strategic testing of a hidden action*. Without relying on any specific parameterizations of the probability distributions of audit evidence, Shibano (1990) confirms comparative results regarding a defalcation game and establishes new comparative results for a misreporting game. As in a defalcation game, audit risk either increases or decreases with the auditee's payoff parameters in a misreporting game.

These early studies clarify the relation between the players' payoff parameters and audit risk and that between the payoff parameters and information acquisition, but do not examine the relation between information acquisition and audit risk. Patterson (1993) is the first study that sheds light on this relation. In her study on the auditor's strategic sampling choice, Patterson (1993) extends Newman and Noel's (1989) game to a defalcation game in which the auditor chooses sample size before gathering costly audit information. On analyzing this game, she finds that, for small sample sizes, audit risk may increase with sample size and reports a numerical example of the anomalous cases. It is difficult, however, to seek the definitive analytical conditions for such cases owing to the complexity of the model.

The current study analyzes two simple audit games in which a manager decides whether to commit fraud and an auditor obtains audit evidence and then decides whether to accept the manager's assertion. Each game is compared to the corresponding benchmark game in which the auditor does not gather any information besides the game structure that is common knowledge. These games are rich enough to observe the intriguing phenomenon that audit risk increases with more information and yet simple enough to analyze the general tendencies of exogenous parameters for such a phenomenon. To the best of the current author's knowledge, there is no study that examines the conditions under which audit risk and its components increase with more information. The purpose of this study is to identify such conditions in two distinct simple settings.

The two games examined in this study, the defalcation game and the misreporting game, are the binary signal versions of the two games analyzed by Shibano (1990). The defalcation game can also be regarded as the binary signal version of Newman and Noel's (1989) game. The binary signal versions have some merit, in that they always have at least one Nash equilibrium for each game and their solutions are expressed as rational functions of exogenous parameters. These properties are convenient for comparative game analyses.

The remainder of the paper is organized as follows. Section 2 describes the defalcation game, identifies its equilibria, and conducts comparative game analysis. Section 3 repeats the same procedure for the misreporting game. Section 4 concludes. The Appendix provides proofs omitted in the text.

2 Defalcation game

2.1 Two types of fraudulent act

Statement of Auditing Standards No. 82 classifies fraudulent acts involving material misstatement of financial statements into two types: (1) defalcation (misappropriation of assets) and (2) intentional misreporting (fraudulent financial reporting). Defalcation can be accomplished in various ways, including embezzling receipts, stealing assets, and causing an entity to pay for goods or services not received. On the other hand, intentional misreporting may involve acts such as manipulation of accounting records or supporting documents, misrepresentation of events or transactions, or intentional misapplication of accounting standards (AICPA, 1997, paras. 3–5).

To distinguish between defalcation and misreporting, the current study considers a defalcation game and a misreporting game separately.[1] Both games have two risk-neutral players: a manager and an auditor of a firm.[2] The essential difference between the two games is whether the manager prepares a financial report. In the defalcation game, the null hypothesis for the audit is determined by exogenous factors such as a prior contract, and a legal or regulatory requirement. In the misreporting game, on the other hand, the process determining the null hypothesis is internalized: the manager prepares a report and the auditor examines whether the report contains any misstatement. Thus, these two games have fundamentally different information structures. The defalcation game is described and analyzed in this section.

2.2 Model description

First, the manager decides whether to commit fraud. His action space is binary, so that his random strategy is fully described by the probability to commit fraud α. Second, the auditor observes a signal $\sigma \in \{h, \ell\}$. When the manager commits fraud, the signal becomes h (high) with probability p_F and becomes ℓ (low) with probability $1 - p_F$. On the other hand, when the manager does not commit fraud, the signal becomes h with probability p_G and becomes ℓ with probability $1 - p_G$. Without loss of generality, p_F is assumed to be greater than p_G. Note that the signal h is interpreted as a bad sign, in that the auditor more frequently observes the signal h when the manager commits fraud than when he does not. Finally, using the observed signal as audit evidence, the auditor decides whether to accept an exogenously determined assertion of the firm. The auditor's strategy is fully described by the two probabilities β_1 and β_2 that she accepts the firm's assertion when the signal is h and ℓ, respectively.

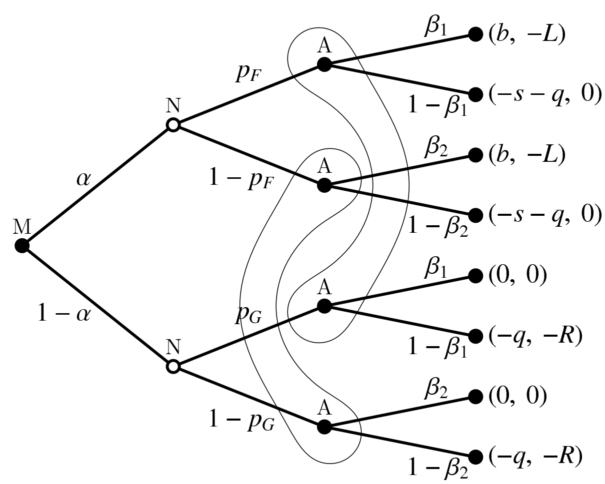
Both players' payoffs are normalized to zero for the case in which the manager does not commit fraud and the auditor correctly accepts the firm's assertion. If the manager does not commit fraud and the auditor falsely rejects the assertion, the manager incurs a rejection penalty q and the auditor incurs a penalty for a type I error (false rejection) R. If the

[1]The author thanks an anonymous referee who suggested the benefit of analyzing the misreporting game.
[2]For convenience, the manager is assumed to be male and the auditor is assumed to be female.

manager commits fraud and the auditor falsely accepts the firm's assertion, the manager receives a bonus b and the auditor incurs a penalty for a type II error (false acceptance) L. If the manager commits fraud and the auditor correctly rejects the firm's assertion, the manager incurs a penalty for revealed fraud s, in addition to a penalty for the auditor's rejection q. Note that if the auditor rejects the firm's assertion, the manager is penalized even when he does not commit fraud. On the other hand, the auditor is penalized only when she makes a mistake. This payoff structure is consistent with previous studies in the literature on strategic auditing (Fellingham, Newman and Patterson, 1989; Shibano, 1990).

Figure 1 shows the game tree of the defalcation game.

Figure 1: Game tree of the model



Symbols M, A, and N represent the manager, auditor and nature, respectively.

2.3 Equilibria

2.3.1 Benchmark defalcation game without audit evidence

As a benchmark, the defalcation game in which the auditor observes no signal is considered first. Since the auditor does not gather any information regarding the manager's action, then the game degenerates to a simple simultaneous game. Let β_0 denote the auditor's probability of accepting the firm's assertion. Then there exists a unique Nash equilibrium in the benchmark game such that $\alpha = \alpha_0^* \equiv R/(L+R)$ and $\beta_0 = \beta_0^* \equiv s/(b+s)$. As is often the case for a simple simultaneous game, the manager's payoff structure determines the auditor's strategy, while the auditor's payoff structure determines the manager's strategy (Fellingham, Newman and Patterson, 1989, 10).

2.3.2 Defalcation game with audit evidence

In the defalcation game with audit evidence, the signal h is more frequently observed when the manager commits fraud than when he does not. As the signal conveys some information regarding the manager's action, the auditor uses the information at equilibrium. A possible strategy for the auditor is the *mechanical strategy* that she rejects the firm's assertion when the signal is h and accepts the assertion when the signal is ℓ, i.e., $(\beta_1, \beta_2) = (0, 1)$. In fact, the mechanical strategy is the auditor's equilibrium strategy if the manager's expected payoff on committing fraud is equal to that for not committing fraud given the auditor's mechanical strategy.

The condition for the case in which the mechanical strategy is the auditor's equilibrium strategy also serves as the criterion for equilibrium classification. For convenience, define δ as the *manager's relative benefit on committing fraud* given the auditor's mechanical strategy such that:

$$\delta \equiv p_F(-s-q) + (1-p_F)b - [p_G(-q)]. \tag{1}$$

The term $p_F(-s-q)+(1-p_F)b$ is the manager's expected payoff on committing fraud when the auditor takes the mechanical strategy, while the term $p_G(-q)$ is the manager's expected payoff on not committing fraud when the auditor uses the mechanical strategy.

Note that if the auditor adopts the mechanical strategy and δ is negative, the manager has an incentive to reduce the probability of committing fraud α. Since $\alpha = 0$ cannot be an equilibrium strategy, then there must be a lower threshold at which the auditor changes strategy. On the other hand, if the auditor uses the mechanical strategy and δ is positive, the manager becomes more inclined to commit fraud. Since $\alpha = 1$ cannot be held at equilibrium, then there must be an upper threshold of the interval of α at which the auditor adopts the mechanical strategy.

Formally, Nash equilibria in the defalcation game are summarized as follows.

Lemma 1 *If $\delta < 0$, there exists a unique Nash equilibrium in the defalcation game such that:*

$$\alpha = \alpha_1^* \equiv \frac{p_G R}{p_F L + p_G R}, \qquad \beta_1 = \beta_1^* \equiv \frac{(-\delta)}{(-\delta) + b}, \qquad \beta_2 = 1. \tag{2}$$

If $\delta > 0$, there exists a unique Nash equilibrium in the defalcation game such that:

$$\alpha = \alpha_2^* \equiv \frac{(1 - p_G)R}{(1 - p_F)L + (1 - p_G)R}, \qquad \beta_1 = 0, \qquad \beta_2 = \beta_2^* \equiv \frac{s}{\delta + s}. \tag{3}$$

If $\delta = 0$, there exist Nash equilibria in the defalcation game such that $\alpha = \hat{\alpha}$, $\beta_1 = 0$ and $\beta_2 = 1$, where $\hat{\alpha}$ can be any number satisfying $\alpha_1^* \leqslant \hat{\alpha} \leqslant \alpha_2^*$.

Proof. See the Appendix.

Lemma 1 states that equilibria in the defalcation game are classified into three cases. The mechanical strategy of the auditor is held at equilibria if and only if $\delta = 0$. In this case, any strategies of the manager between α_1^* and α_2^* can be held at equilibria. The manager's strategies, α_1^* and α_2^*, are the greatest lower bound and the least upper bound of the interval of α for which the auditor adopts the mechanical strategy. They are determined by the signal probabilities, p_F and p_G, and the auditor's payoff parameters, L and R.

If $\delta < 0$ and the auditor uses the mechanical strategy, the manager reduces his probability of committing fraud α. If the auditor adopts the mechanical strategy, her expected payoff is $\alpha(1 - p_F)(-L) + (1 - \alpha)p_G(-R)$. If the auditor always accepts the firm's assertion, her expected payoff is $\alpha(-L)$. When $\alpha < \alpha_1^*$, the payoff $\alpha(-L)$ is greater (i.e., smaller in absolute value) than the payoff $\alpha(1 - p_F)(-L) + (1 - \alpha)p_G(-R)$ and therefore the auditor adopts the unconditional acceptance strategy. If the auditor always accepts the firm's assertion, the manager is more inclined to commit fraud. Therefore, if $\alpha < \alpha_1^*$ and the auditor always accepts the firm's assertion, the manager increases α. On the other hand, if $\alpha > \alpha_1^*$ and the auditor adopts the mechanical strategy, the manager decreases α. This is because δ is negative. To fix the manager at equilibrium, the auditor must randomize her strategy when she observes the signal h. When the auditor adopts the mechanical strategy, the manager's relative benefit for *not* committing fraud is $-\delta$ (> 0). When the auditor always accepts the firm's assertion, the manager's benefit for committing fraud is b. To erase the manager's incentive to deviate from equilibrium, the auditor sets the probability β_1 at $\beta_1^* \equiv (-\delta)/[(-\delta) + b]$, the relative weight of the benefit for not committing fraud. This is the first case in Lemma 1. The second case, where $\delta > 0$, can be explained similarly.

2.4 Comparative game analysis

2.4.1 Audit risk model

Since Lemma 1 provides the closed-form solutions expressed in terms of exogenous parameters, some definitive analytical results are obtained. This subsection discusses how the introduction of audit evidence affects the manager's behavior, the auditor's decision, audit risk and other risks.

Audit risk is defined as the probability that the auditor unknowingly fails to detect material misstatements in financial reporting (AICPA, 1984). Generally Accepted Auditing Standards typically decompose audit risk AR into three components:

$$AR = IR \times CR \times DR, \tag{4}$$

where IR, CR and DR denote inherent risk, control risk and detection risk, respectively. Inherent risk is the probability that the manager's assertion is subject to material misstatements under the assumption that there is no related internal control system. Control risk is the conditional probability that, given that the manager's original assertion contains material misstatements, misstatements are not prevented on a timely basis by the firm's internal controls. Detection risk is the conditional probability that the auditor fails to detect material misstatements that the firm's internal controls fail to prevent.

In auditing practices, assessment of the strength of the auditee's internal controls is an essential part of audit planning, and control risk is accordingly an important component of audit risk. However, introducing internal controls into the analysis substantially complicates the models. Since the aim here is to provide simple models in which audit risk anomalously increases with more information, then the analysis of internal controls is beyond the scope of the current study. [3] As the current study does not focus on internal controls, control risk is hereafter assumed to be one, so that audit risk is the product of inherent risk and detection risk.

2.4.2 Inherent risk

In the defalcation game, inherent risk is the manager's equilibrium strategy. Accordingly, inherent risk is α_0^* in the benchmark case. In the defalcation game with audit evidence, inherent risk IR is as follows:

$$IR = \begin{cases} \alpha_1^* & \text{if} \quad \delta < 0, \\ \hat{\alpha} & \text{if} \quad \delta = 0, \\ \alpha_2^* & \text{if} \quad \delta > 0. \end{cases} \tag{5}$$

Since p_F is greater than p_G, it is easy to verify that $\alpha_1^* < \alpha_0^* < \alpha_2^*$. Hence, if the auditor obtains audit evidence when δ is negative, inherent risk decreases to α_1^*. However, if the

[3]For discussions on internal controls, see Anderson and Young (1988), Newman, Park and Smith (1998), Smith, Tiras and Vichitlekarn (2000) and Patterson and Smith (2006).

auditor obtains audit evidence when δ is positive, inherent risk increases to α_2^*, which is greater than α_0^*. It may be surprising that, in some instances, δ is actually positive and therefore inherent risk increases when the auditor obtains audit evidence. The following proposition shows the general tendencies of exogenous parameters in such instances.

Proposition 1 (Inherent risk)
(1) *For any positive s, q, L, R, p_F and p_G* ($p_G < p_F < 1$), *there exists a finite positive number τ such that $\delta > 0$ for all $b > \tau$.*
(2) *For any positive b, L, R, p_F and p_G* ($p_G < p_F < 1$), *there exists a finite positive number ε such that $\delta > 0$ for all $s < \varepsilon$ and $q < \varepsilon$.*
(3) *For any positive b, q, L, R, p_F and p_G* ($p_G < p_F < 1$) *satisfying* $q/b > (1-p_F)/(p_F-p_G)$, *there does not exist any value of s satisfying $\delta \geqslant 0$.*
(4) *It holds that* $\delta > 0$ *if and only if*

$$\frac{p_F}{p_G} < \frac{b}{b+s+q} \cdot \frac{1}{p_G} + \frac{q}{b+s+q}. \tag{6}$$

Proof. The parameters L and R are irrelevant to the value of δ.
(1) Since $\lim_{b\to\infty} \delta = \infty$, there exists a finite positive number τ such that $\delta > 0$ for all $b > \tau$.
(2) Since $\lim_{s\to 0} \delta = (1-p_F)b - (p_F - p_G)q$, $\lim_{q\to 0}(\lim_{s\to 0} \delta) = (1-p_F)b > 0$. Therefore, there exist finite positive numbers ε_s and ε_q such that $\delta > 0$ for all $s < \varepsilon_s$ and $q < \varepsilon_q$. Setting ε to the smaller of ε_s and ε_q yields the second part of Proposition 1.
(3) Since $\partial\delta/\partial s = -p_F < 0$, δ approaches its supremum when s approaches zero. Observe that the condition that $\lim_{s\to 0} \delta = (1-p_F)b - (p_F - p_G)q < 0$ is equivalent to the condition that $q/b > (1-p_F)/(p_F - p_G)$. This completes the proof of the third part.
(4) Solving $\delta > 0$ for p_F/p_G yields condition (6). *Q.E.D.*

The first two parts of Proposition 1 state that there exists a sufficiently large b to make δ positive and that there exists a combination of sufficiently small s and q to make δ positive. In other words, the introduction of audit evidence increases inherent risk if the manager has a sufficiently strong incentive to commit fraud, or if the manager's penalties for revealed fraud and for auditor rejection are sufficiently small.

To understand the driving force behind this phenomenon, decompose δ into two parts:

$$\delta = [p_F(-s) + (1-p_F)b] - (p_F - p_G)q. \tag{7}$$

The factors that affect the sign of δ are imperfect audit technology and the non-zero rejection penalty. First, to isolate the effect of imperfect audit technology, assume that the rejection penalty q is zero. Then δ is determined by the expected penalty for revealed fraud and the expected bonus for the manager: $\delta = p_F(-s) + (1-p_F)b$. If the audit technology is perfect in fraud detection (i.e., $p_F = 1$), the relative benefit for committing fraud δ is always negative. When p_F is not equal to 1, however, a sufficiently small s or a sufficiently large b yields a positive δ. Thus, imperfect audit technology introduces the possibility that inherent risk anomalously increases when the auditor obtains audit evidence.

Second, the non-zero rejection penalty q decreases the manager's incentive to commit fraud. In the current setting, the manager is always penalized when the auditor rejects the firm's assertion, irrespective of whether the manager commits fraud. If the auditor does not gather information besides the game structure that is common knowledge, the manager's action does not affect the auditor's decision. Accordingly, the expected rejection penalty remains the same whatever action the manager takes. However, if audit evidence is introduced into the game, the manager knows that the auditor will have some information regarding his action and the auditor will differentiate her action depending on whether the signal is h or ℓ. As a result, the manager's expected rejection penalty is smaller when he does not commit fraud than when he does. Consequently, when the rejection penalty q is large, the manager becomes less inclined to commit fraud.

The second effect of the decrease in expected rejection penalty may dominate the first effect of imperfect audit technology. The third part of Proposition 1 indicates that if q is sufficiently large or, equivalently, if b is sufficiently small, the second effect dominates the first effect. This is stronger than the statement that δ is negative when q/b is sufficiently large. Instead, it claims that, whatever value s takes, δ is always negative if q/b is sufficiently large. In other words, if the ratio q/b is sufficiently large, the introduction of audit evidence always reduces inherent risk, irrespective of the value of the penalty for revealed fraud s. Even the smallest possible s cannot offset this effect in such cases.

The fourth part of Proposition 1 indicates that inherent risk increases with more information when p_G is sufficiently large and p_F/p_G is sufficiently small simultaneously. Roughly speaking, if the signal is not very informative, inherent risk tends to increase with more information.

2.4.3 Detection risk

In this study, detection risk is the conditional probability that the auditor fails to detect fraud given that the manager commits fraud. In the benchmark game, the auditor's equilibrium strategy β_0^* represents detection risk. In the game with audit evidence, detection risk DR is expressed as:

$$DR = \begin{cases} \beta_1^* p_F + (1 - p_F) & \text{if} \quad \delta < 0, \\ 1 - p_F & \text{if} \quad \delta = 0, \\ \beta_2^*(1 - p_F) & \text{if} \quad \delta > 0. \end{cases} \tag{8}$$

Regarding detection risk, the following result is proved.

Proposition 2 (Detection risk) *The introduction of audit evidence always increases detection risk in the defalcation game.*

Proof. When $\delta < 0$, one has

$$\beta_1^* p_F + (1 - p_F) - \beta_0^* = \frac{(p_F - p_G)bq}{(b + s)[(-\delta) + b]} > 0. \tag{9}$$

When $\delta = 0$, one has

$$1 - p_F - \beta_0^* = \frac{(1 - p_F)(p_F - p_G)q}{s + (p_F - p_G)q} > 0. \tag{10}$$

When $\delta > 0$, one has

$$\beta_2^*(1 - p_F) - \beta_0^* = \frac{(p_F - p_G)sq}{(b + s)(\delta + s)} > 0. \tag{11}$$

Accordingly, the introduction of audit evidence always increases detection risk in the defalcation game. *Q.E.D.*

This result may be counterintuitive, because obtaining information makes the auditor's decision worse given that the manager commits fraud.

As in the case of inherent risk, the expected rejection penalty plays an important role in this phenomenon. Observe that the rejection penalty q is a factor on the right hand sides in equations (9), (10) and (11). If q is positive, audit evidence always increases detection risk. If q is zero, detection risk does not change.[4] Therefore, the positive rejection penalty q generates this result.

The same argument as in the case of inherent risk applies here. When the auditor obtains audit evidence, the manager knows that she will differentiate her decision depending on the evidence. Hence, the manager's expected rejection penalty is smaller when he does not commit fraud than when he does. Therefore, the manager becomes less inclined to commit fraud. As the game structure is common knowledge, the auditor knows the manager's incentive to reduce the probability of committing fraud. Accordingly, the auditor becomes more generous in substantive testing and hence the introduction of audit evidence always increases detection risk.

[4] This might be a relatively general result. Smith, Tiras and Vichitlekarn (2000) find that detection risk does not change after introducing additional information into the game in which no rejection penalty for the manager exists.

2.4.4 Audit risk

Since detection risk always increases when audit evidence is introduced, audit risk increases if inherent risk increases. Even in the case for which inherent risk decreases, audit risk might increase in some instances. In fact, the increase in detection risk occasionally dominates the decrease in inherent risk. The following proposition clarifies this point.

Proposition 3 *When $\delta < 0$, the introduction of audit evidence increases audit risk if and only if*

$$\frac{p_F}{p_G} < t \equiv \frac{[b(L+R)+sL]q}{(b+s+q)sL}. \tag{12}$$

Proof. The condition $\alpha_1^*[\beta_1^* p_F + (1-p_F)] - \alpha_0^* \beta_0^* > 0$ is equivalent to $p_F/p_G < t$. There exist exogenous parameters simultaneously satisfying $p_F/p_G < t$ and $\delta < 0$. There also exist exogenous parameters simultaneously satisfying $p_F/p_G > t$ and $\delta < 0$. *Q.E.D.*

For the case in which $\delta < 0$, condition (12) requires that the likelihood ratio of the signal h be sufficiently large to reduce audit risk. If the signal perfectly reveals the manager's action (i.e., $p_F = 1$ and $p_G = 0$), the likelihood ratio goes to infinity, so that it exceeds any finite threshold t. Since δ is negative when the signal is perfect, then the perfect signal always reduces audit risk.

For the case in which the signal is not perfect, however, whether audit risk increases depends on the sign of δ and the relation between the likelihood ratio p_F/p_G and the threshold t, determined by the manager's payoff parameters, b, s and q, and the auditor's payoff parameters, L and R.[5] This result is consistent with the numerical example reported by Patterson (1993, 293). She finds a case in which audit risk increases with sample size when the sample size is relatively small. Furthermore, the fourth part of Proposition 1 indicates that inherent risk tends to increase with audit evidence when the evidence is not very informative. These results might collectively indicate that audit risk tends to increase when the auditor observes a signal that is not very informative.

As in the following proposition, the introduction of audit evidence increases audit risk if s is sufficiently small, if L is sufficiently small, or if R is sufficiently large.

[5] If q is zero, detection risk does not change when audit evidence is introduced. Therefore, audit risk increases if and only if inherent risk increases. Observe that t is zero when q is zero.

Proposition 4 (Audit risk)
(1) *For any set of positive b, q, L, R, p_F and p_G ($p_G < p_F < 1$) satisfying $\delta < 0$, there exists a finite positive number ε such that $\alpha_1^*[\beta_1^* p_F + (1-p_F)] - \alpha_0^*\beta_0^* > 0$ for all $s < \varepsilon$.*
(2) *For any set of positive b, s, q, p_F and p_G ($p_G < p_F < 1$) satisfying $\delta < 0$, there exists a finite positive number τ such that $\alpha_1^*[\beta_1^* p_F + (1-p_F)] - \alpha_0^*\beta_0^* > 0$ for all $R/L > \tau$.*

Proof. The following expressions provide the direct proofs:

$$\lim_{s\to 0}\{\alpha_1^*[\beta_1^* p_F + (1-p_F)] - \alpha_0^*\beta_0^*\} = \alpha_1^* \cdot \frac{(p_F - p_G)q}{p_F b + (p_F - p_G)q} > 0, \tag{13}$$

$$\lim_{R/L\to\infty}\{\alpha_1^*[\beta_1^* p_F + (1-p_F)] - \alpha_0^*\beta_0^*\} = \frac{(p_F - p_G)bq}{[(-\delta) + b](b+s)} > 0, \tag{14}$$

Q.E.D.

Recall that the third part of Proposition 1 states that if q/b is sufficiently large, the introduction of audit evidence always reduces inherent risk (i.e., $\delta < 0$) whatever value s takes. The first part of Proposition 4 indicates, however, even in the case in which q/b is too large to make δ non-negative, the introduction of audit evidence increases audit risk if s is sufficiently small. In sum, the condition of s and q for the increase in audit risk is less severe than the condition for the increase in inherent risk.

According to the second part of Proposition 4, the introduction of audit evidence tends to increase audit risk when the auditor has a strong incentive to avoid a type I error (false rejection) and is less concerned about a type II error (false acceptance). The ratio R/L is the *auditor's liability ratio*, which plays an important role in the studies by Newman and Noel (1989) and Shibano (1990). The auditor's payoff parameters L and R do not determine whether inherent risk increases, while they do affect whether audit risk increases. This is because the magnitude of inherent risk depends on the auditor's payoff parameters and the magnitude of inherent risk affects whether audit risk increases when δ is negative. If the increase in detection risk outweighs the decrease in inherent risk, the introduction of audit evidence increases audit risk in spite of the decrease in inherent risk.

2.4.5 Type I risks

Statement of Auditing Standards No. 47 states that the definition of audit risk does not include the probability that an auditor erroneously concludes that financial statements are materially misstated. In such a case, the auditor is supposed to reconsider or extend the audit procedures and request the auditee to reevaluate the appropriateness of the financial statements. These steps are expected to lead the auditor to the correct conclusion (AICPA, 1984, footnote 2). In short, the probability of false rejection is assumed to be reasonably small in auditing practices. It is unclear, however, whether this assumption is correct in strategic environments.

To examine this issue, define type I detection risk and type I audit risk as follows: type I detection risk is the conditional probability of false rejection given that the manager does not commit fraud; and type I audit risk is the probability of false rejection. In the defalcation game, type I detection risk is expressed as:

$$\text{Type I detection risk} = \begin{cases} (1-\beta_1^*)p_G & \text{if } \delta < 0, \\ p_G & \text{if } \delta = 0, \\ p_G + (1-\beta_2^*)(1-p_G) & \text{if } \delta > 0. \end{cases} \tag{15}$$

The introduction of audit evidence always increases (type II) detection risk, while it has the opposite effect on type I detection risk.

Proposition 5 (Type I detection risk) *The introduction of audit evidence always decreases type I detection risk in the defalcation game.*

Proof. When $\delta < 0$, one has

$$(1-\beta_1^*)p_G - (1-\beta_0^*) = -\frac{(p_F - p_G)b(b+s+q)}{(b+s)[(-\delta)+b]} < 0. \tag{16}$$

When $\delta = 0$, one has

$$p_G - (1-\beta_0^*) = -\frac{(p_F - p_G)[(1-p_G)q + s]}{(p_F - p_G)q + s} < 0. \tag{17}$$

When $\delta > 0$, one has

$$p_G + (1-\beta_2^*)(1-p_G) - (1-\beta_0^*) = -\frac{(p_F - p_G)s(b+s+q)}{(b+s)(\delta+s)} < 0. \tag{18}$$

Q.E.D.

This result is intuitive, in that obtaining audit evidence improves the auditor's decision. Audit evidence reduces the conditional probability that the auditor makes a type I error (false rejection) given that the manager does not commit fraud. Observe that type I detection risk decreases with audit evidence, even in the case for which $q = 0$.

Next, type I audit risk is examined. In the current setting, the risk is the product of 1 minus inherent risk and the type I detection risk. Regarding type I audit risk, the following proposition is proved.

Proposition 6 (Type I audit risk) *When $\delta \neq 0$, the introduction of audit evidence always decreases type I audit risk.*

Proof. Since type I detection risk always decreases when the auditor obtains audit evidence, then type I audit risk decreases if the probability that the manager does *not* commit fraud decreases or, equivalently, if inherent risk increases. Therefore, if δ is positive, type I audit risk always decreases.

When δ is negative, type I audit risk decreases as follows:

$$\begin{aligned} &(1-\alpha_1^*)(1-\beta_1^*)p_G - (1-\alpha_0^*)(1-\beta_0^*) \\ &= -(1-\alpha_0^*)(1-\beta_0^*)\frac{(p_F - p_G)[p_F(b+s+q)L + p_G qR]}{(p_F L + p_G R)[(-\delta)+b]} < 0. \end{aligned} \tag{19}$$

Accordingly, the introduction of audit evidence always decreases type I audit risk if $\delta \neq 0$. *Q.E.D.*

Proposition 6 confirms the intuition that extended audit procedures remove type I audit risk. As in auditing practices, type II risks are more problematic than type I risks in the current setting.

2.5 Two-stage formulation of the defalcation game

A simple extension of the defalcation game is to grant the auditor an option to determine whether to observe a costly signal $\sigma \in \{h, \ell\}$ and to inform the manager about her decision before the manager chooses whether to commit fraud.[6] If the auditor chooses not to observe σ, she will decide whether to accept the predetermined assertion with only knowledge of the game structure. If the auditor chooses to observe σ, she will decide whether to accept the assertion based on the realized signal and knowledge of the game structure. In other words, the extended two-stage game is a combination of the benchmark defalcation game without audit evidence and the defalcation game with audit evidence. Let k represent the cost of the signal for the auditor.

[6] This game is fundamentally different from the game in which the auditor determines whether to observe the signal *after* the manager chooses his action. Although the "after" game is in itself interesting, it is not a simple extension of the defalcation game and accordingly it deserves a separate study. Note that the "after" game reverts to Fellingham and Newman's (1985) model when the audit technology becomes perfect (i.e., $p_F = 1$ and $p_G = 0$). Fellingham, Newman and Patterson (1989) also analyze an audit model with costly perfect information.

Proposition 7 (The auditor's decision) *Assume that $\delta \neq 0$. At the beginning of the two-stage defalcation game, the auditor chooses to observe the signal if and only if*

$$\begin{cases} (\alpha_0^* - \alpha_1^*)L > k & \text{if } \delta < 0, \\ (\alpha_2^* - \alpha_0^*)R > k & \text{if } \delta > 0. \end{cases} \tag{20}$$

Proof. Using the auditor's expected payoff $A(\alpha, \beta_1, \beta_2)$ defined in the Appendix, one has $A(\alpha_0^*, \beta_0^*, \beta_0^*) = -\alpha_0^* L = -(1 - \alpha_0^*)R$, $A(\alpha_1^*, \beta_1^*, 1) = -\alpha_1^* L$, and $A(\alpha_2^*, 0, \beta_2^*) = -(1 - \alpha_2^*)R$. When $\delta < 0$, the auditor chooses to observe σ if and only if $A(\alpha_1^*, \beta_1^*, 1) - k$ is greater than $A(\alpha_0^*, \beta_0^*, \beta_0^*)$. When $\delta > 0$, the auditor chooses to observe σ if and only if $A(\alpha_1^*, 0, \beta_2^*) - k$ is greater than $A(\alpha_0^*, \beta_0^*, \beta_0^*)$. These two conditions are equivalent to the conditions in (20).

Q.E.D.

This is perhaps a striking result. As $\alpha_1^* < \alpha_0^* < \alpha_2^*$, the left hand sides of (20) are both positive. When δ is positive and the auditor observes the signal σ, inherent risk and audit risk always increase. Nevertheless, the auditor chooses to observe the signal if the cost of the signal k is sufficiently small. When δ is negative, the auditor chooses to observe the signal if the cost k is sufficiently small. Although inherent risk decreases in this case, audit risk increases if the penalty for revealed fraud s is sufficiently small or if the auditor's liability ratio R/L is sufficiently large. Even in such instances, the auditor chooses to observe the signal σ irrespective of the increase in audit risk.

2.6 Due care and audit liability

In the analysis above, the auditor's liability for audit failure L is assumed to be fixed, irrespective of whether the auditor obtains audit evidence. In the real world, however, the auditor's liability may be reduced if the court finds audit quality to be high. If the auditor proves that she used due care to avoid audit failure, the court may exonerate her from liability. On the other hand, if the court finds that the auditor found a symptom of fraud and accepted the firm's assertion, it may increase her penalty for audit failure. To capture these effects, the defalcation game is modified as follows.[7]

Let L_h and L_ℓ be the auditor's penalties for a type II error when she observes signals h and ℓ, respectively. The auditor's liability is reduced if she used due care to avoid audit failure, while liability is increased if she found a symptom of fraud and failed to detect it, i.e., $L_\ell < L < L_h$.

Since δ is independent of the auditor's payoff parameters, then whether the introduction of

[7]The author thanks an anonymous referee for drawing attention to this issue.

audit evidence increases inherent risk does not depend on L_h and L_ℓ. However, as Lemma 1 shows, inherent risk and overall audit risk depend on the auditor's payoff parameters.

Proposition 8 (Due care consideration) *When the auditor's penalty for a type II error is differentiated, α_1^* and α_2^* are redefined as follows:*

$$\alpha_1^* \equiv \frac{p_G R}{p_F L_h + p_G R}, \qquad \alpha_2^* \equiv \frac{(1-p_G)R}{(1-p_F)L_\ell + (1-p_G)R}. \tag{21}$$

With the newly defined α_1^* and α_2^*, one has $\partial\alpha_1^*/\partial L_h < 0$, $\partial\alpha_1^*[\beta_1^* p_F + (1-p_F)]/\partial L_h < 0$, $\partial\alpha_2^*/\partial L_\ell < 0$, and $\partial\alpha_2^*[\beta_2^*(1-p_F)]/\partial L_\ell < 0$. In words, both inherent risk and overall audit risk decrease with due care consideration if $\delta < 0$, while the risks increase with due care consideration if $\delta > 0$.

Proof. Redefine $A(\alpha, \beta_1, \beta_2)$ in the proof of Lemma 1 using L_h and L_ℓ. Following the same procedure as in the proof of Lemma 1, one has equations in (21). Observe that β_1^* and β_2^* do not contain the auditor's payoff parameters and hence they do not change when L_h and L_ℓ are introduced. The derivatives are readily calculated directly and their signs are readily ascertained. *Q.E.D.*

Proposition 8 states that due care amplifies the difference between the cases for which $\delta > 0$ and $\delta < 0$. If the manager has a strong incentive to commit fraud (i.e., $\delta > 0$), inherent risk and overall audit risk, which increase with audit evidence, further increase when due care is introduced. When δ is negative, inherent risk and overall audit risk decrease with due care consideration. When due care is introduced, inherent risk further decreases with audit evidence. In this case, there is the possibility that audit risk that would increase with audit evidence when due care is ignored actually decreases with audit evidence when due care is introduced. This is because the auditor's liability ratio in Proposition 4 becomes smaller with due care consideration.

3 Misreporting game

3.1 Potentially fraudulent financial reporting

In Section 2, the manager has no choice in determining the firm's assertion that serves as the null hypothesis for the audit. As some results obtained in Section 2 are counterintuitive, it is important to examine whether the same results are found in a different setting. For this purpose, this section analyzes the misreporting game in which the firm's performance is stochastically determined and the manager prepares a financial report after observing the realized performance.

3.2 Model description

As in the defalcation game, the game players are a risk-neutral manager of a firm and a risk-neutral auditor. To facilitate comparison between the defalcation game and the misreporting game, notation similar to, but slightly different from, that in the defalcation game is used for the misreporting game. All variables are reset and redefined in this subsection.

First, nature determines the firm's performance $\pi \in \{h, \ell\}$. Nature chooses h (high) with probability θ and ℓ (low) with probability $1-\theta$. Second, on observing the realized performance π, the manager prepares a financial report $\rho \in \{h, \ell\}$. If the report ρ differs from the realized performance π, the report ρ is said to be fraudulent. For simplicity, the manager is assumed to always choose $\rho = h$ when the firm's performance is h.[8] Let α be the probability that the manager prepares a fraudulent report when the realized performance is ℓ. Then the manager's strategy is fully described by the probability α. Third, the auditor observes a signal $\sigma \in \{h, \ell\}$. When the manager commits fraud, the signal σ differs from the manager's report ρ with probability p_F and coincides with the report ρ with probability $1-p_F$. Since the manager potentially commits fraud only when $\pi = \ell$, then this can be restated as follows: if the firm's performance is ℓ and the manager reports h, the signal σ becomes ℓ with probability p_F and the signal σ becomes h with probability $1-p_F$. When the manager reports honestly, the signal σ differs from the report ρ with probability p_G and coincides with the report ρ with probability $1-p_G$. As in the defalcation game, p_F is assumed to be greater than p_G. Finally, using the observed signal as audit evidence, the auditor decides whether to accept the manager's report. To describe the auditor's strategy, define the four probabilities, $\beta_1, \beta_2, \beta_3$ and β_4, as follows. The auditor accepts the manager's report h with probability β_1 when the observed signal is ℓ and with probability β_2 when the signal is h. The auditor accepts the manager's report ℓ with probability β_3 when the observed signal is h and with probability β_4 when the signal is ℓ. These four probabilities collectively specify the auditor's strategy.

Both players' payoffs are essentially the same as in the defalcation game. The auditor is penalized only if she makes a mistake. The penalties for type I and type II errors are denoted by R and L, respectively. The manager's payoff parameters b, s and q represent the bonus for good performance, the penalty for revealed fraud and the penalty for auditor rejection. Note again that the manager is penalized if the auditor rejects the manager's report, even if he honestly reports the firm's performance. In other words, the manager's payoff is as follows. If $\pi = h$ and $\rho = h$, the manager's payoff is b when the auditor accepts the report and $b-q$ when the auditor rejects the report. If $\pi = \ell$ and $\rho = h$, the manager's payoff is b

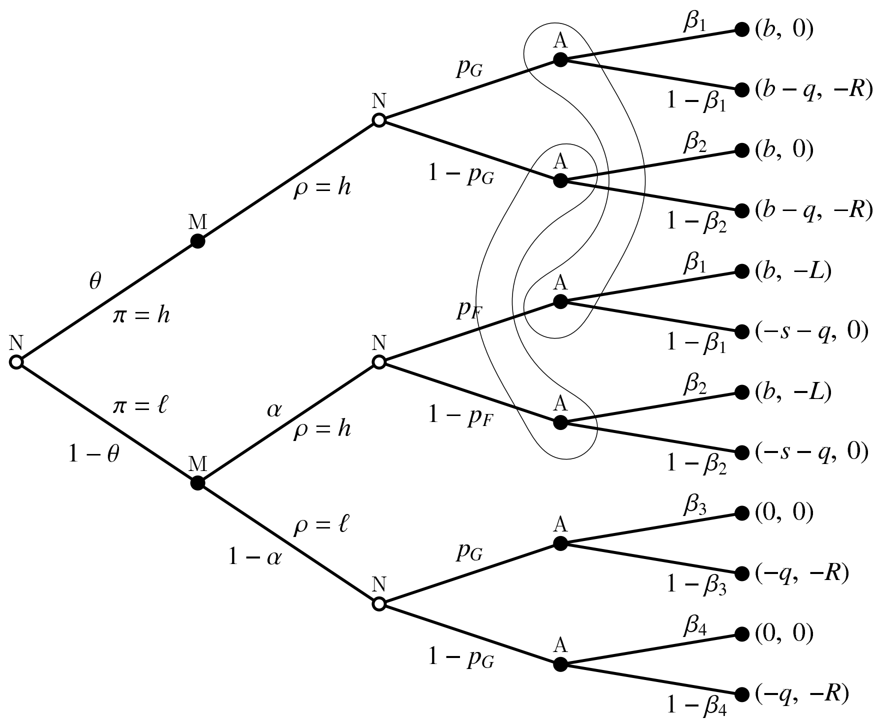
[8] If the rejection penalty q is large, the manager may report ℓ even when the firm's performance is h. This is because the manager can avoid the possible rejection penalty by preparing a modest report ℓ given that the auditor more frequently rejects the manager's report when he reports h than when he reports ℓ. However, such a case is not a focus of the current study. Hence, the possibility of fraudulent reporting is excluded when the firm's performance is h.

when the auditor accepts the report and $-s-q$ when the auditor rejects the report. If $\pi = \ell$ and $\rho = \ell$, the manager's payoff is zero when the auditor accepts the report and $-q$ when she rejects the report.

It is noteworthy that the defalcation game and the misreporting game have fundamentally different information structures, in that the misreporting game does *not* revert to the defalcation game even if θ approaches either zero or one.

Figure 2 shows the game tree of the misreporting game.

Figure 2: Game tree of the model



Symbols M, A, and N represent the manager, auditor and nature, respectively.

3.3 Equilibria

3.3.1 Case in which the manager reports low performance

If the manager reports ℓ, the auditor knows that the actual performance is ℓ. Therefore, irrespective of the realized signal σ, the auditor always accepts the manager's report. Observe that the strategy of accepting the report strictly dominates the strategy of rejecting the report in both cases for which $\sigma = h$ or $\sigma = \ell$. Accordingly, the auditor chooses $\beta_3 = \beta_4 = 1$ at Nash equilibria.

3.3.2 Benchmark misreporting game without audit evidence

To establish a benchmark, consider the misreporting game in which the auditor observes no signal. In this game, the auditor's strategy is fully described by the probability β_0 that the auditor accepts the manager's report h. The benchmark game has a unique Nash equilibrium so that $\alpha = \alpha_0^* \equiv [\theta/(1-\theta)] \cdot (R/L)$ and $\beta_0 = \beta_0^* \equiv (s+q)/(b+s+q)$.

3.3.3 Misreporting game with audit evidence

Next, the misreporting game with audit evidence is examined. In this game, when the manager reports h, the signal ℓ is regarded as a bad sign. The auditor uses this information to decide on the audit opinion.

As in the defalcation game, define δ as the manager's relative benefit for fraudulent reporting given the firm's performance ℓ and the auditor's mechanical strategy such that:

$$\delta = p_F(-s-q) + (1-p_F)b. \tag{22}$$

The relative benefit δ substantially simplifies the description of the equilibria in this game. The following lemma provides Nash equilibria when θ is sufficiently small.

Lemma 2 *When θ is sufficiently small such that:*

$$\theta < \frac{(1-p_F)L}{(1-p_F)L + (1-p_G)R}, \tag{23}$$

Nash equilibria in the misreporting game are described as follows. If $\delta < 0$, there exists a unique Nash equilibrium in the misreporting game such that:

$$\alpha = \alpha_1^* \equiv \frac{\theta}{1-\theta} \cdot \frac{p_G}{p_F} \cdot \frac{R}{L}, \qquad \beta_1 = \beta_1^* \equiv \frac{(-\delta)}{(-\delta)+b}, \qquad \beta_2 = 1. \tag{24}$$

If $\delta > 0$, there exists a unique Nash equilibrium in the misreporting game such that:

$$\alpha = \alpha_2^* \equiv \frac{\theta}{1-\theta} \cdot \frac{1-p_G}{1-p_F} \cdot \frac{R}{L}, \qquad \beta_1 = 0, \qquad \beta_2 = \beta_2^* \equiv \frac{s+q}{\delta+s+q}. \tag{25}$$

If $\delta = 0$, there exist Nash equilibria in the misreporting game such that $\alpha = \hat{\alpha}$, $\beta_1 = 0$ and $\beta_2 = 1$, where $\hat{\alpha}$ can be any number satisfying $\alpha_1^* \leqslant \hat{\alpha} \leqslant \alpha_2^*$.

Proof. See the Appendix.

If θ is large, there may exist corner equilibria.[9] Assumption (23) excludes these atypical cases and simplifies the following analysis. Hereafter, assumption (23) is assumed to hold.

3.4 Comparative game analysis

It is readily verified that $\alpha_1^* < \alpha_0^* < \alpha_2^*$. Since inherent risk in the misreporting game is the product of $1-\theta$ and α, then the sign of δ, as in the defalcation game, determines whether inherent risk increases with audit evidence.

Proposition 9 (Inherent risk)
(1) *For any positive s, q, L, R, p_F and p_G ($p_G < p_F < 1$), there exists a finite positive number τ such that $\delta > 0$ for all $b > \tau$.*
(2) *For any positive b, L, R, p_F and p_G ($p_G < p_F < 1$), there exists a finite positive number ε such that $\delta > 0$ for all $s < \varepsilon$ and $q < \varepsilon$.*
(3) *For any positive b, q, L, R, p_F and p_G ($p_G < p_F < 1$) satisfying $q/b > (1-p_F)/p_F$, there does not exist any value of s satisfying $\delta \geqslant 0$.*
(4) *It holds that $\delta > 0$ if and only if*

$$p_F < \frac{b}{b+s+q}. \tag{26}$$

Proof. Similar to the proof of Proposition 1.

The first two parts of Proposition 9 are essentially the same as the first two parts of Proposition 1. In the last two parts of Proposition 9, the threshold for q/b and condition (26) are slightly different from the counterparts in Proposition 1. This is because the auditor knows that the manager reports honestly when he reports ℓ and hence a manager who reports ℓ does not incur the rejection penalty. Thus, p_G does not appear in the threshold and condition (26).

Proposition 10 (Detection risk) *The introduction of audit evidence does not affect detection risk in the misreporting game.*

[9] If θ is greater than $p_F L/(p_F L + p_G R)$, the unique equilibrium in the game is $(\alpha, \beta_1, \beta_2) = (1, 1, 1)$. If δ is positive and θ is located between $(1-p_F)L/[(1-p_F)L + (1-p_G)R]$ and $p_F L/(p_F L + p_G R)$, the unique equilibrium in the game is $(\alpha, \beta_1, \beta_2) = (1, 0, 1)$.

Proof. When $\delta < 0$, one has $\beta_1^* p_F + (1 - p_F) - \beta_0^* = 0$. When $\delta = 0$, one has $1 - p_F - \beta_0^* = 0$. When $\delta > 0$, one has $\beta_2^*(1 - p_F) - \beta_0^* = 0$. *Q.E.D.*

The introduction of audit evidence always increases detection risk in the defalcation game, but it does not affect detection risk in the misreporting game. The driving force that leads to the increase in detection risk in the defalcation game is the expected rejection penalty. In the misreporting game, however, a manager who reports ℓ does not incur the rejection penalty at equilibrium. Therefore, there is no strategic interaction that increases detection risk.

Since the introduction of audit evidence does not affect detection risk at all, then audit risk increases if and only if inherent risk increases. The sign of the change in audit risk is the same as for that in inherent risk.

Proposition 11 (Type I detection risk) *The introduction of audit evidence always decreases type I detection risk in the misreporting game.*

Proof. When $\delta < 0$, one has $(1 - \beta_1^*)p_G - (1 - \beta_0^*) = -[(p_F - p_G)/p_F][b/(b + s + q)] < 0$. When $\delta = 0$, one has $p_G - (1 - \beta_0^*) = -(p_F - p_G) < 0$. When $\delta > 0$, one has $p_G + (1 - \beta_2^*)(1 - p_G) - (1 - \beta_0^*) = -[(p_F - p_G)/(1 - p_F)][(s + q)/(b + s + q)] < 0$.
Q.E.D.

Thus, the introduction of audit evidence decreases type I detection risk in both the defalcation game and the misreporting game, irrespective of the difference between the information structures of the games.

In the misreporting game, the probability of a type I error does not depend on the manager's strategy α. This is because the auditor knows that a manager who reports ℓ does not commit fraudulent reporting. Since type I detection risk always decreases when the auditor obtains audit evidence, then type I audit risk also always decreases. This confirms the intuition that extended audit procedures effectively reduce type I audit risk.

Assume next that the auditor has an option to choose whether to obtain audit evidence at a cost of k. Then, informed about the auditor's decision and observing the firm's performance π, the manager prepares a report that serves as the null hypothesis for the audit. In this two-stage formulation of the misreporting game, the auditor obtains audit evidence if δ is negative and the cost k is sufficiently small.

Proposition 12 (The auditor's decision) *Assume that $\delta \neq 0$. At the beginning of the two-stage misreporting game, the auditor chooses to observe the signal if and only if $\delta < 0$ and $\theta(p_F - p_G)R/p_F > k$.*

Proof. Using the auditor's expected payoff $A(\alpha, \beta_1, \beta_2)$ defined in the Appendix, one has $A(\alpha_0^*, \beta_0^*, \beta_0^*) = -(1-\theta)\alpha_0^* L = -\theta R$, $A(\alpha_1^*, \beta_1^*, 1) = -(1-\theta)\alpha_1^* L$, and $A(\alpha_2^*, 0, \beta_2^*) = -\theta R$. Accordingly, $A(\alpha_1^*, \beta_1^*, 1) - A(\alpha_0^*, \beta_0^*, \beta_0^*) = \theta(p_F - p_G)R/p_F$ and $A(\alpha_2^*, 0, \beta_2^*) - A(\alpha_0^*, \beta_0^*, \beta_0^*) = 0$. When $\delta > 0$, the auditor does not choose to obtain audit evidence as long as k is positive. When $\delta < 0$ and $\theta(p_F - p_G)R/p_F > k$, the auditor is better off by obtaining audit evidence. *Q.E.D.*

Proposition 12 indicates that the auditor obtains audit evidence only if audit risk decreases with evidence. When δ is negative, the auditor's expected payoff is determined by the product of inherent risk and the penalty for a type II error. In this case, if k is sufficiently small, the auditor is better off with more information, because information reduces inherent risk. When δ is positive, the auditor's expected payoff is determined by the product of the probability that the manager honestly reports h and the penalty for a type I error. If the realized performance is h, the manager always reports h. Therefore, the auditor's expected payoff does not depend on the manager's strategy α. In other words, when δ is positive, additional information conveyed by the signal σ does not change the auditor's expected payoff. When the cost of observing the signal is positive (i.e., $k > 0$), the auditor does not choose to observe the signal.

To analyze the effect of due care, let L_d and L_c be the auditor's penalty for a type II error when she observes a signal that differs from the manager's report and the penalty when she observes a signal that coincides with the report, respectively. The auditor's liability is reduced if she used due care to avoid audit failure, while liability is increased if she found a symptom of fraud and failed to detect it, i.e., $L_c < L < L_d$.

Proposition 13 (Due care consideration) *Assume that θ is sufficiently small such that $\theta < (1-p_F)L_c/[(1-p_F)L_c + (1-p_G)R]$. When the auditor's penalty for a type II error is differentiated, α_1^* and α_2^* are redefined as follows:*

$$\alpha_1^* \equiv \frac{\theta}{1-\theta} \cdot \frac{p_G}{p_F} \cdot \frac{R}{L_d}, \qquad \alpha_2^* \equiv \frac{\theta}{1-\theta} \cdot \frac{1-p_G}{1-p_F} \cdot \frac{R}{L_c}. \tag{27}$$

With the newly defined α_1^* and α_2^*, one has $\partial\alpha_1^*/\partial L_d < 0$, $\partial\alpha_1^*[\beta_1^* p_F + (1-p_F)]/\partial L_d < 0$, $\partial\alpha_2^*/\partial L_c < 0$, and $\partial\alpha_2^*[\beta_2^*(1-p_F)]/\partial L_c < 0$. In words, both inherent risk and overall audit risk decrease with due care consideration if $\delta < 0$, while the risks increase with due care consideration if $\delta > 0$.

Proof. Redefine $A(\alpha, \beta_1, \beta_2)$ in the proof of Lemma 2 using L_d and L_c. Following the same procedure as in the proof of Lemma 2, one has equations in (27). The derivatives are readily calculated directly. *Q.E.D.*

Proposition 13 is essentially the same as Proposition 8. Due care consideration increases inherent risk and overall audit risk when $\delta > 0$, but decreases the risks when $\delta < 0$. The difference between these two cases is amplified by due care consideration.

4 Concluding remarks

The current study confirms that the introduction of audit evidence does not always reduce inherent risk and overall audit risk. If a manager has a sufficiently strong incentive to commit fraud, inherent risk increases with information. This result is robust in the two different settings analyzed in the present study.

Detection risk always increases with audit evidence in the defalcation game, while the risk does not change at all in the misreporting game. In the defalcation game, the fundamental driving force behind the increase in detection risk is the strategic interaction between a manager who is inclined to honest action to reduce the expected rejection penalty and an auditor who expects the manager's behavior. In the misreporting game, the auditor knows that a manager who reports low performance is honest and therefore the manager is not penalized. Hence, the driving force that leads to the increase in detection risk disappears.

As in the misreporting game, if detection risk does not change with audit evidence, overall audit risk increases if and only if inherent risk increases. However, if detection risk increases with audit evidence, as in the defalcation game, the conditions under which audit risk increases with audit evidence are less severe than in the case in which detection risk does not change. In the defalcation game, the auditor's liability ratio affects whether or not audit risk increases with audit evidence by affecting detection risk. If the auditor has a sufficiently strong incentive to avoid false rejection, audit risk increases with information in the defalcation game.

The analysis shows that requiring auditors to obtain (additional) information is not an adequate action to reduce audit risk, at least in some instances. The results found in this study might be relevant to ongoing corporate reform, in that they shed light on what should be done to reduce audit risk. For example, when an auditor provides an audit client with non-audit services, the auditor's penalty for false rejection is generally greater than when she does not provide such services. The analytical results of the present study support the provision of the Sarbanes-Oxley Act of 2002 that prohibits audit firms from providing their audit clients with almost all non-audit services.

The models analyzed here are just two examples of strategic audit games. Accordingly, it is fair to say that the generalizability of the propositions obtained in this study remains to be seen. Furthermore, the analysis is limited in that the payoff parameters are endowed exogenously. If the audit is examined in a broader environment, the results may potentially change (Newman, Patterson and Smith, 2005).

The models in this study, however, are two of the simplest possible acceptance sampling models that capture the basic relation among audit risk and its components, the payoff structures and information obtained by the auditor. Their simple settings isolate the effects of the payoff parameters on audit risk and its components when the auditor obtains more information. In this regard, the study provides two reference points for more generalized audit games in which the payoffs are internalized.

Appendix

Proof of Lemma 1

The manager's expected payoff $M(\alpha,\beta_1,\beta_2)$ and the auditor's expected payoff $A(\alpha,\beta_1,\beta_2)$ in the defalcation game are as follows:

$$\begin{aligned} M(\alpha,\beta_1,\beta_2) =& \alpha\{p_F[\beta_1 b + (1-\beta_1)(-s-q)] + (1-p_F)[\beta_2 b + (1-\beta_2)(-s-q)]\} \\ &+ (1-\alpha)[p_G(1-\beta_1)(-q) + (1-p_G)(1-\beta_2)(-q)]. \end{aligned} \tag{28}$$

$$\begin{aligned} A(\alpha,\beta_1,\beta_2) =& \alpha[p_F\beta_1(-L) + (1-p_F)\beta_2(-L)] \\ &+ (1-\alpha)[p_G(1-\beta_1)(-R) + (1-p_G)(1-\beta_2)(-R)]. \end{aligned} \tag{29}$$

The function $M(\alpha,\beta_1,\beta_2)$ is affine (i.e., linear with an intercept) in α, while the function $A(\alpha,\beta_1,\beta_2)$ is affine in both β_1 and β_2. Hence, the manager's best response correspondence is

$$\begin{aligned} &\alpha = 1 && \text{if} \quad \partial M/\partial\alpha > 0, \\ &0 \leqslant \alpha \leqslant 1 && \text{if} \quad \partial M/\partial\alpha = 0, \\ &\alpha = 0 && \text{if} \quad \partial M/\partial\alpha < 0. \end{aligned} \tag{30}$$

Observe that α_1^* and α_2^* in Lemma 1 solve $\partial A/\partial\beta_1 = 0$ and $\partial A/\partial\beta_2 = 0$, respectively. Using the relation $p_G < p_F$, one has $\alpha_1^* < \alpha_2^*$. Since $\partial^2 A/(\partial\beta_1\partial\alpha) < 0$ and $\partial^2 A/(\partial\beta_2\partial\alpha) < 0$, the auditor's best response correspondence is as follows:

$$\begin{array}{llll} \beta_1 = 1 \quad \text{and} \quad \beta_2 = 1 & & \text{if} & \alpha < \alpha_1^*, \\ 0 \leqslant \beta_1 \leqslant 1 \quad \text{and} \quad \beta_2 = 1 & & \text{if} & \alpha = \alpha_1^*, \\ \beta_1 = 0 \quad \text{and} \quad \beta_2 = 1 & & \text{if} & \alpha_1^* < \alpha < \alpha_2^*, \\ \beta_1 = 0 \quad \text{and} \quad 0 \leqslant \beta_2 \leqslant 1 & & \text{if} & \alpha = \alpha_2^*, \\ \beta_1 = 0 \quad \text{and} \quad \beta_2 = 0 & & \text{if} & \alpha_2^* < \alpha. \end{array} \tag{31}$$

The five cases in (31) are examined as follows.

Case 1: $\alpha < \alpha_1^*$ If $\alpha < \alpha_1^*$, then $\beta_1 = \beta_2 = 1$. In this case, $\partial M(\alpha, 1, 1)/\partial\alpha = b > 0$. Accordingly, the manager's best response is $\alpha = 1$. This violates the condition $\alpha < \alpha_1^*$ because $\alpha_1^* < 1$. There is no Nash equilibrium in this case.

Case 2: $\alpha = \alpha_1^*$ Since $0 < \alpha_1^* < 1$, then if α_1^* is the manager's equilibrium strategy, $\partial M(\alpha, \beta_1, 1)/\partial\alpha$ must be zero. Simplifying $\partial M(\alpha, \beta_1, 1)/\partial\alpha = 0$, one has $(b - \delta)\beta_1 = -\delta$. As direct calculation shows $\delta < b$, one obtains $\beta_1 = \beta_1^*$. The condition $\beta_1^* \geqslant 0$ is equivalent to $\delta \leqslant 0$. When $\delta \leqslant 0$, it holds that $\beta_1^* < 1$. Hence, if $\delta \leqslant 0$, the strategy set $(\alpha, \beta_1, \beta_2) = (\alpha_1^*, \beta_1^*, 1)$ is a Nash equilibrium. Observe that $\beta_1^* = 0$ if and only if $\delta = 0$.

Case 3: $\alpha_1^* < \alpha < \alpha_2^*$ Since $0 < \alpha_1^* < \alpha_2^* < 1$, if $\hat{\alpha}$ satisfying $\alpha_1^* < \hat{\alpha} < \alpha_2^*$ is the manager's equilibrium strategy, $\partial M(\alpha, 0, 1)/\partial\alpha$ must be zero. Recall that δ is defined as $\partial M(\alpha, 0, 1)/\partial\alpha$. Therefore, δ must be zero in this case. On the contrary, if δ is zero, any strategy sets $(\hat{\alpha}, 0, 1)$ satisfying $\alpha_1^* < \hat{\alpha} < \alpha_2^*$ are Nash equilibria.

Case 4: $\alpha = \alpha_2^*$ Since $0 < \alpha_2^* < 1$, if α_2^* is the manager's equilibrium strategy, $\partial M(\alpha, 0, \beta_2)/\partial\alpha$ must be zero. Simplifying $\partial M(\alpha, 0, \beta_2)/\partial\alpha = 0$, one has $(\delta + s)\beta_2 = s$. This equation does not hold if $\delta = -s$ and therefore $\delta \neq -s$ is necessary. In this case, β_2 is equal to β_2^*. The condition $\beta_2^* \geqslant 0$ is equivalent to the condition $\delta > -s$. When δ is greater than $-s$, the condition $\beta_2^* \leqslant 1$ is equivalent to the condition $\delta \geqslant 0$. Observe that $\delta \geqslant 0$ implies that $\delta > -s$ and hence $\delta \neq -s$. Therefore, if $\delta \geqslant 0$, the strategy set $(\alpha, \beta_1, \beta_2) = (\alpha_2^*, 0, \beta_2^*)$ is a Nash equilibrium. Note that $\beta_2^* = 1$ if and only if $\delta = 0$.

Case 5: $\alpha_2^* < \alpha$ If $\alpha_2^* < \alpha$, then $\beta_1 = \beta_2 = 0$. In this case, $\partial M(\alpha, 0, 0)/\partial\alpha = -s < 0$. Accordingly, the manager's best response is $\alpha = 0$. This violates the condition $\alpha_2^* < \alpha$, because $\alpha_2^* > 0$. There is no Nash equilibrium in this case.

The Nash equilibria in Cases 2, 3 and 4 are summarized as in Lemma 1. *Q.E.D.*

Proof of Lemma 2

The manager's expected payoff $M(\alpha,\beta_1,\beta_2)$ and the auditor's expected payoff $A(\alpha,\beta_1,\beta_2)$ in the misreporting game are as follows:

$$\begin{aligned} M(\alpha,\beta_1,\beta_2) =& \theta\{p_G[\beta_1 b + (1-\beta_1)(b-q)] + (1-p_G)[\beta_2 b + (1-\beta_2)(b-q)]\} \\ &+(1-\theta)\alpha\{p_F[\beta_1 b + (1-\beta_1)(-s-q)] \\ &+(1-p_F)[\beta_2 b + (1-\beta_2)(-s-q)]\}. \end{aligned} \tag{32}$$

$$\begin{aligned} A(\alpha,\beta_1,\beta_2) =& \theta[p_G(1-\beta_1)(-R) + (1-p_G)(1-\beta_2)(-R)] \\ &+(1-\theta)\alpha[p_F\beta_1(-L) + (1-p_F)\beta_2(-L)]. \end{aligned} \tag{33}$$

The manager's best response is $\alpha_1 = 1$ if $\partial M/\partial\alpha > 0$, $0 \leqslant \alpha \leqslant 1$ if $\partial M/\partial\alpha = 0$, and $\alpha = 0$ if $\partial M/\partial\alpha < 0$. Observe that α_1^* and α_2^* in Lemma 2 solve $\partial A/\partial\beta_1 = 0$ and $\partial A/\partial\beta_2 = 0$, respectively. Using the relation $p_G < p_F$, one has $\alpha_1^* < \alpha_2^*$. When $\theta < (1-p_F)L/[(1-p_F)L + (1-p_G)R]$, it holds that $0 < \alpha_1^* < \alpha_2^* < 1$.

Since $\partial^2 A/(\partial\beta_1\partial\alpha) < 0$ and $\partial^2 A/(\partial\beta_2\partial\alpha) < 0$, the auditor's best response is (1) $\beta_1 = \beta_2 = 1$ if $\alpha < \alpha_1^*$, (2) $0 \leqslant \beta_1 \leqslant 1$ and $\beta_2 = 1$ if $\alpha = \alpha_1^*$, (3) $\beta_1 = 0$ and $\beta_2 = 1$ if $\alpha_1^* < \alpha < \alpha_2^*$, (4) $\beta_1 = 0$ and $0 \leqslant \beta_2 \leqslant 1$ if $\alpha = \alpha_2^*$, and (5) $\beta_1 = \beta_2 = 0$ if $\alpha_2^* < \alpha$. These five cases are examined as follows.

Case 1: $\alpha < \alpha_1^*$ If $\alpha < \alpha_1^*$, then $\beta_1 = \beta_2 = 1$. Note that $\partial M(\alpha,1,1)/\partial\alpha = (1-\theta)b > 0$. Accordingly, the manager's best response is $\alpha = 1$. This violates the condition that $\alpha < \alpha_1^* < 1$. There is no Nash equilibrium in this case.

Case 2: $\alpha = \alpha_1^*$ If α_1^* is the manager's equilibrium strategy, $\partial M(\alpha,\beta_1,1)/\partial\alpha$ must be zero. Simplifying this relation, one has $(b-\delta)\beta_1 = -\delta$. As direct calculation shows $\delta < b$, one knows that the value $\beta_1 = \beta_1^*$ solves the equation $\partial M(\alpha,\beta_1,1)/\partial\alpha = 0$. The condition $\beta_1^* \geqslant 0$ is equivalent to the condition $\delta \leqslant 0$. When $\delta \leqslant 0$, it holds that $\beta_1^* < 1$. Hence, if $\delta \leqslant 0$, the strategy set $(\alpha,\beta_1,\beta_2) = (\alpha_1^*,\beta_1^*,1)$ is a Nash equilibrium. Observe that $\beta_1^* = 0$ if and only if $\delta = 0$.

Case 3: $\alpha_1^* < \alpha < \alpha_2^*$ If $\hat{\alpha}$ satisfying $\alpha_1^* < \hat{\alpha} < \alpha_2^*$ is the manager's equilibrium strategy, $\partial M(\alpha,0,1)/\partial\alpha$ must be zero. Confirm that δ is equal to $(\partial M(\alpha,0,1)/\partial\alpha)/(1-\theta)$. Therefore,

δ must be zero. On the contrary, if δ is zero, any strategy sets $(\hat{\alpha}, 0, 1)$ satisfying $\alpha_1^* < \hat{\alpha} < \alpha_2^*$ are Nash equilibria.

Case 4: $\alpha = \alpha_2^*$ If α_2^* is the manager's equilibrium strategy, $\partial M(\alpha, 0, \beta_2)/\partial\alpha$ must be zero. Simplifying $\partial M(\alpha, 0, \beta_2)/\partial\alpha = 0$, one has $(\delta + s + q)\beta_2 = s + q$. Since $\delta + s + q = (1 - p_F)(b + s + q) > 0$, then the value $\beta_2 = \beta_2^*$ solves the equation $\partial M(\alpha, 0, \beta_2)/\partial\alpha = 0$. The condition $\beta_2^* \leqslant 1$ is equivalent to $\delta \geqslant 0$. When $\delta \geqslant 0$, it holds that $\beta_2^* > 0$. Therefore, if $\delta \geqslant 0$, the strategy set $(\alpha, \beta_1, \beta_2) = (\alpha_2^*, 0, \beta_2^*)$ is a Nash equilibrium. Note that $\beta_2^* = 1$ if and only if $\delta = 0$.

Case 5: $\alpha_2^* < \alpha$ If $\alpha_2^* < \alpha$, then $\beta_1 = \beta_2 = 0$. In this case, $\partial M(\alpha, 0, 0)/\partial\alpha = -(1 - \theta)(s + q) < 0$. Accordingly, the manager's best response is $\alpha = 0$. This violates the condition that $\alpha_2^* < \alpha$, because $\alpha_2^* > 0$. There is no Nash equilibrium in this case.

The Nash equilibria in Cases 2, 3 and 4 are summarized as in Lemma 2. *Q.E.D.*

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