Technical Note –
Using the CCA Framework to Estimate Potential Losses and Implicit Government Guarantees to U.S. Banks

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This note uses the contingent claims analysis (CCA) framework to estimate potential bank losses (in the event of distress) and the magnitude of implicit government guarantees for the top 17 U.S. commercial banks, all of which have been stress-tested in the context of the SCAP. In addition, it presents potential losses and quantifies the individual banks’ contributions to government contingent liabilities and systemic risk in the event of a systemic bank distress (“tail risks”).

Summary

The following note and charts shows some of the results from applying the methodology presented in Gray and Jobst (2009)¹ to the major US banks.

Estimation method

- **individual CCA**: BSM framework with implied asset volatility derived from equity options and 5-year CDS spreads as basis for the calculation of the alpha-value \((1-(\text{Put option(equity)}/(\text{Put option (CDS)}))\).

- **systemic CCA**: the multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV), estimated via the Linear Ratio of Spacings (LRS) method, and a non-parametric identification of the linear and non-linear time-varying dependence structure. The contribution to systemic (joint tail risk) is derived from the relative weight of the univariate marginal distribution at the specified percentile of the multivariate density.

A. The total potential losses in the event of banking system distress and implicit government guarantees

Methodology

In the CCA framework, the total market value of assets, $A$, at any time, $t$, is equal to the sum of its equity market value, $E$, and its risky debt, $D$, maturing at time $T$.

\[
A(t) = E(t) + D(t)
\]

Asset value is stochastic and may fall below the value of outstanding liabilities. We assume that default occurs when the asset value is insufficient to meet the amount of debt owed to creditors at maturity, i.e., $A$ falls below a given “distress barrier”, $B$, defined as present value of promised payments on debt discounted at the risk free rate. This capital-structure-based evaluation of contingent claims on firm performance implies that a firm defaults if its asset value. Equity value is the value of an implicit call option on the assets, with an exercise price equal to default barrier, $B$. The value of risky debt is equal to default-free debt minus the present value of expected loss due to default. The expected potential loss due to default can be calculated as the value of a put option on the assets with an exercise price equal to $B$.

\[
Risky\ Debt = Default-free\ Debt - Potential\ loss\ due\ to\ default
\]

\[
D(t) = Be^{-(T-t)} - P_e(t)
\]

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2 MKMV defines this barrier equal to total short-term debt plus one-half of long-term debt.

3 We identify contingent liabilities based on the standard Black-Scholes-Merton (BSM) framework of capital structure-based option pricing theory (OPT) (Black and Scholes, 1973; Merton, 1973 and 1974). According to Merton’s reduced-form model, a firm’s outstanding liabilities constitute a bankruptcy level (“default threshold”). Owners of corporate equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off. They also have the option to default if their firm’s asset value (“reference asset”) falls below the present value of the notional amount of outstanding debt (“strike price”) owed to bondholders at maturity. So, corporate bond holders effectively write a European put option to equity owners, who hold a residual claim on the firm’s asset value in non-default states of the world. Bond holders receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky corporate debt (and bearing the potential loss) due to the limited liability of equity owners. The value of the put option is determined by the duration of debt claim, the leverage of the firm, and asset-price volatility.
**Equity value** can be computed as the value of a call option:

\[
E(t) = A(t)N(d_1) - Be^{-rT}N(d_2)
\]

\[
d_1 = \frac{\ln\left(\frac{A}{B}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}
\]

\(r\) is the risk-free rate; \(\sigma\) is the asset return volatility; \(N(d)\) is the cumulative probability of the standard normal density function below \(d\).

**The present value of expected losses associated with outstanding liabilities can be valued as an implicit put option.** This implicit put option is calculated with the default threshold as strike price on the current asset value of each institution. Thus, the present value of expected loss can be computed as:

\[
P_E(t) = Be^{-rT}N(-d_2) - A(t)N(-d_1)
\]

\[
d_1 = d_2 + \sigma\sqrt{T} \quad \text{and} \quad d_2 = \frac{\ln\left(\frac{A}{B}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}
\]

Where \(r\) is the risk-free rate; \(\sigma\) is the asset return volatility; \(N(d)\) is the cumulative probability of the standard normal density function below \(d\). Note that \(N(-d_2)\) is the “risk-neutral” default probability. The calibration of the model uses the value of equity, the volatility of equity, the distress barrier as inputs into two equations in order to calculate the implied asset value and implied asset volatility.\(^4\)

**What fraction of total potential losses is viewed by the markets as being implicitly guaranteed by the government?** Government financial guarantees benefits the bank’s debt holders, but does not affect equity values in a major way. Then the bank’s CDS spreads should capture only the expected loss retained by the bank after accounting for the implicit government guarantee. Hence, the scope of the government guarantee is defined as difference between the total expected loss (the value of a put option \(P_E(t)\) derived from the bank’s

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equity price) and the value of an implicit put option \( P_{\text{CDS}}(t) \) derived from the bank’s CDS spread, where\(^5\)

\[
P_{\text{CDS}}(t) = \left(1 - \exp\left(-\frac{\text{(basis points}}{10000} T \right) \right) e^{-\sigma T},
\]

We denote \( \alpha \) as the fraction of total potential loss covered by the implicit government guarantee. In other words \( \alpha \ t \ P_{E} \ t \) is the fraction of bank default risk covered by the government and \([1-\alpha \ t \] P_{E} \ t \) is the risk retained by the banks and reflected in the CDS prices. Using these relationship is can be seen that:

\[
\alpha \ t = 1 - \frac{P_{\text{CDS}}(t)}{P_{E} \ t}.
\]

**B. Systemic bank distress and associated potential losses**

**Methodology**

As a logical extension to the individual bank analysis, we evaluate the magnitude of systemic risk jointly posed by financial institutions based on a measure for multivariate dependence. In order to assess an institution’s contribution to systemic risk (and the underlying joint default risk), it is necessary to move beyond “singular CCA” and consider the correlation (or more correctly, dependence) structure between the financial institutions.

However, the estimation of systemic risk through correlation becomes exceedingly unreliable in the presence of “fat tails”. If default risk become more commonplace than suggested by the standard assumption of normality, i.e., there is a higher probability of large losses (“negative skewness”) and/or extreme outcomes (“excess kurtosis”), and/or default risk increases in a non-linear way as the market value of assets declines, the concept of conventional correlation is misleading. This is especially true in times of stress, when higher volatility inflates conventional correlation measures automatically. Thus, accounting for both linear and non-linear dependence between higher moments of changes in asset values can deliver important insights about the joint tail risk of multiple entities.

\(^5\) An adjustment factor \( RFV/RMV \) needs to be included, which is the ratio between recovery at face value (RFV) and recovery at market value (RMV), which decreases (increases) the CDS spread in the case of a positive (negative) basis with the corresponding bond spread. We approximate the change in recovery value based on the stochastic difference between the fair value CDS spread and the fair value option adjusted spread reported by MKMV.
This requires moving beyond “singular CCA” by accounting for the dependence structure of individual bank balance sheets and associated contingent claims. In this note, we generate the multivariate density of each bank’s contingent liabilities from the univariate marginals of the market-implied debt guarantees and their non-parametrically defined time-varying dependence. As opposed to the traditional (pairwise) correlation-based approach, this method of measuring “tail dependence” is better suited to analyzing extreme linkages of multiple entities, because it links the univariate marginal distributions in a way that formally captures joint asymptotic tail behavior.

As an integral part of this approach, the marginal distributions fall within the domain of Generalized Extreme Value Distribution (GEV), estimated via the Linear Ratio of Spacings (LRS) method, in order to quantify the possibility of common extreme shocks (Coles et al., 1999; Poon et al., 2003; Stephenson, 2003; Jobst, 2007) as the multivariate density of risk capital at different levels of statistical confidence, such as “fat-tails” using “multivariate extreme value dependence”. This richer framework also allows to quantify the contribution of specific institutions to systemic risk, how this systemic risk affects the government’s contingent liabilities, and how policy measures may influence the size and allocation of this systemic risk (Jobst and Gray, 2009).

As opposed to the traditional (pairwise) correlation-based approach, this method of measuring “tail dependence” is better suited to analyzing extreme linkages of multiple entities. If large risk exposures become more commonplace than suggested by the standard assumption of normality, i.e., there is a higher probability of large losses (“negative skewness”) and/or extreme outcomes (“excess kurtosis”), and default risk increases in a non-linear way as the market value of assets declines, the concept of conventional correlation gives misleading information about intertemporal association. This is especially true in times of stress, when higher volatility inflates conventional correlation measures on techncial grounds. Thus, accounting for both linear and non-linear dependence between higher moments of changes in asset values can deliver important insights about the joint tail risk of multiple entities when estimating systemic risk based on correlation and volatility alone becomes exceedingly unreliable.

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Technical description of systemic banking risk measure

As an alternative to the extreme value copula $C R \xi(y_1,...,y_n)$ that links the $i$th univariate marginal distribution functions $F_i$, describing the dependence between the components of a random vector $X = x^1,...,x^n$, extend the limiting law of normalized maxima

$$\lim_{m \to \infty} P \frac{X_{nm} - b_m}{a_m} / \frac{1}{a_m} = \frac{1}{a_m}$$

as the specification of individual asymptotic tail behavior to the multivariate case $G \chi = \exp - \sum_{i=1}^{n} \eta_i A_y y_i / \sum_{i=1}^{n} \eta_i y_i$. The $i$th univariate marginal distribution $y_i = y_i / x_i = 1 + \xi_i / \mu_i / \sigma_i$, (for $i = m$) converges to generalized extreme value (GEV), with $1 + \xi_i / \mu_i / \sigma_i > 0$, scale parameter $\sigma_i > 0$, location parameter $\mu_i$ and shape parameter $\xi_i$. The dependence function $A$, characterizes the dependence structure of $G \chi$ and is defined on simplex $S_y = \omega \in R^n: \sum_{i=1}^{n} \omega_i = 1$ with $0 \leq \omega$, $\omega = 0$, $\omega = 1$, and $A = 0$, $A = 1$. It is derived non-parametrically via the multivariate logistic model and represents a convex function on $0, 1$ with $A = 0, A = 1$. are obtained under complete dependence and mutual independence respectively. The choice of the empirical distribution function of underlying data to model the marginal distributions at any given point in time avoids problems associated with using specific parameters that may or may not fit these distributions well – a problem potentially exacerbated during stressful periods.\footnote{This approach is distinct from previous studies of joint patterns of extreme behavior. For instance, Longin (2000) derives point estimates of the extreme marginal distribution of a portfolio of assets based on the correlation between the series of individual maxima and minima. However, in absence of a principled standard definition of order in a vectorial space for $n$-dimension asset vectors, the simple aggregation of marginal extremes does not necessarily concur with the joint distribution of the extreme marginal distributions. See also Embrechts et al. (2001) as regards this issue. Embrechts, Paul, Lindskog, F. and A. McNeil (2001), “Modelling Dependence with Copulas and Applications to Risk Management,” Preprint, ETH Zurich. Longin, F. (2000), “From Value at Risk to Stress Testing: the Extreme Value Approach,” Journal of Banking and Finance, Vol. 24, 1097-1130.}
C. Results

- The individual charts of each of the 17 banks shows the measure of alpha (rising to between 0.8 to 0.95 for most banks in early 2009), indicating market perceptions are that the government is taking most of the credit risk. Figure 1 below shows the example of BoA.

Figure 1

Bank of America: Contingent Liabilities from Debt Guarantees

- Banking sector risks were the highest over the first quarter of 2009. A simple summation of the contingent liabilities (alpha times put equity) peaked at US$ 1 trillion in March 2009. However, the simple summation of individual CCA estimates of each bank does not capture intertemporal changes in the dependence structure between U.S. banks. Once the dependence structure is included, the 50% of the multivariate distribution is lower than the simple summation. Key results are shown in Figure 2, gray area is simple summation while the dashed green line is the 50%

1/ The alpha-value is defined as 1-(Put Option on CDS/Put Option on Equity).
percentile, the difference being the “diversification effect.” The 99th percentile spiked in April (over 2 trn) and has since fallen back.

Figure 2
Contribution of each bank to the 99th percentile is shown in Figure 3. Citigroup was the largest contributor to systemic risk shocks (at the 99th percentile) in the U.S. banking sector in March 2009, with Morgan Stanley being a large contributor in late 2008.

Figure 3

United States: Financial Sector - Multivariate Density of Contingent Liabilities (“Systemic CCA”) with conditional individual bank contribution at the 99th percentile 1/

Figure 4

United States: Financial Sector - Multivariate Density of Contingent Liabilities (“Systemic CCA”) with conditional individual bank contribution at the 99th percentile 1/

- Figure 4 shows that the banks that “failed the US stress tests” contributed the most to the government’s contingent liabilities.

Sample period: 01/02/2008-10/19/2009 (482 obs.) of individual contingent liabilities of sample banks. Source: IMF staff estimates. 1/ The multivariate density is generated from univariate marginals that conform to the Generalized Extreme Value Distribution (GEV), estimated via the Linear Ratio of Spacings (LRS) method over an estimation window of 60 working days, and a non-parametric identification of the time-varying dependence structure.
Percentage contributions are in Table 1

Table 1

**United States: Contingent Liabilities - Systemic risk contribution 1/**

*(In percent, average contribution (since starting date) to tail risk of total contingent liabilities at the 99th percentile level, based on univariate marginal extreme value and the unconditional marginal impact on dependence structure)*

<table>
<thead>
<tr>
<th>Banks with SCAP-identified capital need</th>
<th>since Sept. 14,</th>
<th>since Jan. 1, 2009</th>
<th>since Apr. 1, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Citigroup</td>
<td>9.57</td>
<td>11.33</td>
<td>7.54</td>
</tr>
<tr>
<td>Bank of America</td>
<td>5.73</td>
<td>6.16</td>
<td>7.76</td>
</tr>
<tr>
<td>Fifth Third Bancorp.</td>
<td>3.51</td>
<td>3.48</td>
<td>3.68</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>11.03</td>
<td>11.15</td>
<td>7.55</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>3.93</td>
<td>4.48</td>
<td>4.20</td>
</tr>
<tr>
<td>Suntrust</td>
<td>3.86</td>
<td>3.75</td>
<td>3.72</td>
</tr>
<tr>
<td>Regions Financial Corp.</td>
<td>4.92</td>
<td>4.44</td>
<td>4.61</td>
</tr>
<tr>
<td>PNC Bank</td>
<td>8.47</td>
<td>8.00</td>
<td>9.01</td>
</tr>
<tr>
<td>Key Corp.</td>
<td>4.74</td>
<td>4.33</td>
<td>4.41</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td><strong>55.76</strong></td>
<td><strong>57.12</strong></td>
<td><strong>52.49</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Banks without SCAP-identified capital need</th>
<th>since Sept. 14,</th>
<th>since Jan. 1, 2009</th>
<th>since Apr. 1, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Street Corp.</td>
<td>5.47</td>
<td>4.83</td>
<td>5.81</td>
</tr>
<tr>
<td>American Express Bank</td>
<td>3.97</td>
<td>3.77</td>
<td>3.82</td>
</tr>
<tr>
<td>Capital One Financial Corp.</td>
<td>5.61</td>
<td>6.42</td>
<td>7.62</td>
</tr>
<tr>
<td>US Bancorp.</td>
<td>4.91</td>
<td>5.47</td>
<td>6.27</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>7.40</td>
<td>8.25</td>
<td>8.85</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>6.45</td>
<td>4.32</td>
<td>4.09</td>
</tr>
<tr>
<td>Bank of New York Mellon</td>
<td>6.48</td>
<td>5.89</td>
<td>7.05</td>
</tr>
<tr>
<td>BB&amp;T</td>
<td>3.94</td>
<td>3.94</td>
<td>4.01</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td><strong>44.24</strong></td>
<td><strong>42.88</strong></td>
<td><strong>47.51</strong></td>
</tr>
</tbody>
</table>

1/ Each bank’s percentage share is based on its time-varying contribution to the multivariate density of total contingent liabilities from the banking sector at the 99th percentile. The multivariate probability distribution is generated from univariate marginals and a time-varying dependence structure based on generalized extreme value.