Asset Trading and Valuation with Uncertain Exposure

Juan Carlos Hatchondo
Richmond Fed

Per Krusell
IIES & Princeton

Martin Schneider
Stanford
Motivation

• “Well functioning financial markets”
  – prices aggregate dispersed information...
  – agents efficiently share exposure to aggregate risk
Motivation

• With complete markets & agreement among agents
  – individual risk exposures equated in equilibrium...
  – ... to aggregate exposure of representative agent...
  – ... which determines risk premia
Defining exposure

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• \( X_{t+1} = \) asset payoff,
• \( M^i_{t+1} = \) agent \( i \)'s intertemporal MRS \( t + 1 \) vs \( t = \) “relative hunger”
• \( -M^i_{t+1} = \) “relative pleasure”

• exposure of agent \( i \) to risk \( X_{t+1} \) := \[
\frac{cov_t(-M^i_{t+1}, X_{t+1})}{var_t(X_{t+1})}
\]

(positive exposure to \( X \ = \) “more pleasure when \( X \) is high”)
Defining exposure

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  (positive exposure to $X = \text{“more pleasure when } X \text{ is high”}$)

- Euler equations with complete markets & agreement
  \[ P_t = \frac{1}{R_t}E_t[X_{t+1}] \quad - \quad \{ \text{cov}_t \left( -M^i_{t+1}, X_{t+1} \right) \} \]
  discounted expected payoff risk premium
Defining exposure

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- Euler equations with complete markets & agreement

$$P_t = \frac{1}{R_t} E_t[X_{t+1}] - \text{var}_t(X_{t+1}) \left\{ \frac{\text{cov}_t(-M_{t+1}^i, X_{t+1})}{\text{var}_t(X_{t+1})} \right\}$$

discounted expected payoff

payoff risk

exposure
Motivation

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- This paper
  - news about aggregate exposure arrive as dispersed information
  - agents see only initial individual exposures ...
    ... that are informative about aggregate exposure
  - trading, prices, measured risk premia?
Overview of Model

- Two period exchange economy with incomplete markets
  - assets contingent on aggregate risk factor realized tomorrow
  - LRT utilities; endowments tradable

- Private information
  - news about asset payoffs
  - endowments & preferences → initial exposures to risk factor

- Two aggregate shocks today
  - pooled news about asset payoffs:
    - “how many agents get good news”
    - private news informative about pooled news
  - aggregate shock to distribution of initial exposures
    - “how many agents have high initial exposure”
    - = “aggregate exposure” of (full info) representative agent
    - individual exposure informative about aggregate exposure

- Rational expectations equilibrium
Nonrevealing Equilibria

• Prices do not reflect all agents’ pooled information
  – about news → relevant for expected asset payoffs
  – about aggregate exposure → relevant for risk premia
    e.g. low price because bad news or high aggregate exposure!

• Equilibrium beliefs depend on initial exposure
  – agent w/ high initial exposure more optimistic
    1. believes that aggregate exposure is high
    2. views price as better news (than does low-exposure agent)

• Trading does not equate exposure across agents
  – in contrast to full info case
  – less reallocation of exposure by agents with same signals about payoffs
  – more “speculative” trading by agents with same exposure, different signals
Asset Pricing Implications

• Shocks to aggregate exposure matter more
  – price always lower if higher aggregate exposure (higher risk premium)
  – with asymmetric info, price drop mistaken for low expected payoff!
    ⇒ price falls even more

• Joint behavior of volume & risk premia in a crisis
  – increase in (i) aggregate exposure and (ii) uncertainty about exposures
    ⇒ low volume & high risk premia, low prices

• More excess return predictability
  prices respond more to shocks that are not payoff news
    ⇒ econometrician observes more time variation in risk premia

• Wealth effects & belief aggregation
  – e.g. log utility: price reflects wealth-weighted average belief
  – correlation of wealth and exposure matters for risk premia
Related Literature

- **Existence & efficiency in economies with asymmetric information**
  Radner, Laffont, Green, Lucas

- **Quasi-complete economies**
  DeMarzo-Skiadas

- **Exponential utility & normal shocks**
  noise traders: Grossman-Stiglitz, Hellwig, Admati
  uncertain endowments: Diamond-Verrecchia, Ganguli-Yang
  dynamics: Wang, Albuquerque-Bauer-Schneider

- **Heterogeneous beliefs and asset prices**
  Detemple-Murthy, Calvet-Grandmont-Lemaire, Jouini-Napp

- **Adverse selection in financial markets**
Now what?

Paper considers different setups

1. discrete states, $u$ smooth
   (choose values appropriately for nonrevelation)

2. continuous & discrete states, $u$ linear risk tolerance
   (cannot have fully informative equilibrium!)

3. normal shocks, $u$ exponential

This talk: consider case 2 with identical log utilities
Computational algorithm available
Model with log utility

Continuum of agents and 2 dates. Timing:

1. Nature draws distribution $\mu$ of agent types $\theta$
   
   - Type determines endowment & information
   
   - Agents observe type & asset prices, but not $\mu$

   Agents trade assets

   - Assets = claims on $\tau \in \{\tau_1, \tau_2\}$ realized at date 2 ($\tau_1 > \tau_2$)
   
   - Markets incomplete: no assets contingent on aggregate shock $\mu$

2. “Tradable risk factor” $\tau$ realized

   Assets pay off & agents consume
Agent Problem

- Type $\theta$ determines
  - endowment $\omega(\theta) = (\omega_1(\theta), \omega_2(\theta))$, depends on tradable risk $\tau$ only
  - signal $s(\theta)$

- $\hat{\delta}$ = subjective probability of high state $\tau = \tau_1$, given $\theta$, prices

- Agent solves

$$\max_{(c_1, c_2)} \left\{ \hat{\delta} \log c_1 + (1 - \hat{\delta}) \log c_2 \right\}$$

s.t. $pc_1 + (1 - p)c_2 = p\omega_1(\theta) + (1 - p)\omega_2(\theta)$.

where $p =$ price of claim on high state $\tau = \tau_1$. 
Distribution of Types

- Endowments differ across types only if initial exposure to \( \tau \)-risk differs
  - initial exposure to \( \tau \)-risk is 1-1 with \( e_\tau (\theta) := \log \frac{\omega_1(\theta)}{\omega_2(\theta)} \)
  - positive exposure to \( \tau \) \( \iff \) endowment higher if \( \tau \) hi \( (\tau = \tau_1 > \tau_2) \)

- Four different types:
  - Exposure \( e_\tau \) high or low: \( \bar{e} > e \)
  - Signal \( s \) good or bad: \( \bar{s} > s \)

- Type distribution \( \mu \) parameterized by \( \delta, \varepsilon \)
  - \( \delta \) agents get good signal \( \bar{s} \), and \( \delta = \text{Prob} (\tau = \tau_1 | \mu) \)
  - \( \varepsilon \) agents get high exposure \( \bar{e}_\tau \)
  - \( e_\tau, s \) independent across agents given \( \mu \)

- Distribution of aggregate shocks \( \delta, \varepsilon \)
  - \( \varepsilon \in \{\varepsilon^l, \varepsilon^h\} \), with \( \varepsilon^l < \varepsilon^h \) and \( \text{Pr} (\varepsilon = \varepsilon^h) = \eta \)
  - \( \delta \in [0, 1] \) with conditional density \( f (\delta; \varepsilon) \).
Rational Expectations Equilibrium

Price function $\tilde{P}(\delta, \varepsilon)$ & allocation $c(\theta, \delta, \varepsilon)$ s.t. for every $\delta, \varepsilon$

1. Agent’s optimize given price $p = \tilde{P}(\delta, \varepsilon)$ and belief $\hat{\delta}$
   (belief formed by Bayes rule given $\theta, p$ and knowledge of $\tilde{P}$)

2. Markets clear
   $$\sum_{\theta \in \Theta} \mu(\theta) c(\theta, \delta, \varepsilon) = \sum_{\theta \in \Theta} \mu(\theta) \omega(\theta, \delta, \varepsilon) =: \Omega(\varepsilon).$$
Types of equilibria

- $\delta (\delta, \varepsilon) = \text{wealth weighted average of beliefs } \hat{\delta}(\theta)$

- With log utility, equilibrium price can be written as

$$\frac{p}{1-p} = \frac{\delta (\delta, \varepsilon)}{1 - \delta (\delta, \varepsilon)} \frac{\Omega_2(\varepsilon)}{\Omega_1(\varepsilon)}$$

- Full information:
  - $\delta (\delta, \varepsilon) = \delta$,
  - representative agent pricing
  - price decreasing in aggregate exposure, or $\log \frac{\Omega_1(\varepsilon)}{\Omega_2(\varepsilon)}$
    (aggregation of initial exposure works more generally for LRT class)

- Asymmetric information
  - can’t have fully informative equilibrium (agents learn $\delta$ from price)
  - can’t distinguish news $\delta$, aggregate exposure (& hence $\varepsilon$) from price
  - consider nonrevealing equilibria with $\tilde{P}$ continuous & strictly increasing in $\delta$
Proposition. For all $\delta$, $\tilde{P}(\delta, \varepsilon^l) > \tilde{P}(\delta, \varepsilon^h)$ (higher aggregate exposure lowers prices)
Proposition. Individual beliefs \( \hat{\delta}(\theta) \) satisfy

1. *holding the signal* \( s \) *fixed, agents with higher exposure to* \( \tau \)-*risk* \( \bar{e} \)
   *are more optimistic that* \( \tau \) *high*)

2. *holding exposure fixed, agents with the good signal* \( \bar{s} \)
   *are more optimistic that* \( \tau \) *high*)

Implications for trading, compared to full info

- **Part 1** \( \implies \) *less reallocation of exposure* (same signal, different demand)
  with full info: equates exposure across agents

- **Part 2** \( \implies \) *more speculative trading* (same endowment, different demand)
  with full info: zero
Amplified Shifts in Aggregate Exposure

**Proposition.** For every $\delta \in (0, 1)$,

$$\tilde{P}(\delta, \varepsilon^l) > \tilde{P}_{FI}(\delta, \varepsilon^l) > \tilde{P}_{FI}(\delta, \varepsilon^h) > \tilde{P}(\delta, \varepsilon^h).$$

*(price depends more strongly on aggregate exposure under asymmetric information)*

Recall

$$\frac{\tilde{P}(\delta, \varepsilon)}{1 - \tilde{P}(\delta, \varepsilon)} = \frac{\tilde{\delta}(\delta, \varepsilon)}{1 - \tilde{\delta}(\delta, \varepsilon)} \frac{\Omega_2(\varepsilon)}{\Omega_1(\varepsilon)}.$$

- direct effect of exposure same in full- and asymmetric-info cases
- with asymmetric info, $\varepsilon$ also moves average belief $\tilde{\delta}$
- for example, if aggregate exposure increases, price falls as with full info, and
  - belief effect: price could reflect good news, $\tilde{\delta} > \delta \Rightarrow$ price lower
  - wealth effect: wealth of high exposure, optimistic agents falls $\Rightarrow$ price lower
Interpretation: Credit Crisis

• “Market freeze” & “fire sale prices” in frictionless markets?

• Labels
  —agents = banks
  —risk $\tau = \text{aggregate shock that affects payoff on mortgages}$
  —individual exposure = sensitivity of cash flow to $\tau$ is private info

• Initial equilibrium: $\varepsilon = \varepsilon^l$, exposures known
  — prices aggregate dispersed info about $\tau$
  — exposures equated by trading MBS

• Comparative static: $\varepsilon = \varepsilon^h$ and individual exposures private info
  — banks estimate $\varepsilon^h$ from own exposure & price
  — less trading between exposed and non-exposed
  — price low (might reflect bad news about payoffs)
  — price lower than what is predicted by observer
    who knows $\delta$ & assumes “worst-case” exposure $\varepsilon^h$:

$$\tilde{P}(\delta, \varepsilon^h) < \tilde{P}_{FI}(\delta, \varepsilon^h).$$
Wealth Effects and Average Beliefs

- Consider econometrician who
  - sees many realizations from model
  - correctly assumes log preferences
  - estimates joint distribution of $\tau, p$ & consumption from data
  - assumes full info & tests representative agent Euler equation

- Euler equation does not hold: price reflects wealth-weighted average belief!

- If low exposure agents are wealthier, average belief is more pessimistic than estimated distribution → find “risk premium puzzle”

- If high-exposure agents are wealthier, the opposite results obtains.

- In addition, find “time variation in risk aversion”: prices look too optimistic when high, too pessimistic when low
Predictability & Excess Volatility

- Why do low prices predict high excess returns?

- Observed in many markets (e.g. stocks, bonds)

- Econometrician looks at data from the model
  
  Runs regression of excess return on price

- With full or asymmetric info, regression beta $< 0$, $R^2 > 0$

- With asymmetric information, $|\beta|$, $R^2$ strictly larger
  
  - analytical results with exponential utility & normal shocks
  
  - numerical results with log utility; wealth distribution matters

- Exposure shocks always move premia more with asy info
Conclusion

• This paper
  – news about aggregate exposure arrive as dispersed information
  – agents see only initial individual exposures ...
    ... that are informative about aggregate exposure

• Agents disagree in equilibrium
  – agent w/ high initial exposure more optimistic
    1. believe aggregate exposure high
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• Asset pricing implications:
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