OBJECTIVES OF THE COURSE

Courses offered in Statistics can be broadly categorized as methodological with emphasis on applications or theory. The course Introduction to the Theory of Probability is a THEORY course. It is designed to present the major ideas of probability distribution theory assuming a mathematical background consisting of two semesters of calculus (and ideally one semester of linear algebra). The course should serve as background preparation for many courses including applied statistics, stochastic processes, mathematical finance, and actuarial science. In particular, the course will cover concepts needed for 50% of the first actuarial exam: Mathematical Foundations of Actuarial Science (the remaining 50% of the exam is Calculus). It is the prerequisite course for Stochastic Processes I, Statistics and Regression Analysis, and is a required course in the Stern Ph.D. Core. The course emphasis will be the formulation of probability models in order to describe underlying generating processes. All concepts will be illustrated with examples and presented to the students in the form of problems to be solved. Required material will be presented in class and students will be urged to participate in the development of these concepts.

Course Pre-Requisites

V63.0121 Calculus I and V63.0122 Calculus II
A. REQUIRED TEXT

You may select one of the three texts for the course. The homework problems will be from Hogg, McKean, and Craig 7th edition.

- **(R) Rice, J.A.** (1995) "Mathematical Statistics and Data Analysis", second edition, California: Duxbury Press. When appropriate, topics are motivated and illustrated with practical problems. This is the least rigorous of the three books.

B. GRADING

Problem assignments will be given at the end of each session and will be submitted the following week for review. All assignments **MUST** be completed. Each week two problems from the homework will be graded. The final grade is computed as:

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom Participation</td>
<td>10%</td>
</tr>
<tr>
<td>Graded Homework Problems</td>
<td>45%</td>
</tr>
<tr>
<td>Final Examination</td>
<td>45%</td>
</tr>
</tbody>
</table>

C. READING AND ASSIGNMENT LIST

The discussion topics will be indicated for each text. Homework problems will be assigned from (HC).

**COURSE SYLLABUS:**

**READINGS AND LIST OF HOMEWORK PROBLEMS**

1. General Concepts of Probability

(HC) Chapter 1, Sections 1-4

(HT) Chapters 1, 2

(R) Chapter 1

Homework set 1 in notes (attached)

Page 28: 5, 10, 30, 31
2. Random Variables, Probability Functions

And Distribution Functions

<table>
<thead>
<tr>
<th>Section</th>
<th>References</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(HC) Chapter 1, Sections 5-7</td>
<td></td>
<td>38: 1, 2, 4, 5, 8</td>
</tr>
<tr>
<td>(HT) Chapter 3, Sections 2-3</td>
<td></td>
<td>43: 1, 2, 3</td>
</tr>
<tr>
<td>(HT) Chapter 4, Pgs. 180-184</td>
<td></td>
<td>50: 3, 7, 10, 20, 21, 22, 23</td>
</tr>
<tr>
<td>(R) Chapter 2, Pgs. 33-35 and 46-48</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Mathematical Expectation

<table>
<thead>
<tr>
<th>Section</th>
<th>References</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(HC) Chapter 1, Sections 8-10</td>
<td></td>
<td>56: 2, 3, 7</td>
</tr>
<tr>
<td>(HT) Chapter 3, Sections 2-3</td>
<td></td>
<td>64: 1-8, 17, 18</td>
</tr>
<tr>
<td>(HT) Chapter 4, Pgs. 184-188</td>
<td></td>
<td>72: 2, 3</td>
</tr>
<tr>
<td>(R) Chapter 4, Sections 1-2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Multivariate Distributions

<table>
<thead>
<tr>
<th>Section</th>
<th>References</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>(HC) Chapter 2</td>
<td></td>
<td>83: 6, 7, 8</td>
</tr>
<tr>
<td>(HT) Chapter 11, Sections 1-3</td>
<td></td>
<td>92: 1, 2.</td>
</tr>
<tr>
<td>(R) Chapter 3, Sections 1-6</td>
<td></td>
<td>100: 1, 2, 4, 9, 10</td>
</tr>
<tr>
<td>(R) Chapter 4, Sections 3-4</td>
<td></td>
<td>108: 1, 3, 7</td>
</tr>
<tr>
<td>(R) Chapter 4, Sections 3-4</td>
<td></td>
<td>116: 2, 4, 8, 9</td>
</tr>
<tr>
<td>(R) Chapter 11, Sections 1-3</td>
<td></td>
<td>125: 3, 9</td>
</tr>
</tbody>
</table>
5. Special Distributions

(HC) Chapter 3, Sections 1-2 (Discrete)

(HC) Chapter 3, Section 3 and 6 (Normal Related)

(HC) Chapter 3, Section 4 (Normal 4.1 only)

(HC) Chapter 3, Section 5 (Multivariate Normal)

(HT) Chapter 3, Sections 4-7

(HT) Chapter 4, Sections 2-4

(HT) Chapter 11, Sections 4-5

(R) Chapter 2, Sections 1-2

Page 146: 1-6, 18, 19
Page 154: 1-5, 8
Page 164: 1-5, 18-21
Page 175: 2-6, 9-14
Page 186: 2, 5, 10

Extra Problems in Notes (pages 47 and 48) plus:

1. Let $X_1$, $X_2$, and $X_3$ be random variables with equal variances but with correlation coefficients $r_{12} = 0.3$, $r_{13} = 0.5$ and $r_{23} = 0.2$. Find the correlation coefficient of the linear functions $Y = X_1 + X_2$ and $Z = X_2 + X_3$.

2. Find the variance of the sum of 10 random variables if each has variance 5 and if each pair has correlation coefficient 0.5
6. Distribution of Functions of Random Variables

(HC) Chapter 4

(HT) Chapter 3, Section 8

(HT) Chapter 4, Section 5

(HT) Chapter 5, Sections 1-3

(R) Chapter 2, Sections 3-4

(R) Chapter 3, Sections 7-8

(R) Chapter 4, Sections 5-6

(R) Chapter 6

(R) Chapter 11, Section 6

Page 195: 8 - 11

Page 201: 4, 5, 9, 16

Let $X_1$ and $X_2$ be independent random variables. Let $X_1$ and $Y = X_1 + X_2$ have chi-square distributions with $r_1$ and $r$ degrees of freedom, respectively. Here $r_1 < r$. Show that $X_2$ has a chi-square distribution with $r - r_1$ degrees of freedom.

Hint: Write $M(t) = E(e^{t(x_1+x_2)})$ and make use of the independence of $X_i$.

7. Limit Theorems

(HC) Chapter 5

(HT) Chapter 5, Sections 4-6

(R) Chapter 5

Page 236: 5, 6

Page 313: 1 - 3, 5, 7, 9

8. Point Estimation

(HC) Chapter 6, Section 1

(HT) Chapter 6, Section 1

(R) Chapter 8, Section 1, up to Pg. 258

Page 326: 2, 6, 8