The more we know, the less we agree:
Public announcements, higher-order expectations and rational inattention

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Environments with higher-order expectations

- Decisions often influenced by higher-order expectations (agents’ opinion about the opinion of others) rather than the agents’ own opinion about fundamentals
- In such environments: public announcements have double role:
  1. information about the fundamental
  2. information about the opinion of others
- In financial markets: ⇒ frenzy of trading around announcements
In this paper:

1. In some Gaussian structures, public announcement leads to
   A. **Polarized HOE**: increased dispersion in higher order expectations (not first)
   B. **Contrarian HOE**: the more optimistic about fundamental \( \Rightarrow \) the more pessimistic about others’ expectation about the fundamental

2. standard, Grossman-Stiglitz, OLG-dynamic model with rational inattention: the two properties arise endogenously

3. Two properties explain hectic, volatile, informative trading around public announcements
Polarized and Contrarian HOEs: Example

- Apple = old project + new project: $\theta = \theta_o + \theta_n$,
- early, $x_o^j = \theta_o + \epsilon^j$, and late analyst, $x_n = \theta_n + \nu$
  - without public signal private signals are uninformative:
    \[
    \int_0^1 \left| E(E(\theta|x_n)|x_o^j) - \bar{E}(E(\theta|x_n)|x_o^j) \right| dj = \\
    = \int_0^1 \left| E(\hat{a}_\theta x_n|x_o^j) - \bar{E}(\hat{a}_\theta x_n|x_o^j) \right| dj = 0
    \]
  - with public signal, $y = \theta + \eta$, higher $x_o^j \rightarrow$ lower guess of $x_n$: contrarian and polarized HOEs
    \[
    \int_0^1 \left| E(E(\theta|x_n,y)|x_o^j,y) - \bar{E}(E(\theta|x_n,y)|x_o^j,y) \right| dj = \\
    = \int_0^1 \left| E(a_\theta x_n + c_\theta y|x_o^j,y) - \bar{E}(a_\theta x_n + c_\theta y|x_o^j,y) \right| dj = \\
    = |a_{x_n} a_\theta| \int_0^1 |\epsilon^j| dj > 0
    \]
Polarized and Contrarian HOEs: Statistics

- 2 groups of agents: 1, 2 unit mass of agents in each group (in paper $T$ groups)
- group 2 forecasts $\theta$ fundamental, group 1 forecasts the forecast of group 2
- $y, x_1^i, x_2^i, \theta \sim N(0, \cdot)$ general, but symmetric covariance structure:

\[
\sigma_x^2, \sigma_\theta^2, \sigma_y^2, \rho_{x,x}, \rho_{x,x'}, \rho_{x,y}, \rho_{\theta,x}, \rho_{\theta,y} > 0
\]

where $\rho_{x,x'}$ across groups
Observations

1. (Milgrom, 1980): Suppose "positive signals":

\[ a_\theta \equiv \frac{\partial E (\theta | x_1^j, y)}{\partial x_1^j}, \quad c_\theta \equiv \frac{\partial E (\theta | x_1^j, y)}{\partial y} \]  > 0

then there is no polarization in first-order expectations:

\[ \int_0^1 \left| E (\theta | x_1^j, y) - E (\theta | x_1^j, y) \right| dj < \int_0^1 \left| E (\theta | x_1^j) - E (\theta | x_1^j) \right| dj \]
Observations

2. Add $\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta}{\sigma_{\theta}^2 \rho_{x,y} (1 - \rho_{x,y}^2) + a_\theta}$, then polarized HOE:

$$\int_0^1 \left| E(E(\theta|x_2^i, y)|x_1^i, y) - E(E(\theta|x_2^i, y)|x_1^i, y) \right| \, di >$$

$$\int_0^1 \left| E(E(\theta|x_2^i)|x_1^i) - E(E(\theta|x_2^i)|x_1^i) \right| \, di$$

3. Same condition implies contrarian HOE:

$$\frac{\partial E(\theta|x_1^i, y)}{\partial x_1^i} \frac{\partial E(E(\theta|x_2^i, y)|x_1^i, y)}{\partial x_1^i} < 0$$
Announcements on financial markets

- Multi period, CARA-Normal Grossman-Stiglitz with endogenous choice of information
- Two, unit mess, groups of myopic traders, OLG structure
- Noisy supply of assets, $u_t$, in each period

Diagram:

- Period -1: Information choice
  - $\sigma^2_{x_i}, \sigma^2_{\nu_0}, \sigma^2_{\nu_n}$
  - $p_0$

- Period 0: Early traders
  - Early traders
  - $y$

- Period 1: Early traders
  - Early traders
  - $z_o^i, z_o^i$
  - $p_1$

- Period 2: Late traders
  - Late traders
  - $\theta$
  - $p_2$
• fundamental value: $\theta = \theta_o + \theta_n$ (old project, new project)
• each trader receive private signals
  • Early traders:
    \[ x^i = \theta_o + \epsilon^i \]
  • Late traders:
    \[ z_o^i = \theta_o + \nu_o^i \]
    \[ z_n^i = \theta_n + \nu_n^i. \]
• w.p. $\pi$ there is an announcement in period 1: $y = \theta + \eta$
• in period 0, each trader finds out whether there will be an announcement
• all factors and noise terms are i.i.d normal
Information choice

- In period -1 agents allocate attention across factors: early traders pick $\sigma_{\varepsilon_i}^2$, late traders pick $\sigma_{\nu_o}^2$, $\sigma_{\nu_n}^2$ to maximize the expectation of

$$\pi \max_{d_i(I^i, \mathcal{P}_t^a)} E \left( -e^{-\gamma W_t^i} \mid I^i, \mathcal{P}_t^a \right)$$

$$+ (1 - \pi) \max_{d_i(I^i, \mathcal{P}_t^n)} E \left( -e^{-\gamma W_t^i} \mid I^i, \mathcal{P}_t^n \right)$$

- early trader can learn only about current project (no information yet about next project)
- late trader can decide which project to learn about
Information processing constraint

- The choice is subject to the *information processing constraint*:

\[
H([\theta_o, \theta_n]) - H\left([\theta_o, \theta_n] | I^i\right) \leq \frac{\kappa}{2}
\]

- \(H(\cdot)\) is entropy of a random variable
- for late traders this translates to:

\[
\ln \frac{\sigma_{\theta_n}^2 + \sigma_{v_n}^2}{\sigma_{v_n}^2} \frac{\sigma_{\theta_o}^2 + \sigma_{v_o}^2}{\sigma_{v_o}^2} \leq \kappa
\]

- or fraction of attention on old project:

\[
\lambda^i \equiv \frac{1}{\kappa} \ln \frac{\sigma_{\theta_o}^2 + \sigma_{v_o}^2}{\sigma_{v_o}^2}
\]
The RE equilibrium for given information structure

- by standard methods (1. assume linear prices, 2. derive optimal strategies, 3. find fixed point of coefficients)
- REE equilibrium portfolio holdings in period 1 and 0

\[
\begin{align*}
    d_1^i &= \frac{E(p_2|x^i, y, q_1, q_0) - p_1}{\gamma \text{var}(p_2|x^i, y, q_1, q_0)} = \frac{\tau_1}{\delta_1} (x^i - q_1) \\
    d_0^i &= \frac{E(p_1|x^i, q_0) - p_0}{\gamma \text{var}(p_1|x^i, q_0)} = \frac{\tau_0}{\delta_0} (x^i - q_0)
\end{align*}
\]

- \(q_t\): residual information in price \(p_t\), over and above information in public signal and past prices
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- \( q_t \): residual information in price \( p_t \), over and above information in public signal and past prices

\[ q_1 = \theta_o - \frac{\delta_1}{\tau_1} u_1 \]

\[ q_0 = \theta_o - \frac{\delta_0}{\tau_0} u_0 \]

- finding the equilibrium amounts to finding the fixed point in \( \frac{\tau_t}{\delta_t} \)
Trading volume and information content of prices

- change of positions between periods 0 and 1 (time-series)

\[
V^{01} = \frac{1}{2} E_u \left( \int |d_{1,i} - d_{0,i}| \, di \right) = \left| \frac{\tau^a_1}{\delta_1} - \frac{\tau^a_0}{\delta_0} \right| \frac{\sigma_\varepsilon}{\sqrt{2\pi}}
\]
Trading volume and information content of prices

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- or positions in period 1 with and without announcement (cross-section)

\[ V^{11} = \frac{E_u \left( \int |d^a_{1,i}| \, di \right)}{E_u \left( \int |d^n_{1,i}| \, di \right)} = \left| \frac{\tau^a_1}{\delta_1} \right| \left| \frac{\tau^n_1}{\delta_1} \right| \]

- information content of prices: compare variance in \( q_t \) with and without announcement: \( C \equiv \frac{(\tau^a_1)^2}{(\tau^n_1)^2} \)
Trading intensities with different informational choices
Remarks

- large $V^{11}, C$ because of polarized second-order expectations:
  - holding is disagreement over variance
    $$d^i = \frac{E(\theta|I, P_t) - p}{\gamma \text{Var}(\theta|I, P_t)}$$
  - in standard models disagreement decreases after announcement, two opposite forces
  - here: go into same directions, large effect
Remarks

- large $V^{11}$, $C$ because of polarized second-order expectations:
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  - in standard models disagreement decreases after announcement, two opposite forces
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- large $V^{01}$, because of contrarian second-order expectations:
  - early traders in period 0, bet on $y$, forecast depends on first-order expectations
  - early traders in period 1, bet on $p_2 \approx$ expectation of late traders, forecast depends on second-order expectations
Attention allocation: REE with endogenous learning

- trading volume is large when private information sets are independent across periods
- why would a late trader specialize on the other factor?
- simple (but slightly wrong) intuition: learn about $\theta_n$, because past price already reveals a lot on $\theta_o$
• However: in REE equilibrium
  • current prices also reveal information: if other late traders learn about $\theta_n$, you might not want to
  • past prices are informative only if early traders trade aggressively
  • early traders’ trading intensity depends on information content in $p_2$ (which depends on the attention allocation of late traders)

• **Result:** For any fixed set of parameters, there are $\sigma_{\eta}^2, \pi$ that if $\sigma_{\eta}^2 < \sigma_{\eta}^2$ and $\pi > \bar{\pi}$ than late traders learn **only** about $\theta_n$
Matching stylized facts

- Known that trading volume increases around announcements
- Puzzling: should not in a representative agent model or in standard noisy REE model
- With some modification (He-Wang (1995), Kim-Verrecchia (1991)), some volume but mostly of the type $V^{01}$
Matching stylized facts

- Known that trading volume increases around announcements
- Puzzling: should not in a representative agent model or in standard noisy REE model
- With some modification (He-Wang (1995), Kim-Verrecchia (1991)), some volume but mostly of the type $V^{01}$
- Kandel-Pearson (1995): polarized expectations necessary to match facts
  - Enforced by high-frequency evidence: announcement is followed by hectic and informative trading ($V^{11}, C$ are large)
- But:
  - Polarization in higher-order expectations is sufficient
  - It is generated endogenously by rational inattention
Related literature

- public announcement and trading volume (previous slide)
Conclusion

- Separated private information sets $\Rightarrow$ public announcement leads to polarized and contrarian HOEs
  - public announcement connects information sets $\Rightarrow$ makes private signals relevant to guess other private signals
- In an OLG-dynamic model rational inattention $\Rightarrow$ separated private information sets
  - late traders prefer to learn privately on elements they cannot learn publicly
  - past prices reveal information of past traders publicly
- contrarian and polarized HOE implies hectic and informative trading around announcements