What Ties Return Volatilities to Price Valuations and Fundamentals?

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Volatility and the Macroeconomy

After more than 25 years of research on volatility, the central unsolved problem is the relation between the state of the economy and aggregate financial volatility. The number of models that have been developed to predict volatility based on time series information is astronomical, but the models that incorporate economic variables are hard to find. Using various methodologies, links are found but they are generally much weaker than seems reasonable. For example, it is widely recognized that volatility is higher during recessions and following announcements, but these effects turn out to be a small part of measured volatility. [Engle and Rangel RFS 2008]

• E.g. NBER recession dummies explain 5% of variation of future stock and bond return volatility

• In this paper we argue that the relation between volatility and the macroeconomy is in fact very complex and goes beyond the simple boom-bust business cycle variation.
  – We focus on the relation between volatilities and price-valuations;
  – Uncertainty and learning dynamics generate important non-linearities, missed by simple models.
  – Use information in prices to inform about future volatilities and comovement.
Correlation between volatility and P/E ratio strongly time varying, flipping to positive at times

- Defies conventional wisdom and most macro-finance asset pricing models (e.g. habit formation, long run risk)

What economic mechanism generates this time variation?
• Whatever mechanism generates variation in relation between P/E ratio and volatility is likely at play also for Treasury bonds.
The correlation between stocks and bonds is also strongly time varying:

\[
\text{Covariance Stock, 5-Year Bond}
\]

- Is this consistent with the dynamics of the correlation between volatilities and price valuations?
- What is the economic mechanism joining all these facts?
Bond and Stock Comovement

- The correlation between stocks and bonds is also strongly time varying:

![Covariance Stock, 5-Year Bond](image)

- Is this consistent with the dynamics of the correlation between volatilities and price valuations?
- What is the economic mechanism joining all these facts?

In this paper we argue that learning about regular and unusual fundamental states leads to a non-monotonic relation between volatilities and prices, and explains all these facts.
• Inflation:

\[
\frac{dQ_t}{Q_t} = \beta_t \, dt + \sigma_Q \, dW_t,
\]
The Model

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\frac{dQ_t}{Q_t} = \beta_t \, dt + \sigma_Q \, dW_t,
\]

- Earnings:

\[
\frac{dE_t}{E_t} = \theta_t \, dt + \sigma_E \, dW_t,
\]
The Model

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- Signals:
  \[ \frac{dS_t}{S_t} = \theta_t \, dt + \sigma_S \, dW_t \]
The Model

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• Investors’ preferences represented by state price density:
\[
\frac{dM_t}{M_t} = -k_t \, dt - \sigma_M \, dW_t,
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The Model

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\[
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\]

– Constant Prices of Risk: \( \sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3}, \sigma_{M,4}) \)

– Conditional Real Rate: \( k_t = \alpha_0 + \alpha_\theta \theta_t + \alpha_\beta \beta_t \)
The Model

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The Model

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  - Drift vector **Unobserved**: \( \nu_t = (\beta_t, \theta_t, -k_t, \theta_i)' \)
  - Volatility Matrix: \( \Sigma = (\sigma'_Q, \sigma'_E, -\sigma'_M, \sigma'_S)' \).
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  - $\nu_t$ jumps between $n$—states $\{\nu^1, \ldots, \nu^n\}$, with generator matrix $\Lambda$, 
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- **Investors’ Beliefs**: Define $\pi_{it} = \text{Prob} (\nu_t = \nu^i | \mathcal{F}_t)$. Then
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- **Investors’ Beliefs:** Define \( \pi_{it} = \text{Prob}(\nu_t = \nu^i|\mathcal{F}_t) \). Then

\[
d\pi_{it} = \mu_i(\pi_t)dt + \sigma_i(\pi_t)d\tilde{W}_t,
\]

- in which \( d\tilde{W}_t = \Sigma^{-1} \left[ \frac{dX}{X} - E_t \left[ \frac{dX}{X} \right] \right] = \text{normalized (Brownian) news vector, and} \)

\[
\mu_i(\pi_t) = [\pi_t \Lambda]_i,
\]

\[
\sigma_i(\pi_t) = \pi_{it} [\nu^i - \mathcal{V}(\pi_t)]' (\Sigma')^{-1}
\]

\[
\mathcal{V}(\pi_t) = E_t [\nu_t] = \sum_{j=1}^n \pi_{jt} \nu^j.
\]
The Model

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  $$\mu_i(\pi_t) = [\pi_t \Lambda]_i,$$
  $$\sigma_i(\pi_t) = \pi_{it} \left[ \nu^i - \nu(\pi_t) \right]' (\Sigma')^{-1},$$
  $$\nu(\pi_t) = E_t \left[ \nu_t \right] = \sum_{j=1}^{n} \pi_{jt} \nu^j.$$

  $\sigma_i(\pi_t)$ describes the magnitude of revisions from news;
Asset Prices

- P/E Ratio:

\[
\frac{P_t}{E_t}(\pi_t) = \sum_{j=1}^{n} C_j \pi_{jt}; \quad C = A^{-1} \cdot 1_n
\]

\[
A = \text{Diag}(k^1 - \theta^1 + \sigma_M \sigma_E', \cdots, k^n - \theta^n + \sigma_M \sigma_E') - \Lambda.
\]
Asset Prices

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- Define \( C_j \) = Conditional P/E ratios = \( E \left[ \int_t^\infty \frac{M_t E_t}{M_t E_t} d\tau \bigg| \nu_t = \nu^i, F_t \right] \)
**Asset Prices**

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  - Define \(C_j = \) Conditional P/E ratios = \(E \left[ \int_t^{\infty} \frac{M_t}{E_t} E_t d\tau | \nu_t = \nu^i, F_t \right]\)

- **Bond Price:**
  \[
  B_t(\pi_t, \tau) = \sum_{i=1}^{n} \pi_{it} B_i(\tau), \quad B(\tau) = \Omega e^{\omega \tau} \Omega^{-1} 1_n
  \]
  - where \(\Omega\) and \(\omega\) are the matrix of eigenvectors and the eigenvalues of
  \[
  \hat{\Lambda} = \Lambda - \text{Diag}(k^1 + \beta^1 - (\sigma_M + \sigma_Q) \sigma'_Q, \ldots, k^n + \beta^n - (\sigma_M + \sigma_Q) \sigma'_Q)
  \]
\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^{n} C_i \pi_{it} (\nu_i - \varpi(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^{n} C_i \pi_{it}} \]
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\]

- Inflation, earnings, kernel news affect stock vols but with \textbf{endogenous weights.}
\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \sum_{i=1}^n \frac{C_i \pi_{it} (\nu_i - \Psi(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^n C_i \pi_{it}} \]

- Inflation, earnings, kernel news affect stock vols but with endogenous weights.

- These endogenous weights change over time.
  - E.g. Assume \( i, j \) such that \( \pi_i \approx 1 - \pi_j \), Then

\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{(C_i - C_j) \pi_{it} (1 - \pi_{it}) (\nu^i - \nu^j)'(\Sigma')^{-1}}{P/E(\pi_t)} \]
Stock Volatility

\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^n C_i \pi_{it} (\nu_i - \nu(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^n C_i \pi_{it}} \]

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- \( \implies \) If \( C_i \approx C_j \), even if high uncertainty, volatility is low;
Stock Volatility

\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^{n} C_i \pi_{it} (\nu_i - \mathcal{V}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^{n} C_i \pi_{it}} \]

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- \( \Rightarrow \) If \( C_i \approx C_j \), even if high uncertainty, volatility is low;
- \( \Rightarrow \) If \( C_i >> C_j \), even medium-low uncertainty may yield high volatility.
Stock Volatility

\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^{n} C_i \pi_{it} (\nu_i - \mathcal{V}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^{n} C_i \pi_{it}} \]

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- \( \Rightarrow \) If \( C_i \approx C_j \), even if high uncertainty, volatility is low;
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- \( \Rightarrow \) regression of volatility on fundamental uncertainty will have low \( R^2 \) even if model is good.
Stock Volatility

\[ \sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^n C_i \pi_{it} (\nu_i - \mathbf{v}(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^n C_i \pi_{it}} \]

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- \( \Longrightarrow \) regression of volatility on fundamental uncertainty will have low \( R^2 \) even if model is good.

- For bonds, replace \( C_i \) by \( B_i(\tau) \). Do not vary as much.
Estimation Methodology

- Only inflation and earnings (fundamentals) are observed by the econometrician.

- Formulate simulated Likelihood of observing fundamentals at discrete points of time (quarterly) with daily sampled shocks

- Signals and kernel shocks affect this likelihood function

- In SMM procedure use the scores of this SML function and overidentifying moments which are:
  - P/E ratio, 3 Treasury Bond Yields (3m, 1y, 5y)
  - Realized volatilities of Stock, 1-Year and 5-Year T. Bond
  - Realized covariances of Stock, 1-Year, and 5-Year T. Bond
Parameter Estimates

• Model has 23 non-zero parameters

• Inflation Drifts: \((D, L, M, H) = (-0.1\%, 2\%, 4\%, 8.1\%)\), which we call the Deflation, and Low, Medium, and High states of inflation, respectively.

• Earnings Drifts: \((L, H, N) = (-5.0\%, 2.5\%, 6.1\%)\), which we call the Low, High, and New economy growth rates of earnings.
Parameter Estimates

- Model has 23 non-zero parameters

- Inflation Drifts: \((D,LI,MI,HI) = (-0.1\%, 2\%, 4\%, 8.1\%)\), which we call the Deflation, and Low, Medium, and High states of inflation, respectively.

- Earnings Drifts: \((LG,HG,NG) = (-5.0\%, 2.5\%, 6.1\%)\), which we call the Low, High, and New economy growth rates of earnings

- 12 States overall. However only 6 of them have positive probabilities of occurring in sample
### Model Implied Transition Matrix and Asset Prices

#### Implied 1-Year Transition Probability Matrix

<table>
<thead>
<tr>
<th></th>
<th>(D-LG)</th>
<th>(LI-HG)</th>
<th>(MI-LG)</th>
<th>(MI-HG)</th>
<th>(HI-LG)</th>
<th>(LI-NG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D-LG)</td>
<td>0.801</td>
<td>0.138</td>
<td>0.027</td>
<td>0.004</td>
<td>0.029</td>
<td>0.000</td>
</tr>
<tr>
<td>(LI-HG)</td>
<td>0.012</td>
<td>0.951</td>
<td>0.002</td>
<td>0.031</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
<td>(MI-LG)</td>
<td>0.012</td>
<td>0.027</td>
<td>0.656</td>
<td>0.121</td>
<td>0.184</td>
<td>0.000</td>
</tr>
<tr>
<td>(MI-HG)</td>
<td>0.000</td>
<td>0.001</td>
<td>0.080</td>
<td>0.906</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td>(HI-LG)</td>
<td>0.013</td>
<td>0.004</td>
<td>0.184</td>
<td>0.0158</td>
<td>0.783</td>
<td>0.000</td>
</tr>
<tr>
<td>(LI-NG)</td>
<td>0.012</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0131</td>
<td>0.000</td>
<td>0.972</td>
</tr>
</tbody>
</table>

#### Implied 5-Year Transition Probability Matrix

<table>
<thead>
<tr>
<th></th>
<th>(D-LG)</th>
<th>(LI-HG)</th>
<th>(MI-LG)</th>
<th>(MI-HG)</th>
<th>(HI-LG)</th>
<th>(LI-NG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D-LG)</td>
<td>0.345</td>
<td>0.420</td>
<td>0.075</td>
<td>0.066</td>
<td>0.087</td>
<td>0.005</td>
</tr>
<tr>
<td>(LI-HG)</td>
<td>0.037</td>
<td>0.793</td>
<td>0.022</td>
<td>0.120</td>
<td>0.011</td>
<td>0.017</td>
</tr>
<tr>
<td>(MI-LG)</td>
<td>0.032</td>
<td>0.084</td>
<td>0.291</td>
<td>0.289</td>
<td>0.304</td>
<td>0.001</td>
</tr>
<tr>
<td>(MI-HG)</td>
<td>0.009</td>
<td>0.022</td>
<td>0.188</td>
<td>0.675</td>
<td>0.105</td>
<td>0.000</td>
</tr>
<tr>
<td>(HI-LG)</td>
<td>0.037</td>
<td>0.053</td>
<td>0.304</td>
<td>0.160</td>
<td>0.445</td>
<td>0.000</td>
</tr>
<tr>
<td>(LI-NG)</td>
<td>0.039</td>
<td>0.017</td>
<td>0.011</td>
<td>0.054</td>
<td>0.008</td>
<td>0.869</td>
</tr>
</tbody>
</table>

#### Implied Stationary Probabilities, P/E Ratio, and Treasury Yields

<table>
<thead>
<tr>
<th>No.</th>
<th>State</th>
<th>$\pi^*$</th>
<th>$C$</th>
<th>$i_{0.25}$</th>
<th>$i_1$</th>
<th>$i_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(D-LG)</td>
<td>0.038</td>
<td>13.209</td>
<td>0.008</td>
<td>0.012</td>
<td>0.026</td>
</tr>
<tr>
<td>2.</td>
<td>(LI-HG)</td>
<td>0.232</td>
<td>17.537</td>
<td>0.042</td>
<td>0.042</td>
<td>0.044</td>
</tr>
<tr>
<td>3.</td>
<td>(MI-LG)</td>
<td>0.174</td>
<td>9.832</td>
<td>0.078</td>
<td>0.082</td>
<td>0.088</td>
</tr>
<tr>
<td>4.</td>
<td>(MI-HG)</td>
<td>0.352</td>
<td>12.256</td>
<td>0.076</td>
<td>0.076</td>
<td>0.078</td>
</tr>
<tr>
<td>5.</td>
<td>(HI-LG)</td>
<td>0.170</td>
<td>8.545</td>
<td>0.142</td>
<td>0.136</td>
<td>0.115</td>
</tr>
<tr>
<td>6.</td>
<td>(LI-NG)</td>
<td>0.034</td>
<td>34.114</td>
<td>0.042</td>
<td>0.042</td>
<td>0.042</td>
</tr>
</tbody>
</table>
Investors' Beliefs

State 1 (Deflation, Low Growth)

State 2 (Low Inflation, High Growth)

State 3 (Medium Inflation, Low Growth)

State 4 (Medium Inflation, High Growth)

State 5 (High Inflation, Low Growth)

State 6 (Low Inflation, New Economy Growth)
Valuation and Volatilities: Data versus Model Fit

P/E Ratio

Stock Volatility

5-Year Yield

5-Year Bond Volatility
A $V-$Shaped Relation between Stock Volatility and P/E Ratio

- Model suggests a $V-$shape, but also high volatility for medium P/E ratio.
A \( V \)-Shaped Relation between Stock Volatility and P/E Ratio

- Model suggests a \( V \)-shape, but also high volatility for medium P/E ratio.
  - Due to high uncertainty about deflation \( \Rightarrow \) High volatility and medium P/E ratios.
  - Is this prediction true in the data?
A $V-$Shaped Relation between Stock Volatility and P/E Ratio

- Remove all data observations when the probability of deflation is greater than 20%
- Kernel regression:
Stock Return Volatility and P/E Ratio: Data versus Model

Correlation Between Stock Volatility and P/E Ratio

-1 -0.5 0 0.5 1

Why Does the Volatility/PE ratio Correlation May Turn Positive?

- After period of learning, in which uncertainty is low
  - Good news about the economy / inflation,
    - $\ast \implies$ push up P/E ratio.
    - $\ast \implies$ increase uncertainty.
  - Effects amplified by higher persistence of new state.
Why Does the Volatility/PE ratio Correlation May Turn Positive?

• After period of learning, in which uncertainty is low
  – Good news about the economy / inflation,
    * → push up P/E ratio.
    * → increase uncertainty.
  – Effects amplified by higher persistence of new state.

• E.g. the late 1990s
  – Increase in probability to enter a NG state
## Model Implied Transition Matrix and Asset Prices

### Implied 1-Year Transition Probability Matrix

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<tr>
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<td>0.027</td>
<td>0.004</td>
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</tr>
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<td>0.951</td>
<td>0.002</td>
<td>0.031</td>
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</tr>
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<td>(MI-LG)</td>
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<td>0.121</td>
<td>0.184</td>
<td>0.000</td>
</tr>
<tr>
<td>(MI-HG)</td>
<td>0.000</td>
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<td>0.080</td>
<td>0.906</td>
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</tr>
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<td>0.0158</td>
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<td>0.000</td>
</tr>
<tr>
<td>(LI-NG)</td>
<td>0.012</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0131</td>
<td>0.000</td>
<td>0.972</td>
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</table>

### Implied 5-Year Transition Probability Matrix

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<tr>
<th></th>
<th>(D-LG)</th>
<th>(LI-HG)</th>
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<td>0.345</td>
<td>0.420</td>
<td>0.075</td>
<td>0.066</td>
<td>0.087</td>
<td>0.005</td>
</tr>
<tr>
<td>(LI-HG)</td>
<td>0.037</td>
<td>0.793</td>
<td>0.022</td>
<td>0.120</td>
<td>0.011</td>
<td>0.017</td>
</tr>
<tr>
<td>(MI-LG)</td>
<td>0.032</td>
<td>0.084</td>
<td>0.291</td>
<td>0.289</td>
<td>0.304</td>
<td>0.001</td>
</tr>
<tr>
<td>(MI-HG)</td>
<td>0.009</td>
<td>0.022</td>
<td>0.188</td>
<td>0.675</td>
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<td>(HI-LG)</td>
<td>0.037</td>
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<td>0.869</td>
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### Implied Stationary Probabilities, P/E Ratio, and Treasury Yields

<table>
<thead>
<tr>
<th>No.</th>
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<th>$\pi^*$</th>
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A \( V \)–Shape Relation between Bond Volatility and Yields

**Bond Yield and Volatility (Data)**

**Bond Yield and Volatility (Model)**
Bond Return Volatility and Yield: Data versus Model
Why Does the Bond Volatility/Yield Correlation Turn Negative?

- In the 1970s: Increase in probability of being in a persistent high inflation state
  - $* \implies \text{High yields.}$
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\implies \textit{Same economic mechanism (learning) accounts for diametrically opposite relation between yields volatility in different historical periods.}
When Does Stock-Bond Correlation Become Negative?

- The deflation, low growth state (D, LG) has low $P/E$ and low yield $i_t$

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- $\Rightarrow$ An increase in probability to end up in a deflation / low growth period decrease $P/E$ and increase bond prices $B(t, \tau)$.

- Because agents are learning, these variation in the probability is stochastic, increasing the negative covariance between $P/E$ and $B(t, \tau)$

- According to the model, such probabilities increased substantially at the beginning of 2000 and again at the end of the decade.
• As for likelihood functions, simulate shocks to state variables at daily frequency and aggregate quadratic variation up to forecast horizon

\[
V^M(T_1, T_2, t) = \frac{1}{S} \sum_{j=1}^{M} \sigma_A(\pi^{(s)}_{T_1+jh}) \sigma_A(\pi^{(s)}_{T_1+jh})' h,
\]

• Report Results:

\[
\text{Vol}(t + 1, t + k) = \beta_0 + \beta_1 \text{Vol}(t - k + 1, t) + \beta_2 V^*(t + 1, t + k; t) + \beta_3 X(t) + \varepsilon(t + 1, t + k),
\]
## 1-Year Ahead Forecasts: Stock Volatility

| No. | Sample       | Const. | Vol \((t - 4, t)\) | \(V^*(t + 1, t + 4)\) | NBER\((t)\) | \(R_{S}^{(-)}(t)\) | \(r(t)\) | Term\((t)\) | \(\sigma_I(t)\) | \(\sigma_E(t)\) | \(\sigma_{PF}^i(t)\) | \(\sigma_{PF}^e(t)\) | \(\bar{R}^2\) |
|-----|--------------|--------|-------------------|-------------------|----------|----------------|--------|----------|---------------|---------------|----------------|----------------|---------|--------|
| 1   | 1962-2008    | 0.082  | 0.429            |                   |          |               |        |          |               |               |               |               | 0.153   |        |
|     |              |        |                   | \(\text{[4.072]}^\ast\) | \(\text{[2.873]}^\ast\) |          |          |          |               |               |               |               |         |        |
| 2   | 1962-2008    | -0.008 | 1.393            |                   |          |               |        |          |               |               |               |               | 0.416   |        |
|     |              |        |                   | \(\text{[-3.373]}^\ast\) | \(\text{[8.877]}^\ast\) |          |          |          |               |               |               |               |         |        |
| 3   | 1962-2008 (EC)| -0.008 | 1.241            |                   |          |               |        |          |               |               |               |               | 0.542   |        |
|     |              |        |                   | \(\text{[-3.319]}^\ast\) | \(\text{[8.821]}^\ast\) |          |          |          |               |               |               |               |         |        |
| 4   | 1962-2008    | -0.007 | 0.123            | 1.259             |          |               |        |          |               |               |               |               | 0.419   |        |
|     |              |        |                   | \(\text{[-3.087]}^\ast\) | \(\text{[1.986]}\) |          |          |          |               |               |               |               |         |        |
| 5   | 1962-2008    | 0.171  |                   | -0.037            | -0.459   | 0.195         | -0.447 | -0.989   | -0.16         |               |               |               | 0.057   |        |
|     |              |        |                   | \(\text{[4.511]}^\ast\) | \(\text{[-2.060]}\) | \(\text{[-1.621]}\) | \(\text{[0.609]}\) | \(\text{[-0.442]}\) | \(\text{[-0.639]}\) | \(\text{[-0.105]}\) |          |          |         |        |
| 6   | 1962-2008    | -0.004 | 1.435            | -0.016            | -0.359   | -0.183        | -1.235 | -0.428   | -0.274         |               |               |               | 0.45    |        |
|     |              |        |                   | \(\text{[-1.477]}\) | \(\text{[9.161]}^\ast\) | \(\text{[-1.170]}\) | \(\text{[-1.735]}\) | \(\text{[-0.736]}\) | \(\text{[-1.578]}\) | \(\text{[-0.364]}\) | \(\text{[-1.185]}\) |          |          |         |        |
| 7   | 1962-2008    | -0.039 | 0.063            | 1.407             | -0.017   | -0.339        | -0.175 | -1.281   | -0.525         | -0.293        |               |               | 0.447   |        |
|     |              |        |                   | \(\text{[-1.408]}\) | \(\text{[0.782]}\) |          |          |          |               |               |               |               |         |        |
| 8   | 1971-2008    | 0.166  |                   |                   |          |               |        |          |               |               |               |               | 0.069   | -0.236 |
|     |              |        |                   | \(\text{[7.816]}^\ast\) | \(\text{[8.626]}^\ast\) |          |          |          |               |               |               |               |         |        |
| 9   | 1971-2008    | -0.12  | 1.461            |                   |          |               |        |          |               |               |               |               | 2.499   | -0.075 |
|     |              |        |                   | \(\text{[-3.206]}^\ast\) | \(\text{[7.760]}^\ast\) |          |          |          |               |               |               |               |         |        |
| 10  | 1971-2008    | -0.114 | 0.139            | 1.417             |          |               |        |          |               |               |               |               | 2.425   | -0.139 |
|     |              |        |                   | \(\text{[-3.147]}^\ast\) | \(\text{[2.483]}\) |          |          |          |               |               |               |               |         |        |

\(\ast\) Statistically significant at the 5% level.
## 1-Year Ahead Forecasts: 5-Year Bond Volatility

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<th>Const.</th>
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<th>$\text{Vol}^*(t+1, t+4)$</th>
<th>NBER$(t)$</th>
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<th>$\sigma_I(t)$</th>
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## 1-Year Ahead Forecasts: Covariance Stocks and 5-Year Bond

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Why is Model Successful?

- Model uses information aggregation property of asset prices to extract investors’ beliefs.

- Equilibrium asset pricing model gives time varying weights to inflation and earnings uncertainties. These have low $R^2$ on their own. Engle and Rangel (2008) find differing impact of fundamentals on returns in spline-GARCH model. In particular model figures out:

  - For stocks, usually negative relationship between returns and volatility reversed occasionally, such as the late 1990s due to uncertainty of new economy,

  - Effect of inflation (rates) on volatility changed sign in the current decade

- Simple linear regressions on macroeconomic variables miss these dynamic effects.
Weights Given to Earnings and Inflation Shocks

Stock Volatility Weights

5-Year Bond Volatility Weights
Conclusions

• What ties return volatilities to price valuations and fundamentals?
  – Investors’ learning about the current earnings growth/inflation state.
  – Learning dynamics about fundamentals provides a unified framework that contemporaneously explains
    1. the major variation in the volatility of stocks and bonds in the last 50 years;
    2. their non-monotonic, \( V \)-shaped relation with stocks and bonds valuation ratios
    3. the time variation in their comovement.
  – The learning mechanism is also supported by the model’s success in predicting future volatilities and covariances

• We propose a new, equilibrium-based, structural form methodology for understanding the fluctuations and predictability of volatilities and covariances of asset returns.
  – Our estimation methodology differentiates the information set of the econometrician from the one of market participants;
  – Model-based, closed-form formulas for prices and second moments enables us to extract the investors’ signals about fundamentals.
    * Large improvement in volatility forecasting availability, especially for stocks.
Model versus Lags Across Forecasting Horizons

Stock Volatility Forecast

5-Year Bond Volatility Forecast

Covariance Stock and 5-Year Bond Volatility Forecast
Survey and Model Uncertainty

Inflation Uncertainty

Earnings Uncertainty
Uncertainty, PE ratio, and Stock Return Volatility

Inflation Uncertainty and Data P/E Ratio

Inflation Uncertainty and Model P/E Ratio

Earnings Uncertainty and Data P/E Ratio

Earnings Uncertainty and Model P/E Ratio

Stock Volatility and P/E Ratios (Data)

Stock Volatility and P/E Ratios (Model)
Stock Return Volatility and P/D Ratio: Long Sample

Volatility versus P/D Ratio: 1885Q3 – 2009Q2