Decentralized Trading with Private Information

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Motivation I: Efficiency

- Many asset markets are decentralized
- Poor information on market prices and transactions
- Does decentralized trading in such markets lead to efficiency?
Motivation II: Information

- Grossman-Stiglitz paradox: in REE prices are fully revealing, no incentives to collect info
- In many markets significant resources are spent collecting the information
- What incentives do agents have to collect information and how can it be utilized
Decentralized Trading with Private Information

Economy

- Agents are heterogeneous with respect to asset holdings
- Some agents have superior information about the value of the assets
- Bilateral matchings
- Bayesian learning based on trades
- Inferences drawn from history of trades
- No information on other trades
- Key feature: distribution of beliefs about the value of the asset is endogenous and changes over time
Decentralized Trading with Private Information

Main results of the paper

- **Efficiency:** The long run allocations are Pareto Efficient, coincide with competitive equilibrium allocations with fully revealing prices.

- **Information:** Value of information is positive initially, eventually zero.

- **Trading Dynamics:** Informed agents buy more of the valuable asset early and sells some of it back at higher prices once information dissipates among agents.
Literature

- Glosten-Milgrom, etc: models of OTC markets where all agents observe *all transactions*

- This paper: different agents observe different transactions
  - also Duffie-Manso
Environment

• Continuum of agents divided in a finite number of types $I$.
• 2 assets
• Type $j \in I$ starts with initial endowment $x = (x_{0j}^1, x_{0j}^2) \in R_+^2$.
• Types and endowments are symmetric in two assets
• Total endowment of each asset is normalized to 1
Information

- Two states of the world, 1 and 2.
- In state $i$: asset $i$ pays 1 unit of consumption good, asset $-i$ pays nothing.
- Utility $u(x)$ is strictly concave, differential, bounded from above, $u(0) = -\infty$.
- Information is revealed in two stages:
  1. Ex-ante probability of state $i$ is 0.5, common knowledge.
  2. Randomly chosen fraction $m$ of all agents privately observe a common signal $s \in \{s_1, s_2\}$. Probability of state $i$ occurring is $\phi(s_i)$. Let $\phi(s_2) = 1 - \phi(s_1)$ and $\phi(s_1) \in (0.5, 1)$.
Benchmark: Rational Expectations Equilibrium

- Informed and uninformed agents who start with the same endowment consume the same allocations.
- Prices fully reveal which state of the world occurs.
- All final bundles have $x^1 = x^2$.
- Asset prices $p^j(s)$ in state $s$ satisfy
  \[
  \frac{p^1(s)}{p^2(s)} = \frac{\phi(s)}{1 - \phi(s)}
  \]
Bargaining and Consumption

- Bargaining follows Gale
  - After some observe $s$, all agents are randomly paired and one from the pair is randomly chosen to make a take it or leave it offer
  - Agents can make only feasible for him offers, the other one can accept only feasible for him offers

- After a round of bargaining
  - w.p. $\gamma$ a new round of matching and bargaining begins
  - w.p. $(1 - \gamma)$ the game ends, state $i$ occurs with probability $\phi(s_i)$ and agents consume their endowments

- Agents observe only offers that occur during their match.
Equilibrium

- We consider a symmetric Perfect Bayesian Equilibrium of this game. Symmetric means that allocations are symmetric for states $s_1$ and $s_2$. 
Notation

• $h_t$ is history in period $t$ that an agent might observe

• Beliefs of uninformed agents

$$\delta(h_t) = \frac{\Pr(h_t, s_1)}{\Pr(h_t, s_1) + \Pr(h_t, s_2)}$$

• Let

$$\pi(\delta) = \delta \phi + (1 - \delta)(1 - \phi).$$

• Expected utility and value function

$$U(x, \delta(h_t)) \equiv E\{u(x_t|h_t)\}$$

$$= \pi(\delta(h_t))u(x^1_t(h_t)) + (1 - \pi(\delta(h_t)))u(x^2_t(h_t))$$

$$V(h_t) \equiv v(x_t|h_t) = (1 - \gamma)U(x, \delta(h_t)) + \gamma E[v(x_{t+1}|h_{t+1})|h_t].$$
Value of information

• If uninformed agent could costlessly observe $s$ in period $t$, his utility would be

$$\delta(h_t)v^i(x_t, t; s_1) + (1 - \delta(h_t))v^i(x_t, t; s_2)$$

• Value of information

$$\delta(h_t)v^i(x_t, t; s_1) + (1 - \delta(h_t))v^i(x_t, t; s_2) - v(x_t|h_t) \geq 0$$

• **Theorem 1**: the value of information is zero in the long run:

$$\delta_t v^i(x_t, t; s_1) + (1 - \delta_t)v^i(x_t, t; s_2) - v(x_t|h_t) \to 0$$
When information has no value?

- Value of information goes to zero if
  - i All uninformed agents learn fully:
    * \( \delta_t \to \{0, 1\} \)
  - ii Some uninformed agents do not learn but there are no gains from trade if they learned

- Theorem 2 (Efficiency in the long run:) For all \( s \), \( x^1_t - x^2_t \to 0 \) and \( \kappa(s) = \phi(s)/(1 - \phi(s)) \).

- These are the same allocations that would constitute a CE (although not for the same endowments as agents have in period 0).
Informational rents

- **Theorem 3**: Value of information is strictly positive if and only if \( x_{0,i}^1 \neq x_{0,i}^2 \) for some \( i \)

- When start at a Pareto efficient allocation
  - Milgrom-Stokey result applies
  - No trade in equilibrium
  - Value of information is zero

- In other cases, informed agents can exploit their information
General case: Dynamics of trading

- Informed and uninformed start with average endowment of good 1 equal to average endowment of good 2.
- Both end up with average endowment of good 1 equal to average endowment of good 2, but informed are better off:
  - Informed must have on average more of both goods.
  - This is only possible if informed agents buy more of the more valuable asset early on and sell some of it back to uninformed agents at higher prices once information dissipates.
Static Example

- To understand strategies of different types of agents, consider a one shot model first
- Let $I = 2$
- Assume that fraction of informed is negligible
Equilibrium strategies

- **Uninformed** proposer makes an offer to make uninformed receiver indifferent between accepting and rejecting.
- **Poor informed** proposer mimics uninformed.
- **Rich informed** reveals his information by making a small offer by a price *higher than full info price*.
  - Allocations are *ex-post inefficient*. 
Case 1: Informed knows that state 1 is more likely
Case 2: Informed knows that state 2 is more likely
Dynamic Economy

- Assume $u(x) = -\exp(-x)$
  - Agent’s state depends on $(x^1 - x^2, \delta, t)$.
- Half of agents start with endowment $(2, 0)$, half with $(0, 2)$.
- Informed agents get signal $s_1$
Trades of the informed

- **Blue line**: rich informed. **Blue dotted line**: signal is public info
- **Red line**: poor informed. **Blue dotted line**: signal is public info
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Convergence to efficient allocations

- Solid line: benchmark economy
- Dotted line: signal is public info
Lessons from the dynamic example

- Convergence to efficiency is slower under asymmetric information
- Hump-shaped behavior for informed
  - Buy "too much" of the more valuable good while uninformed do not know the signal
  - Sell back to uninformed that good at a higher price once information spreads
- Very rich informed sell their endowment slowly
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Conclusion

- All information is revealed in the long run and allocations coincide with CE allocations
- Informed agents obtain higher payoff than uninformed agents
- Informed agents buy more valuable good early on from uninformed and sell it back at a higher price once the information spread through the market