

# A Theory of Pyramidal Ownership and Family Business Groups\*

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## **Abstract**

We provide a rationale for the use of pyramidal ownership (the control of a firm through a chain of ownership relations) that departs from the traditional argument of separating ownership and control. With a pyramidal structure a family uses a firm it already controls to set up a new firm. This allows the family to access the entire stock of retained earnings of the firm it controls and to share the security benefits of the new firm with the other existing shareholders of the original firm. Therefore, pyramids are more attractive when internal funds are important (e.g., due to poorly functioning capital markets) and when the security benefits of the new firm are low; conditions that we show hold in an environment with poor investor protection. We also analyze the creation of family business groups (a collection of multiple firms under the control of a single family). Business groups flourish when external markets are poorly developed because, in such cases, internal resources from the existing firms provide the family with a financing advantage vis-a-vis other competing entrepreneurs. Thus, the model predicts that in countries with poor investor protection family business groups should be common and they should be organized as pyramids. Because our model departs from the traditional argument for pyramids as a device to separate ownership and control, it can differentiate between pyramids and dual class shares even in situations in which the same deviation from one share-one vote can be achieved with either method. Unlike the traditional argument, our model is consistent with recent empirical evidence that some pyramidal firms are associated with small deviations between ownership and control. We also argue that pyramids can be an efficient organizational structure for the family if the availability of internal funds is sufficiently important, even though pyramids are associated with high levels of cash flow diversion. Other predictions of the model are consistent with systematic and anecdotal evidence on pyramidal business groups.

Key words: pyramids, business groups, family firms, investor protection, ownership structure, dual-class shares.

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# 1 Introduction

Many firms in the world have a controlling shareholder, usually a family or the State (La Porta, Lopez-de-Silanes and Shleifer, 1999). In several countries, single individuals or families control a large number of firms; an organization typically referred to as a *family business group*.<sup>1</sup> The controlling family often organizes the ownership of the group member firms in a *pyramidal structure*.<sup>2</sup> In such a structure the family achieves control of the constituent firms by a chain of ownership relations: the family directly controls a firm, which in turn controls another firm, which might itself control other firms, and so forth.

Despite the ubiquity of pyramidal business groups, there is surprisingly no formal theory that explains their existence. There are, however, some informal arguments. The traditional one is that a pyramid allows a family to achieve control of a firm with a small cash flow stake.<sup>3</sup> For instance, a family that directly owns 50% of a firm, which in turn owns 50% of a different firm, achieves control of the latter firm with an *ultimate* cash flow stake of only 25%. Securing control through such arrangements is beneficial for the family when private benefits of control are large. Indeed, there is evidence that the controlling family extracts considerable private benefits from the pyramidal group member firms. For instance, Bertrand, Mehta and Mullanaithan (2002) show that the controlling family benefits by diverting cash flows from firms in which its stake is small to firms in which its stake is larger.

Nevertheless, a more detailed examination of the available data on the characteristics of pyramidal ownership structures reveals some facts that cannot be adequately explained by the traditional view. This view suggests that pyramids are created to minimize the cash flow stake that is necessary to achieve control. Thus, it predicts that pyramidal firms should always be associated with a

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<sup>1</sup>The term business group is sometimes used more broadly in the literature to refer to other types of corporate groupings in which the member firms are tied together by common ethnicity of the owners, interlocking directorates, school ties, etc. An example of this type of group is the Japanese keiretsu, an organization in which individual managers have a great degree of autonomy in their firms but coordinate their activities through the President Council and a common Main Bank (Hoshi and Kashyap, 2003). Another example is the horizontal financial-industrial groups in Russia, ‘which are more properly industry alliances’ (Perotti and Gelfer, 2001, p. 1604). *Family* business groups, however, are groups in which member firms are controlled by the same family, such as the groups in Western Europe, Latin America, and East Asia. See Khanna (2000) for a discussion of the definition of business groups.

<sup>2</sup>See, among others, Claessens, Djankov, and Lang (2000) for the evidence on East Asia, Faccio and Lang (2002) and Barca and Becht (2001) for Western Europe, Khanna (2000) for emerging markets, and Morck, Strangeland and Yeung (2000) for Canada.

<sup>3</sup>This argument goes back at least to the beginning of the 20th century. Berle and Means (1932) and Graham and Dodd (1934) use this argument to explain the creation of pyramids in the U.S. in the early 20th century.

substantial separation between ownership and control. Even though there are a number of examples in the literature in which firms in pyramidal groups are characterized by considerable separation between ownership and control (see for example Claessens, Djankov and Lang, 2000), there are many other cases in which the separation achieved is minimal and does not seem to warrant the use of a pyramid (see our discussion in section 2.2).

Moreover, even the cases in which pyramids *do* seem to separate ownership and control are not entirely explained by the traditional view. The reason is that pyramids are not the only way to achieve this separation. For example, the family can achieve any degree of separation by directly owning the firm and issuing shares with no voting rights. In such a case, why would a family choose to control a firm through a pyramid instead of issuing dual-class shares? Yet, despite this apparent equivalence, the empirical evidence indicates that pyramids are much more common throughout the world than dual class shares (La Porta, Lopez-de-Silanes and Shleifer, 1999). This does not appear to be caused by restrictions to the use of dual class shares. Although these restrictions set an effective upper bound to the deviation from one share one vote that can be achieved with dual class shares, many pyramidal firms have deviations that fall *below* this permitted upper bound (again, see section 2.2 for discussion and examples). All this evidence suggests that considerations other than control of voting rights motivate the creation of pyramidal business groups.

In this paper we present a theory that jointly explains why business groups exist (i.e., why a single family controls multiple independent firms) and why these business groups are organized as pyramids (as opposed to an ownership structure in which group firms are owned directly by the controlling family). The model provides a rationale for the existence of pyramids that does not rely on separating cash flow from voting rights. It can thus explain why pyramids arise even in situations in which the family can use dual-class shares to facilitate control. The model can also explain why firms controlled through pyramids sometimes have substantial deviations between ownership and control, while other times the separation is minor. Also, the model is consistent with other anecdotal and empirical evidence regarding the characteristics of pyramidal business groups.

The model has two key ingredients. The first one is the assumption of limited investor protection. If investor protection is poor, the family extracts private benefits from the firms it controls at the expense of minority shareholders. The second ingredient is the assumption that business groups are created over time, that is, the family initially sets up a firm and, at some point in the

future, the opportunity to set up another firm arises.

When this opportunity arises, the family must decide on the ownership structure of the business group. In a *pyramidal* structure the new firm is owned by all the shareholders of the original firm. As a result, the family shares the security benefits of the new firm with non-family shareholders of the existing firm, but it has access to the entire stock of retained earnings (cash) of the original firm.<sup>4</sup> We consider an alternative ownership structure in which the family controls the new firm by directly holding its shares. We refer to this direct ownership structure as a *horizontal structure*. Under this structure, non-family shareholders of the existing firm have no rights to the cash flows of the new firm, and thus the family captures the entire security benefits of the new firm. However, the family has access only to its share of the retained earnings of the original firm.<sup>5</sup>

The level of investor protection plays a crucial role in the choice of structure. Poor investor protection leads to high diversion of cash flows, which makes the pyramidal structure more attractive for two reasons. First, diversion increases the family's private benefits of control, at the expense of a reduction in security benefits.<sup>6</sup> Because in a pyramidal structure the family shares the security benefits with non-family shareholders, while in the horizontal structure it keeps them entirely, high diversion increases the family's payoff under the pyramidal structure relative to the payoff under the horizontal structure (payoff advantage). Second, high diversion makes it more difficult to finance the new firm with external investors as they anticipate the level of diversion and discount the terms at which they are willing to provide finance. Thus, the family's ability to use the entire stock of retained earnings of existing group firms when it chooses the pyramid becomes more valuable (financing advantage).

In addition to the level of investor protection, certain firm characteristics influence the choice of structure. In particular, we show that firms with high investment requirements and/or low profitability are more likely to be set up in pyramids. The argument is similar to that described in the previous paragraph. Because of their characteristics, these types of firms generate lower security benefits for investors. Thus, at the same time the family achieves a higher payoff *and* finds

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<sup>4</sup>Security benefits represent the fraction of the firm's returns that is not diverted by the family and thus accrues to all shareholders. The remaining part (the diverted value) represents a private benefit of control for the family.

<sup>5</sup>Graham and Dodd (1934) argue that the ability to use the resources of an already established firm to set up or acquire new firms was one of the reasons for the existence of pyramids in the U.S. in the early 1900's (see p. 564).

<sup>6</sup>There is a large empirical literature providing evidence that private benefits of control are larger in poor investor protection countries. See Zingales (1994), Nenova (1999) and Dyck and Zingales (2004).

it easier to finance these firms if it uses a pyramid to set them up.

The analysis above assumes that the family is the only party that has the ability to set up and control the new firm. That is, it assumes the existence of a business group. However, we also analyze the conditions under which the business group *itself* appears, that is, the conditions that allow the family to control the new firm. As it turns out, these conditions are very similar to those that are conducive to the creation of pyramids. A firm is more likely to be added to a business group when its investment requirements are high, its profitability is low, and also when investor protection is poor. In such cases, it is difficult for an outside, less wealthy entrepreneur to finance investments in the external market. As a result, families that already own successful firms are the only ones with the financial resources to set up the new firm, regardless of whether the family is the most efficient owner.

In sum, in our model the appearance of business groups and the use of pyramidal structures are natural ramifications of an environment with poor investor protection. When investor protection is poor, business groups appear because the internal resources from the other firms that the family owns provide it with a comparative advantage vis-a-vis new entrepreneurs. At the same time, the use of a pyramidal structure allows the family to maximize these internal resources. Finally, poor investor protection also makes the pyramidal structure more attractive because high diversion reduces the security benefits (which the family gets to share when it chooses the pyramid) and increases the private benefits (which the family keeps entirely). Thus, our model predicts that business groups should adopt a pyramidal ownership structure and that these pyramidal business groups should be very prevalent in poor investor protection countries, an implication that appears to be consistent with available empirical and anecdotal evidence (e.g., La Porta, Lopez-de-Silanes and Shleifer, 1999).

We show that the *observed* ultimate ownership is lower in pyramids. The reason is that the types of firms that are set up in pyramids (firms with high financing requirement and low revenues, as explained above) are those for which a substantial fraction of the shares needs to be sold. This result relies on a selection effect. Absent this selection effect, however, our model can predict lower or higher ultimate ownership for a particular firm that is randomly selected to be set up in a pyramid, instead of in a horizontal structure. This might appear counter-intuitive since pyramids reduce the ultimate ownership due to the chain of ownership. However, this argument ignores an

opposing effect. When a family sets up a firm in a pyramid, it has access to the entire stock of earnings of the original firm and thus can sell fewer shares. A priori, it is not clear which effect dominates.

Because observed ultimate ownership is lower, diversion is higher in firms that belong to pyramids. As explained above, this is not because the use of a pyramidal structure mechanically reduces ownership concentration. Rather, it is because the family uses pyramids to set up firms in which it needs to sell a larger fraction of the cash flows. The family retains lower ultimate ownership concentration and hence diverts more. Thus, our model is also consistent with evidence that shows significant expropriation of investors in firms that belong to pyramidal structures (Bertrand, Mehta and Mullanaitan, 2002, and Johnson et al., 2000).

It is important to note that, in our model, we assume that there are no legal restrictions to the use of dual-class shares. With this assumption, any deviation from one share-one vote generated with the use of pyramids can also be achieved by directly holding shares in the firm (horizontal structure) and issuing an appropriate mix of dual class shares. For this reason, in our framework, arguments for the appearance of pyramids that rely on separation between ownership and control predict an equivalence of the two structures that we consider. However, because in our model pyramids are not used to separate ownership and control, but rather to allow the family to maximize its internal sources of financing and to share the security benefits of new firms, they can still be optimal in this environment. That is, in our model, pyramids are not equivalent to direct ownership with the (potential) use of dual class shares, even if there are no legal restrictions to their use. This helps explain why in some cases pyramids are used despite the fact that the opportunities for separating cash flow and votes with dual class shares are not exhausted.

Another implication of the fact that we do not focus on separation of ownership and control as a rationale for pyramids is that our model does not necessarily require –as the traditional argument does– a small ultimate ownership concentration and consequently a substantial separation between ownership and control in pyramidal firms.<sup>7</sup> In fact, depending on the financing requirements, our model is consistent with families holding either large or small ultimate ownership stakes in pyramidal firms, leading to either minor or substantial separation of ownership and control. Thus our model

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<sup>7</sup>The selection argument above only suggests that families should hold smaller ownership stakes in firms that they control through pyramids, *relative* to firms that they own directly. This is not incompatible with high observed ownership stakes in pyramidal firms, in an *absolute* sense.

can explain why in some pyramidal firms –but not in all– deviations from one-share-one-vote appear to be minor.

The fact that the pyramidal ownership structure is only created if the family is expected to divert a significant fraction of the cash flows produced by the new firm has important valuation implications for existing non-family shareholders of the business group. We show that existing shareholders always lose when a new firm is added to the business group through the creation of a pyramid. This observation raises the following questions. Why would shareholders agree to buy into the business group, if they know that a pyramid might be formed in the future? Arguably, the family will end up paying for future expropriation through reduced share prices today. However, if families internalize the costs of all future expropriation associated with pyramidal ownership, then why do they allow such a structure to persist?<sup>8</sup>

In order to tackle these questions, we extend the model to analyze the optimal contracting at the time in which the first firm in the business group is set up. We assume that shareholders who initially invest in the family firm rationally foresee the possibility that a pyramid will be formed in the future. The simplest case is one in which the family cannot commit not to set up a pyramid in the future if the opportunity arises. In such a case, we show that the pyramid can arise if the first firm is profitable enough. The intuition is straightforward. The family compensates shareholders for the future costs of pyramiding by transferring a large enough fraction of the value of the first firm to initial shareholders (through a reduced share price). Because the business group is created over time, once the first firm is set up and the family has sold some of its shares it faces the incentives described above and will set up the pyramid if the opportunity arises. Thus, absent a good commitment device (e.g., a provision in the corporate charter), the family might end up using pyramids, irrespective of whether they are efficient in an “ex-ante” sense.

However, in our model pyramids are *not always ex-ante inefficient*. Because retained earnings relax financing constraints, and because in a pyramidal structure the family has access to a greater pool of internal funds, there are situations in which the family needs to use pyramids in order to add new firms to the group. This argument suggests a reason why contractual mechanisms to rule

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<sup>8</sup>Arguments based on the traditional view of pyramids as a mechanism to facilitate control and expropriation give rise to a similar question. See for example Bebchuk, Kraakman, and Triantis (2000). Bertrand and Mullanaitan (2003) conjecture that one reason why shareholders might get “trapped” into pyramids is that these might be constructed through acquisitions, and the shareholders of the target firms might have little choice if the controlling shareholder decides to sell.

out pyramids might be absent even if it is feasible to enforce them. We also argue that pyramids are more likely to be ex-ante optimal for the family if investor protection is poor, because in such cases the family is more likely to need the additional internal funds in order to add new firms to the business group.

In the next section (section 2), we discuss the related literature on pyramids and business groups. The presentation of the model is organized as follows. We start our analysis in section 3 by considering a version of the model in which the family already owns a given firm and has to decide on the structure to use (pyramidal or horizontal) to set up a new firm. In section 4 we analyze the conditions that give rise to a business group, that is, the conditions under which the family who already owns a firm can set up the new firm (as opposed to ownership by an outside entrepreneur). In sections 3 and 4 we assume that diversion entails no costs. This assumption makes diversion insensitive to the firm's ownership structure, and simplifies the analysis considerably. In section 5 we relax this assumption, and derive implications regarding variations in diversion and ownership concentration in different structures. Section 6 analyzes the origin of the business group and discusses the optimality of committing to a particular organizational structure. Our theory generates a number of empirical implications, which we discuss in detail in section 7. Section 8 concludes the paper.

## **2 Related Literature**

We consider first the literature on why groups exist, and then we discuss the arguments about pyramidal ownership structure in business groups. Our goal is not only to review the literature, but also to highlight some important questions that are not well explained by existing theories.

### **2.1 Business groups**

Some authors explain the presence of business groups as an efficient organizational form that adds value to member firms. Leff (1978) and, more recently Khanna and Palepu (1997, 1999), argue that business groups substitute for missing markets (e.g. labor and financial markets). Aoki (1984) argues that business groups act as a risk sharing mechanism. Ghatak and Kali (2001) explain business groups as an arrangement that alleviates external credit rationing through mutual debt guarantees, and Kim (2004) shows that these mutual debt guarantees increase the probability of

a bailout. According to these arguments, business groups are more likely to arise in developing countries because these countries are characterized by poor institutional arrangements that prevent the creation of markets.

Another potential benefit of groups is that they are better positioned to lobby the governments for favors (Pagano and Volpin, 2001).<sup>9</sup> Such political economy arguments suggest that business groups should be more prevalent in countries where the government has a stronger role in allocating resources, or in more corrupt countries. Other benefits of groups include the fact that they “prop up” (inject money) failing firms (Friedman, Johnson, and Mitton, 2003) and that a group’s deep pockets serve a strategic role in product market competition (Cestone and Fumagalli, 2004).

There is mixed empirical evidence on the direct valuation effects of group membership. Khanna and Rivking (1999) study 15 countries, and find that in only 3 of them affiliation with a business group adds value to member firms. They find, however, that affiliation does not decrease value in any of the cases. Khanna and Palepu (2000) analyze the performance of business groups in India and find that only the members of the largest groups have valuations higher than those of independent firms. Member firms of medium-sized Indian groups have valuations below their independent counterparts. Fisman and Khanna (2000) show that group-affiliated Indian firms are more likely to locate in less developed states than unaffiliated firms, consistent with the hypothesis that Indian business groups help overcome difficulties that impair production in less developed states. Claessens et al. (2002) find some evidence that firms in business groups organized as pyramids (especially those in which the divergence between votes and cash flows is the greatest) have lower Tobin’s Q. Finally, for a sample of East Asian countries, Claessens, Fan, and Lang (2002) find that group affiliation has on average no effect on firm valuation. They find, however, that there are benefits and costs for different types of member firms: The oldest and slowest-growing firms in the group appear to benefit at the expense of younger, fastest-growing firms.

There is also recent empirical evidence that business groups are associated with increased expropriation of minority shareholders by the controlling shareholder, and might thus be detrimental to firm value. Bertrand, Mehta, and Mullanaithan (2002) find evidence consistent with expropriation in Indian groups, from firms in which the controlling shareholder holds a small equity stake to

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<sup>9</sup>While political clout might be value-enhancing for group firms, it might be detrimental for overall financial and economic development. For example, Rajan and Zingales (2002, 2003) argue that wealthy incumbents (such as the families that control business groups) have incentives to lobby for regulations that impede financial development.

firms in which the controlling shareholder holds a large stake. Bae, Kang, and Kim (2002) find that Korean chaebols use M&A transactions between member firms to expropriate shareholders of the bidder firm and benefit the controlling family. These findings suggest that agency conflicts between controlling and minority shareholders in business groups might offset some of their benefits.

## 2.2 Pyramidal ownership

The conventional wisdom for the existence of pyramids starts with the assumption that families value control because of the large private benefits associated with it. According to this view, the pyramid is a device that allows the family to separate ownership from control. By using pyramids, the family can maximize the amount of capital under its control because they allow the family to retain control in a large number of firms while retaining only a small cash flow stake in each one of them. Because the only reason why the controlling shareholder should use a pyramid is to separate ownership from control, the traditional view predicts that, in pyramidal firms, the deviation from one share-one vote should be large and the concentration of ultimate cash flow rights small.

Consistent with the traditional view, there are a number of examples in the literature in which a family has achieved substantial deviation from one share-one vote through the use of pyramids (see the examples presented in La Porta, Lopez-de-Silanes and Shleifer, 1999, and Claessens, Djankov and Lang, 2000). However, a problem for this view is that there are many other cases in which the separation achieved is small and does not warrant the use of a pyramid. For example, Franks and Mayer (2002) find in their sample of German firms that, in 69% of the firms controlled through pyramids, the controlling shareholder could have achieved the same level of control by simply holding shares directly in the firm. The authors conclude that, in Germany, pyramids are *not* used as a device to achieve control.<sup>10</sup> In a study of ownership and control of Chilean firms, Lefort and Walker (1999) find that the controlling shareholder owns more cash flow than necessary to achieve control. They compute the ultimate cash flow ownership of the controlling shareholder in all the members of a pyramidal group and find this ‘integrated’ ownership to be on average 57%. Thus,

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<sup>10</sup>Franks and Mayer define 25%, 50% and 75% as critical control levels and argue that voting power between any of these critical levels provide the same degree of control. They show that in 69% of their sample of pyramidal firms, the cash flow and control rights do not straddle a control threshold. To see that, when this is the case, the pyramid is not used to separate ownership and control, consider the following example. A family’s ultimate cash flow rights in a firm that belong to a pyramid are 55% and his voting rights are 70%. If the same controlling party held 55% of the shares directly in the firm, he would have 55% of the votes (assuming one-share-one-vote). Because 55% and 70% are between the same two critical levels, direct holdings in the firm and the pyramid provide the controller with the same degree of control.

the separation of ownership and control achieved through pyramids is minimal. Attig, Fischer and Gadhoun (2003) find that, in Canada, the cash flow stake of the controlling shareholder in a pyramid is, on average, 31.78% while the controlling stake is only a bit higher, 41.68%. Faccio and Lang (2002) report that both dual class shares and pyramids are commonly used in Western European countries. However, they find large deviations between ownership and control in only a few of the Western European countries they analyze. Demirag and Serter (2003) report similar findings for Turkey, where cash flow and voting rights appear to be closely aligned despite the widespread prevalence of pyramidal structures. Finally, Valadares and Leal (2001) draw a similar picture for Brazil, where according to the authors pyramids do not appear to be a mechanism to deviate from one-share-one-vote.

An alternative way for a family to achieve control while minimizing its cash flow stake is to hold shares directly in the firm and issue dual class shares. The family simply needs to retain a class of stock that concentrates the majority of the votes but that represents a sufficiently small fraction of the cash flow rights. Another problem with the traditional view is that it only provides a rationale for separating cash flow from voting rights, but does not have any implication as to the optimal mechanism to achieve this deviation.<sup>11</sup> However, if pyramids and dual class shares are equivalent methods to separate ownership from control, why is it that pyramids are much more common throughout the world (La Porta, Lopez-de-Silanes and Shleifer 1999)? One possible explanation is that some countries impose restrictions to the use of dual class shares that, in effect, set an upper bound to the deviation that can be achieved. Nevertheless, we observe pyramids even when restrictions to the issuance of shares with superior voting rights are not binding. For example, in Italy, Bianchi, Bianco, and Enriques (2001) measure the ultimate ownership in each firm that belongs to a pyramid, compute the number of units of capital that the controlling shareholder controls with one unit of his own capital, and average this ratio for all the firms in a pyramid. As a benchmark, consider a family who holds directly 50% of the cash flows and votes in a firm. In this case the ratio is 2. The family can increase this ratio because Italian law allows the issuance of 50% of the firm's capital in non-voting shares (savings shares or *azioni di risparmio*). If the family

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<sup>11</sup>There are other theories that provide a rationale for separating cash flow from voting rights. Gomes (2000) shows that separation of cash flow and voting rights might have reputation benefits. Bebchuk (1999) argues that an initial owner might want to separate cash flow and voting rights to prevent potential raiders from seizing valuable control. These models, however, do not provide arguments for which mechanism should be used to achieve this separation.

uses the maximum fraction of dual class shares and retains 50% of the voting shares (i.e., 25% of the total capital), it can achieve a ratio of 4. Bianchi, Bianco and Enriques find that, while some pyramids allow the controlling shareholder to control a large amount of capital (e.g., the ratio for the De Benedetti group is 10.33 and that for the Agnelli group is 8.86), the ratio for other groups is below 4, and sometimes even below 2 (e.g., for the Berlusconi group, it is 3.66, and for the Pirelli group it is only 1.95).

The empirical evidence discussed above suggests that there must be reasons for the existence of pyramids, over and above the separation of ownership and control. Regulatory or tax considerations might help explain some of this evidence. Indeed, taxes on inter-company dividends do seem to affect the incidence of pyramidal structures (Morck, 2003). Others have suggested that pyramidal structures can be used as an elusive tool to hide the identity of the ultimate owner from either the market or the state (Bianchi, Bianco, and Enriques, 2001).

### 3 Pyramidal and horizontal structures

In this section we present a framework to analyze pyramidal business groups. The model has three dates. At date 0 a family sets up a firm (firm A), keeping a fraction  $\alpha$  of its shares. At date 1, firm A generates cash flows of  $c$ , and the opportunity to set up another firm (firm B) arises.

We start by assuming that the ownership concentration of the family in firm A ( $\alpha$ ) is given, but we endogenize it in section 6. Firm B requires an investment  $i$  at date 1 and generates a revenue  $r$  at date 2. We assume that  $r > i$ , that is, firm B is positive NPV. We also assume that the family has all its wealth invested in firm A.

In this section, we assume that the family is the only party who can set up firm B. In the next section, we consider competition from an entrepreneur. In that section, we analyze the conditions under which the family –and not the entrepreneur– sets up firm B. When this happens, we say that a business group (with member firms A and B) is created.

The family chooses the optimal ownership structure for firm B. Firm B can be owned by the shareholders of firm A (the pyramidal case), or alternatively it can be owned directly by the family (the horizontal structure). In a pyramidal structure, the family sets up firm B as a partial subsidiary of firm A and thus can use the cash  $c$  in firm A to set up firm B. In an horizontal structure, the

family itself sets up firm B. In this case, the family has access only to its personal wealth of  $\alpha c$ .<sup>12</sup> In both structures the family can also sell a stake in firm B to raise additional funds.

We assume that there are no legal restrictions to the use of dual class shares. This assumption ensures that the family can always retain control of firm B, irrespective of the structure it chooses and its ultimate ownership. Control allows the family to divert cash from firm B into their pockets.<sup>13</sup> We start our analysis by assuming that diversion entails no cost and that the level of investor protection limits the amount of diversion that can take place (similar formulations of the diversion technology can be found in Pagano and Roell, 1997, and in Burkart and Panunzi, 2002). In particular, we let  $d$  be the maximum amount (as a fraction of the revenue) that can be diverted. Better investor protection leads to less diversion, that is,  $\partial d/\partial k < 0$ . Because diversion is costless, the owner of firm B always diverts the maximum possible regardless of the ownership structure. In this version of the model, investor protection is the only determinant of diversion.<sup>14</sup>

In section 5 we introduce a cost of diversion (similar to that in Burkart, Gromb, and Panunzi 1998 and Shleifer and Wolfenzon 2002). In the presence of this cost, diversion depends on ultimate ownership. As we show in section 5 this new assumption allows us to derive additional implications from the model, but it does not change the substance of the implications of the basic model, so we start our analysis with this simpler model.

We calculate the family's payoff from each structure (horizontal or pyramidal), and analyze whether the structure is feasible (that is, whether the family can raise sufficient funds to set up firm B). We then characterize the region of the parameter space over which each structure is chosen.

### 3.1 Horizontal structure

The family has cash of  $\alpha c$  available from its ownership in firm A. It sells  $1 - \beta_H$  of firm B to the public and keeps a fraction  $\beta_H$  (its ultimate ownership). Investors expect the family to divert a

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<sup>12</sup>If the family could costlessly divert the cash of firm A and use the entire  $c$  to set up his own firm, the horizontal structure would clearly dominate. However, at date 0, it would be impossible for the family to sell any shares of firm A as investor would expect to get nothing in return. As a result all firms would be financed entirely by the family, with no outside shareholders. We could consider an intermediate case in which the family diverts some of the cash of firm A. Our results would be unchanged under the assumption that, if the family wants to use the diverted funds from firm A to set up firm B, it can only do so at market prices.

<sup>13</sup>We assume that the diversion opportunities are the same regardless of the structure the family uses. If the family retains control regardless of structure, its set of feasible actions and hence its diversion *opportunities* should be the same. *Actual* diversion, however, is determined by the incentives that the family faces in each structure.

<sup>14</sup>For simplicity, we assume that there is no direct diversion from firm A. We can show that allowing for this possibility does not change our results, as long as the family can only contribute the diverted funds to firm B at market prices.

fraction  $d$  of the cash flows. The total cash available to the family for investment is:

$$R_H = \alpha c + (1 - \beta_H)(1 - d)r \quad (1)$$

The maximum investment that can be financed in a horizontal structure is given by  $\bar{R}_H = \max_{\beta_H \in [0,1]} R_H = \alpha c + (1 - d)r$ . In this simplified model, because diversion does not depend on ultimate ownership, the family can fully disperse ownership in firm B without increasing diversion.

Conditional on  $\bar{R}_H \geq i$ , the family gets the entire NPV of the project and so his payoff is:

$$U_H = \alpha c + NPV \quad (2)$$

where, because diversion is costless, the NPV of the project is  $r - i$ .

### 3.2 Pyramidal structure

In this case, firm A sets up firm B. Firm A has access to a stock of cash,  $c$ , plus the proceeds of the sale of a fraction  $1 - \beta_P$  of firm B. The ultimate ownership of the family in firm B is then  $\alpha\beta_P$ . The total cash available for investment is:

$$R_P = c + (1 - \beta_P)(1 - d)r. \quad (3)$$

The maximum investment that can be financed in the pyramidal structure by:  $\bar{R}_P = \max_{\beta_P \in [0,1]} R_P = c + (1 - d)r$ .

Finally, conditional on  $\bar{R}_P \geq i$ , the family's payoff is:

$$\begin{aligned} U_P &= \alpha\beta_P(1 - d)r + dr \\ &= \alpha c + NPV - (1 - \alpha)[(1 - d)r - i] \end{aligned} \quad (4)$$

The payoff differences between the horizontal and the pyramidal structures can be seen in expressions 2 and 4. In an horizontal structure, there are only two types of firm B shareholders: the family and new investors in firm B. New investors in firm B price in the anticipated diversion and get the market return on their investment. As a result the entire NPV is captured by the family. In the pyramidal ownership, there are three types of firm B shareholders: the family, new investors and non-family shareholders of firm A. Because new investors get the market return, the NPV is divided between the family and the non-family shareholders of firm A. The NPV is not

distributed in proportion to the stakes in firm A because only the family has access to the diverted amount. Only the non-diverted NPV  $((1-d)r - i)$  is divided in proportion to the stakes in firm A. That is, non-family shareholders of firm A get  $(1-\alpha)[(1-d)r - i]$  and the family receives the rest.

The family's payoffs in equations 2 and 4 are conditional on the project being taken. The choice between the horizontal and pyramidal structures is also driven by differences in the financing capacity in the two structures,  $\bar{R}_H$  and  $\bar{R}_P$ . Because of the retention of internal funds  $c$  the financing capacity in the pyramid is greater,  $\bar{R}_H < \bar{R}_P$ .

### 3.3 Choice of structure

The following result fully characterizes the choice of structure in this version of the model.

**Result 1** *If the non-diverted NPV of firm B,  $(1-d)r - i$ , is positive, the family always chooses the horizontal structure. If the non-diverted NPV of firm B is negative and the pyramid is feasible ( $\bar{R}_P > i$ ), the family chooses the pyramid. If  $(1-d)r < i$  and  $\bar{R}_P < i$  firm B is not set up by the family.*

The proof of this result, as well as all other proofs, is in the appendix. When the non-diverted NPV is positive, firm B can be financed in either structure. In this case the maximum amount that external investors contribute  $(1-d)r$  is sufficient to pay the investment cost,  $i$ . In terms of payoffs, however, the family prefers the horizontal structure. If the family sets up the pyramid, it shares this positive non-diverted NPV with the non-family shareholders of firm A, whereas if it chooses the horizontal structure it gets to keep this amount. Therefore, in this case, the horizontal structure is chosen.

When the non-diverted NPV is negative, the family prefers the pyramid because it gets to share the negative non-diverted NPV with the other shareholders of firm A. In this case, however, firm B is not always feasible because the maximum amount external investors contribute is less than the set up costs. Therefore, the family chooses the pyramidal structure whenever it is feasible.<sup>15</sup>

**Result 2** *Assume that  $\bar{R}_P \geq i$ , such that firm B is feasible under the pyramidal structure. Given this condition, firm B is less likely to be owned through a pyramid when*

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<sup>15</sup>There is never a case in which the family prefers the pyramid but the only feasible structure is the horizontal. The reason is that the pyramidal structure is feasible whenever the horizontal structure is (because  $\bar{R}_H < \bar{R}_P$ ).

- *Firm B generates higher revenues*
- *Firm B requires a smaller investment*
- *Investor protection increases*

This result follows from the fact that the non-diverted NPV is higher and so more likely to be positive when profitability increases, investment decreases or investor protection is stronger. Because the non-diverted NPV is more likely to be positive, the family is more likely to use a horizontal structure both because its payoff is higher, and because it becomes easier to finance the project.<sup>16</sup>

As we argued above, this simple model identifies a rationale for pyramids that is unrelated to considerations about control of voting rights. Our assumption that there are no legal restrictions to the use of dual-class shares implies that the family can use either structure to achieve control, regardless of how small a cash flow stake it wants to hold. In this framework, any argument for the existence of pyramids that relies on separation of ownership and control cannot make any prediction as to which structure the family should use. Because in our model pyramids are not used to separate ownership from control, but rather to allow the family to maximize its internal sources of financing and to share the security benefits of new firms, they can be optimal in this environment. That is, in our model, pyramids are not equivalent to direct ownership with the (potential) use of dual class shares, even when there are no legal restrictions to the use of dual class shares.

## 4 Business groups

We define a business group as an organization in which a family owns and controls more than one firm. In the last section we assumed that the family is the only party with the ability to set up firm B. This effectively means that we assumed the existence of a business group. In this section we investigate the conditions under which a business groups arises.

We introduce the possibility that, at date 1, there is an alternative owner for firm B (whom we call the entrepreneur). The set up cost of firm B for the entrepreneur is also  $i$ . We assume that the entrepreneur is a better manager than the family, so that under his control revenues of firm B are

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<sup>16</sup>We condition on firm B being feasible under the pyramidal structure because, empirically, only the set of projects that are feasible under the least restrictive conditions will be observable.

$(1+t)r$  with  $t > 0$ , and that he has no personal wealth. Thus, the only advantage of the family is its higher financing capacity due to the accumulation of internal funds in the existing firms it owns (that is, the cash flow  $c$  of firm A).<sup>17</sup>

We assume that the market in question only allows for one firm and that, if the entrepreneur can raise sufficient funds, he will be the only one to enter the market because of his higher productivity. If he cannot raise the necessary funds, then the family sets up firm B using any of the two structures described in the last section. Given this assumption, we can prove the following result.

**Result 3** *Business groups are less likely to arise when*

- *Firm B generates higher revenues*
- *Firm B requires a smaller investment*
- *Investor protection increases*

The comparative advantage of the family is that they have accumulated wealth, and thus do not need to rely as much on external capital markets. As investor protection improves, the comparative advantage of the family eventually disappears and the entrepreneur is able to set up his firm. The entrepreneur is also more likely to raise the necessary funds to set up firm B when firm B's NPV is large.

Notice that the conditions that are conducive to the formation of business groups are also conducive to the formation of pyramids (see results 2). In fact, in this simple model we can prove the following stark result.

**Result 4** *If a business group arises, it is always organized as a pyramid*

Competition from the entrepreneur eliminates the region of the parameter space in which a horizontal structure arises. Thus, in our model, there is an endogenously derived equivalence between business groups and pyramids. The intuition for this result is that horizontal structures only appear when the non-diverted NPV of firm B is positive, because in such cases the family does not want to share the positive NPV of firm B with the existing shareholders of firm A. However,

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<sup>17</sup>Notice that the solution of the model when the family is the best owner is trivial. Because the family has higher wealth, it will always own firm B.

under such conditions entrepreneurial finance is possible, because the fraction of the profits of firm B that can be pledged to outside investors,  $(1 - d)(1 + t)r$ , is bigger than the investment  $i$ . Thus, the situations in which an horizontal structure is optimal are precisely the situations in which the business group loses its financing advantage over the entrepreneur.

It is worth discussing in more detail what is novel regarding the results in this section. The idea that business groups arise in countries with poor investor protection because external financing is more limited is not new. This idea is related to the arguments in Leff (1978) and Khanna and Palepu (1997, 1999) that we discussed in section 2. However, these authors have not considered the optimal choice of ownership structure in a business group. Result 4 suggests that, if business groups are created to substitute for financial markets that are curtailed by poor investor protection, they *should also be organized as pyramids*.<sup>18</sup> This result can only be derived because we model both business groups and pyramids in an integrated framework. In section 7 we discuss in greater detail the empirical implications of this result.

## 5 Ultimate ownership and diversion

The simple framework we have used so far generates several results about the conditions under which business groups appear and the type of structures they use (pyramidal vs. horizontal). However, because we assumed that diversion is independent of ownership concentration, the family (or the entrepreneur) can fully dilute ownership without any implications for value. Thus the previous model is not well suited to address the question of concentration of cash flow rights in pyramidal firms. Furthermore, because diversion is the same irrespective of the organizational form, the model does not have predictions for the relationship between the pyramidal organizational form and diversion.

In this section, we extend the model to endogenize diversion and to allow for optimal choice of the ownership concentration of firm B. We embed the framework of sections 3 and 4 in a simplified version of Shleifer and Wolfenzon's (2002) model of optimal ownership concentration in an economy with poor investor protection. We assume that when a fraction  $d$  of the cash flows is diverted, the

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<sup>18</sup>Notice that if the family is the most efficient owner of firm B ( $t < 0$ ), it becomes possible that the business group is organized horizontally because firm B is always owned by the family in this case (even when its NPV is very high). Furthermore, the relationship between investor protection and the prevalence of business groups also breaks down. Thus, our model only predicts that the business groups that are set up for financing reasons have pyramidal ownership.

family pays a cost (one can think of this as waste involved in the diversion process) that equals a fraction  $c(d, k)$  of the cash flow of the firm. We assume that  $c(0, k) = 0$ ,  $c_d > 0$ ,  $c_{dd} > 0$ , and  $c_{dk} > 0$ . We interpret the parameter  $k$  as the level of investor protection: a high cost of diversion (high  $k$ ) corresponds to good investor protection.

In this version of the model, the fraction of cash flows diverted from firm B depends on the ultimate ownership concentration that the family retains. Because of the importance of ultimate ownership, we perform a change of variables. We let  $x_H$  and  $x_P$  be the family's ultimate ownership in firm B. That is, we define  $x_H \equiv \beta_H$  and  $x_P \equiv \alpha\beta_P$ . This change allows us to switch attention from the direct ownership to the ultimate ownership of firm B.

We start the analysis by characterizing optimal diversion at date 2, and then we work backwards. At date 2, the family can divert from firm B to its pockets. The family maximizes:

$$\max_{d \in [0,1]} x(1-d)r + dr - c(d, k)r$$

For  $x \in \{x_H, x_P\}$ . The level of diversion,  $d(x, k)$ , satisfies the first order condition  $c_d(d(x, k), k) = 1 - x$ . It follows from the properties of  $c(\cdot, \cdot)$  that diversion is decreasing in ownership concentration ( $d_x < 0$ ) and in the level of investor protection ( $d_k < 0$ ).

For future reference, we define  $NPV(x) = r - i - c(d(x), k)r$  as the total value generated by firm B when the ultimate ownership concentration is  $x$ . Note that  $NPV'(x) > 0$  because higher ultimate ownership concentration reduces diversion and consequently reduces the total cost of diversion.

Moving back to date 1, we consider the family's choice of horizontal versus pyramidal structure. As in section 3, for each structure, we calculate the family's payoff and whether the structure is feasible. In addition, we calculate the optimal ultimate ownership concentration of firm B. We then characterize the region of the parameter space over which each structure is chosen.

## 5.1 Horizontal structure

The total cash available for investment in the horizontal structure is:

$$R_H = \alpha c + (1 - x_H)(1 - d(x_H))r. \quad (5)$$

The only difference with the expression for  $R_H$  derived in section 3 is that, in the above expression, expected diversion and hence the share price depend on ultimate ownership concentration.

There are two opposing effects of increasing ownership concentration,  $x_H$ . On the one hand, the direct effect is that fewer shares are sold, reducing the amount collected. On the other hand, with higher ownership concentration, diversion is lower and as a result the price per share is higher, increasing the amount collected.

Conditional on  $\bar{R}_H \geq i$ , the family's payoff at the end of date 2 is given by:

$$U_H = \alpha c + NPV(x_H). \quad (6)$$

Because new shareholders break even, the family gets the entire NPV of the project. Since  $NPV(x_H)$  is increasing, the family chooses the highest possible ownership concentration that satisfies  $R_H(x_H) \geq i$ . Thus  $x_H^*$  is either equal to 1 (when  $\alpha c > i$ ), or is given by the highest level of ownership concentration that satisfies:

$$R_H(x_H^*) = i \quad (7)$$

## 5.2 Pyramidal structure

The expression for the amount raised is given by:

$$R_P = c + \left(1 - \frac{x_P}{\alpha}\right) (1 - d(x_P))r. \quad (8)$$

where  $1 - \frac{x_P}{\alpha} = 1 - \beta_P$  is the fraction of shares of firm B sold to new shareholders. Again, the only difference with the expression in section 3 is that the price per share depends on ultimate ownership concentration.

Conditional on  $\bar{R}_P \geq i$ , the family's payoff (from the perspective of date 1) is given by:

$$U_P = \alpha c + NPV(x_P) - (1 - \alpha)[(1 - d(x_P))r - i] \quad (9)$$

Increasing ownership concentration reduces diversion and increases the NPV of the project. However, lower diversion also means that the family has to share a greater fraction of the NPV with existing shareholders (the term  $(1 - \alpha)[(1 - d(x_P))r - i]$  goes up). Does the family have an incentive to induce diversion by lowering ownership concentration? We show that this is not the case in the following proposition.

**Result 5** *The optimal ultimate ownership concentration,  $x_P^*$ , is the highest value that satisfies the financing requirement  $R_P(x_P) \geq i$ . If  $c > i$ ,  $x_P^* = \alpha$  (i.e., no shares of firm B are sold), and if*

$c < i$ ,  $x_P^*$  is the maximum value of  $x_P$  that satisfies

$$R_P(x_P^*) = i \quad (10)$$

The proof of this result is based on a time inconsistency in the family's optimal amount of diversion. The family bases its diversion decision on its ex-post stake in firm B,  $x_P = \alpha\beta_P$ . However, from the viewpoint of date 1, the family gets a fraction  $\alpha$  of the non-diverted revenue (see equation 9). Because the ex-post stake is lower, the family always diverts too much from the perspective of date 1. To minimize diversion, the family chooses the highest ultimate ownership concentration possible.

### 5.3 Choice of structure

In this section we compare the ultimate ownership concentration that the family chooses with each structure. We focus on the case that  $c < i$ , so that the family needs to sell some shares of firm B in either structure.

A convenient way to think about the funds that can be raised with each structure is to focus on the different types of shareholders of firm B. In an horizontal structure, there are only two types: the family and the new shareholders of firm B. When the family keeps a fraction  $x_H$  of firm B, the remaining fraction  $1 - x_H$  is sold to new shareholders who pay the market price that reflects expected expropriation,  $(1 - d(x_H))r$ . In a pyramidal structure, there are three types of shareholders: the family, non-family shareholders of firm A and new shareholders of firm B. If the family's ultimate ownership is  $x_P$ , the remaining fraction,  $1 - x_P$ , is distributed between non-family shareholders of firm A and new shareholders of firm B. Non-family shareholders of firm A indirectly own  $(1 - \alpha)\beta_P = \frac{x_P}{\alpha} - x_P$  and new shareholders hold  $1 - \beta_P = 1 - \frac{x_P}{\alpha}$  (note that  $(\frac{x_P}{\alpha} - x_P) + (1 - \frac{x_P}{\alpha}) = 1 - x_P$ ). New shareholders pay the market price, while the 'implied' price paid by non-family shareholders is  $\frac{(1-\alpha)c}{(1-\alpha)\beta_P} = \frac{\alpha c}{x_P}$  (of the total  $c$  that firm A contributes, only  $(1 - \alpha)c$  belongs to non-family shareholders who receive  $(1 - \alpha)\beta_P$  of the shares). Indeed, to reflect the contribution by different types of shareholders, equation 8 can be re-written as:

$$R_P = \alpha c + \underbrace{\left(\frac{x_P}{\alpha} - x_P\right)}_{\text{Fraction held by non-family shareholders}} \underbrace{\left(\frac{\alpha c}{x_P}\right)}_{\text{Implied price}} + \underbrace{\left(1 - \frac{x_P}{\alpha}\right)}_{\text{Fraction held by new shareholders}} \underbrace{(1 - d(x_P))r}_{\text{Market price}} \quad (11)$$

In the pyramidal structure the family has access to the entire stock of earnings of firm A. However, as equations 11 and 5 indicate, this does not necessarily translate into a financing advantage. The reason is that, in the pyramidal structure non-family shareholders receive shares, which could have been sold to the market if the horizontal structure had been used instead. The pyramidal structure has a financing advantage only when the ‘implied price’ paid by non-family shareholders is higher than the market price. This happens when diversion is high enough or, more precisely, when the non-diverted NPV of firm B is negative. Because new shareholders break even (zero NPV), a negative non-diverted NPV means that the implied price paid by non-family shareholders is higher than the market price.<sup>19</sup>

In addition to this financing advantage, the pyramidal structure provides the family with a payoff advantage when the non-diverted NPV of firm B is negative. The reason is that the family gets to share the non-diverted NPV with non-family shareholders of firm A. In sum, when diversion is high enough to make the non-diverted NPV of firm B negative, the pyramid provides the family with both a financing and a payoff advantage. We show this more rigorously in Result 7 below. We start, however, with an intermediate result.

**Result 6** *Let  $(\bar{\alpha}, \bar{c}, \bar{r}, \bar{i}, \bar{k})$  be parameters such that  $x_H^* = x_P^* = \bar{x}$ . Then, it must be that  $(1-d(\bar{x}))\bar{r} = \bar{i}$  and  $U^H(\bar{x}) = U^P(\bar{x})$ .*

In words, if the parameters are such that ultimate ownership is the same in both structures, then it must be the case that the non-diverted NPV of firm B is zero and that the payoff to the family is the same regardless of structure. The intuition for this result is as follows. In both structures, the family sells enough shares to raise the same amount  $i$ . When ultimate ownership concentration is the same in both structures ( $x_H^* = x_P^* = \bar{x}$ ) the difference in the amount raised in the pyramidal and in the horizontal structure lies in how the remaining  $1 - \bar{x}$  is distributed. As explained above, in the horizontal structure the entire  $1 - \bar{x}$  is sold to the market whereas in the pyramidal structure part of the  $1 - \bar{x}$  is sold to new shareholders and part is ‘sold’ to non-family shareholders of firm A. The only way for both these structures to raise the same amount of money is that the implied price paid by non-family shareholders be equal to the market price. Because new shareholders who

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<sup>19</sup>If the non-diverted NPV is negative, we have:  $(1 - d(x_P^*))r < i = c + (1 - \frac{x_P^*}{\alpha})(1 - d(x_P^*))r$ , or rearranging,  $\frac{\alpha c}{x_P^*} > (1 - d(x_P^*))r$ .

pay the market price break even (zero NPV), so do the non-family shareholders of firm A. That is, the non-diverted NPV must be zero. The result about payoffs follows directly from this equality.

**Result 7** *Let  $(\bar{\alpha}, \bar{c}, \bar{r}, \bar{i}, \bar{k})$  be parameters such that  $x_H^* = x_P^* = \bar{x}$ . Take parameters  $(\bar{\alpha}, \bar{c}, \bar{r}, i, \bar{k})$ . For  $i > \bar{i}$ , the ultimate ownership in a pyramid is higher ( $x_P^*(i) > x_H^*(i)$ ), the non-diverted NPV of firm B is negative under both structures and, if feasible, the pyramidal structure is chosen (identical results hold when  $r < \bar{r}$  and when  $k < \bar{k}$ ). For  $i < \bar{i}$ , the ultimate ownership in a pyramid is lower ( $x_P^*(i) < x_H^*(i)$ ), the non-diverted NPV of firm B is positive under both structures and the horizontal structure is always chosen (identical results hold when  $r > \bar{r}$  and when  $k > \bar{k}$ ).*

Take the result on  $i$ . As the required investment  $i$  increases above  $\bar{i}$ , ultimate ownership in either structure decreases because more shares need to be sold to raise the necessary capital. That is,  $x_H^*(i) < \bar{x}$  and  $x_P^*(i) < \bar{x}$ . This implies that the non-diverted NPV of project B is negative regardless of the structure because lower ultimate ownership leads to more diversion. Also, as discussed above, when the non-diverted NPV of firm B is negative the pyramid has a financing advantage because the implied price paid by non-family shareholders of firm A is higher than the market price. Thus, when using the pyramidal structure the family needs to sell fewer shares to new shareholders than when it uses the horizontal structure (that is,  $x_P^*(i) > x_H^*(i)$ ). Finally, the family chooses the pyramidal structure for two reasons. Because the non-diverted NPV is negative, choosing the pyramid allows the family to share this value with non-family shareholders of firm A. And second, because ultimate ownership is higher in the pyramidal structure, there is less waste associated with diversion. The intuition for the case when  $i$  is below  $\bar{i}$ , and for  $r$  and  $k$  is similar.

As the above discussion illustrates, the model with endogenous ownership produces essentially the same results regarding pyramidal and horizontal structures that we described in section 5. Pyramids are chosen only when the existing shareholders of firm A lose with the addition of firm B. Otherwise, the family prefers to set up a horizontal structure. Furthermore, pyramids are more likely to be chosen when investor protection and firm B's NPV are low.

As Result 7 indicates, the relative size of the family's ultimate cash flow stake in the pyramidal and in the horizontal structure depends on the parameter values. When diversion is high, the pyramid has the financing advantage and so the ultimate ownership is higher in the pyramidal structure. Conversely, when diversion is low, the market price is higher than then implied price

and the family needs to sell fewer shares in the horizontal structure. As a consequence, the family can keep a higher ultimate ownership concentration.

Result 7 cannot be taken to the data because it compares the hypothetical values of ownership concentration when each structure is chosen. It does not compare the observed (or chosen) ownership concentration levels as the parameters vary. The following result establishes that relation.

**Result 8** *Suppose that different structures are the result of variation in  $r$ ,  $i$  or  $k$  (one by one). The ultimate ownership concentration level observed in any pyramidal structure is lower than the ultimate ownership observed in any horizontal structure. It follows directly that diversion from firm  $B$  is higher in pyramids.*

Suppose that there is variation in the required investment,  $i$ . As we show in the discussion following result 7, the pyramidal structure will be chosen for  $i > \bar{i}$  and, in this case,  $x_P^*(i) < \bar{x}$ . The horizontal structure is chosen when  $i < \bar{i}$  and, in this case,  $x_H^*(i) > \bar{x}$ . Therefore, pyramids will have an ultimate ownership below  $\bar{x}$  and horizontal structure will have an ultimate ownership above  $\bar{x}$ .

The point that ultimate ownership in a pyramid is low has been made informally by many authors. The traditional argument is that the chain of control mechanically reduces ultimate ownership. As Result 7 indicates, however, if one compares the ultimate ownership a firm would have under a pyramid and under a horizontal structure, it is not obvious that ultimate ownership would be lower in a pyramidal structure. One key feature that the traditional argument ignores is that a family has access to a higher amount of cash when it sets up a firm in a pyramid, and consequently, it might need to sell fewer shares to finance the new firm. Our argument in Result 8 is different. The pyramid is set up when the investment required is high, revenues are low and/or investor protection is low. But in these situations more shares need to be sold to finance the set up costs of the new firm. This explains the fact that ultimate ownership is lower in a pyramidal structure than in a horizontal structure.

A similar argument holds for diversion. The traditional view is that diversion is higher in a pyramidal structure because the chain of control reduces the family's ultimate ownership. This argument ignores the fact that families are interested in reducing diversion because its cost falls back to them. Our model is consistent with the empirical observation that there is more diversion

in pyramids, while at the same time families try to minimize diversion. Because families choose pyramids when the investment required is high, revenues are low and/or investor protection is low, they end up selling more shares to finance the new project than in the situations in which the firm chooses the horizontal structure (low investment, high revenues and/or high investor protection).

Result 8 shows that the observed ultimate ownership in pyramids is lower than in horizontal structures. It does not imply that ownership concentration should be low in a pyramid in an *absolute* sense. In fact, the threshold of ultimate ownership at which the family switches to the pyramidal structure,  $\bar{x}$ , can be quite high. If, for instance, this threshold is strictly above 50%, some pyramidal firms will have ultimate ownership concentration around 50%. Clearly, for these firms, the family could have achieved the same degree of control by simply holding shares directly. Yet, the family chooses the pyramids because of its financing and payoff advantages described above. Depending on the parameter values, our model can also predict very small ownership concentration in pyramidal firms. Thus, unlike the traditional argument for pyramids, our model accommodates both high and low ownership concentration in pyramidal firms. We see this as a strength of the model because the empirical evidence indicates that both cases are common. Moreover, we can analyze which characteristics make it more likely that a pyramid will be associated with large ultimate ownership concentration.

**Result 9** *The threshold of ultimate ownership below which the pyramid is chosen,  $\bar{x}$ , is decreasing in investor protection.*

Recall that at the threshold  $\bar{x}$  the non-diverted NPV of firm B is zero. When investor protection is poor, the incentives to mitigate diversion must come from ownership concentration. Consequently, a high ownership concentration is needed to achieve a zero non-diverted NPV. As a consequence, our model predicts that in poor investor protection countries, it will be easier to observe pyramids with large ultimate cash flow stakes.

The main message of our previous results on business groups continue to hold in this setting as well. Suppose that there is an entrepreneur who can set up firm B. The advantage of the entrepreneur over the family is that the entrepreneur generates more cash flows from firm B. The disadvantage is that the entrepreneur has no wealth, whereas the family does. The entrepreneur is thus able to set up firm B when this firm is very profitable (i.e., high  $r$  and low  $i$ ) or when investor

protection is high. These are precisely the conditions that need to hold for the family to choose an horizontal structure, according to result 7. Thus, the presence of a potential entrepreneur reduces the region over which horizontal structures are observed.

## 6 Ex-ante optimality of pyramids

Sections 3 and 5 show that whenever the pyramidal structure is chosen, shareholders of firm A realize a negative return because they share the negative non-diverted NPV with the family. This raises the question of whether shareholders will agree to buy into firm A at date 0 when they know that a pyramid might be formed. The simple answer is that, if shareholders rationally anticipate the future formation of the pyramid, the price they pay for the group's shares at date 0 should be discounted to reflect the future expected expropriation. If this is the case, shareholders will be ready to participate. On the other hand, the fact that the family ends up paying for future expropriation through reduced share prices today raises yet another question: Why does a family choose an ownership structure that magnifies agency conflicts when the family itself internalizes all these costs?

To analyze these questions, we extend the model with exogenous diversion of section 3 to date 0. We assume that at date 0, firm A needs an investment of  $i_A$  and generates revenues of  $r_A > i_A$  at date 1. Similarly, the revenue and the investment of firm B, which can be set up at date 1, are represented by  $r_B$  and  $i_B$ . For simplicity, we assume that there is no diversion of the cash flows of firm A, but the analysis can be easily extended to a case in which there is diversion (see footnote 14). We also assume that the family has no wealth at date 0, and we do not consider competition by the entrepreneur, neither at date 0, nor at date 1.<sup>20</sup> Finally, we assume that shareholders who invest at date 0 have rational expectations regarding the future creation of the pyramid.

Suppose first that the family cannot commit at date 0 not to set up the pyramid at date 1. In this case, the following result shows that pyramids will arise under certain conditions.

**Result 10** *Suppose that  $r_A + (1 - d)r_B > i_A + i_B$  and that  $(1 - d)r_B < i_B$ . In this case, the family*

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<sup>20</sup>If there is competition at date 0, the family will own firm A if it is the most efficient owner, or if the family is sufficiently more wealthy than the entrepreneur. On the other hand, if the entrepreneur owns firm A, the entrepreneur "becomes" the family in date 1, from the perspective of our model. Competition at date 1 has the same effect it had in section 3: it eliminates horizontal structures if the pledgeable income of firm B is larger than the required investment.

sets up firm A at date 0 and uses a pyramid to set up firm B at date 1. Shareholders of firm A break even from the perspective of date 0.

As discussed in section 3, the condition that  $(1 - d)r_B < i_B$  implies that, at date 1, the family prefers the pyramidal structure.<sup>21</sup> The condition that  $r_A + (1 - d)r_B > i_A + i_B$  is the required condition for the financing of firm A. Intuitively, the first firm that the family sets up must be profitable enough in order to compensate initial shareholders for the future expropriation associated with pyramids. If this condition holds, the group's shares can be priced low enough such that initial shareholders break even and the family can raise enough to finance firm A.

Therefore one potential reason why pyramids arise in this case is that the business group is not created at one particular point in time. Rather, the family sets up the group's member firms over time. Once their first firm is set up and the family has sold some of its shares, it faces the incentives described in sections 3. A pyramid will then arise if the new firm's non-diverted NPV is low enough.

Under the assumption that the family cannot commit at date 0 to use a particular structure in the future, result 10 shows that the pyramid might appear irrespective of whether it is *ex-ante* optimal for the controlling family. Result 10 does not rule out the possibility that the family might benefit from a mechanism (such as a contract or a charter provision) that allows it to commit not to form pyramids. From the perspective of date 0, the family bears all the costs of future expropriation associated with pyramids.<sup>22</sup> Thus, its expected payoff might be higher if such a contractual commitment is possible.<sup>23</sup>

Importantly, however, our model also suggests an ex-ante benefit of pyramids. When this benefit is high enough, the family may not want to rule out pyramids by contract *even when* it can do so. We argue that there are cases in which the only way the family can set up both firms A and B is by allowing pyramidal structures. Ruling out pyramids might eliminate the possibility of setting

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<sup>21</sup>Notice that in this case competition from the entrepreneur at date 1 is moot because he cannot finance firm B.

<sup>22</sup>In fact, in the version of the model we analyze in this section, pyramids do not have higher *deadweight* costs relative to horizontal structures. This is because diversion of cash flows from firm B is assumed to be costless. Thus, conditional on the family being able to finance both firms, the family's ex-ante payoff is identical under pyramidal or horizontal structures. We can show, however, that in the model with endogenous diversion (section 5), ruling out pyramids ex-ante will generally decrease ex-post diversion and associated deadweight costs (see also Wolfenzon, 1999).

<sup>23</sup>Arguably, a credible commitment mechanism is less likely to exist in countries with poor investor protection, in which contractual mechanisms are probably less effective. This observation might help explain why pyramids appear in poor investor protection countries, even if they are ex-ante inefficient for the family.

up firm B. Since firm B is positive NPV, this is inefficient from an ex-ante perspective.

This type of situation arises if there is uncertainty regarding the cash flow produced by firm A. Suppose that the revenue generated by firm A is  $r_A = \bar{r}_A - \Delta$  with probability  $\frac{1}{2}$ , and  $r_A = \bar{r}_A + \Delta$  with probability  $\frac{1}{2}$ . The following result holds:

**Result 11** *Suppose that the following conditions hold:*

$$\left(1 - \frac{i_A}{\bar{r}_A}\right) \Delta + \bar{r}_A - i_A + (1-d)r_B - i_B < 0 \quad (12)$$

and:

$$\bar{r}_A - i_A + \frac{1}{2} [(1-d)r_B - i_B] > 0 \quad (13)$$

*In this case, it is not optimal for the family to rule out pyramids at date 0.*

This result merits some detailed explanation. Consider first the case in which the pyramid is ruled out. In this case, firm B must be set up in a horizontal structure and so shareholders of firm A get the entire expected revenue  $\bar{r}_A$  at date 1. As a result, firm A can always be financed (because  $\bar{r}_A > i_A$ ) and the relevant constraint becomes the date 1 or ex-post financing constraint for firm B. At date 1, the family receives  $(1 - \frac{i_A}{\bar{r}_A})\Delta + \bar{r}_A - i_A$  from its holdings in firm A in the high cash flow state and can raise up to  $(1-d)r_B$  by selling shares of firm B. Condition 12 states that, considering these two sources of finance, the family falls short of the required investment,  $i_B$ , even in the high cash flow state.

Consider now the case in which the pyramid is not ruled out, and suppose that non-family shareholders in firm A expect the pyramid to be set up only when the cash flows are high. In such a case, the maximum amount of cash flows that accrue to firm A's shareholders is given by firm A's own expected cash flows,  $\bar{r}_A$ , plus the expected non-diverted NPV of firm A's investment in firm B,  $\frac{1}{2} [(1-d)r_B - i_B]$ . If  $\bar{r}_A + \frac{1}{2} [(1-d)r_B - i_B] - i_A > 0$ , the family can set up firm A, even when shareholders expect firm B to be set up in a pyramid in the high cash flow state. The pyramid also makes it easier for the family to set up firm B. If condition 13 holds, firm B can always be set up in the high cash flow state. The reason is that the family can raise more than the necessary investment to finance firm A at date 0, and keep the rest as cash holdings in firm A.<sup>24 25</sup>

<sup>24</sup>We show in the appendix that the family can never raise enough funds to set up firm A at date 0, and set up the pyramid in both states at date 1. The reason is that, if the family does so, the maximum shareholders of firm A could get from their investment is  $\bar{r}_A - i_A + (1-d)r_B - i_B < 0$ . Thus, they will not contribute any funds to firm A.

<sup>25</sup>Raising extra cash does *not* increase the financing capacity at date 1 if the pyramid is ruled out, because a

If the two conditions of Result 11 hold, ruling out pyramids prevents the family from setting up firm B. Because firm B is a positive NPV firm, it is optimal for the family not to do so. Clearly, when investor protection is perfect (i.e.,  $d = 0$ ), these conditions do not hold. In this case, firm B can be set up even when pyramids are ruled out. A necessary condition for these conditions to hold is then that investor protection be imperfect. Moreover, an increase in  $d$  reduces the left hand side of equation 12 by more than it reduces the left hand side of equation 13. The intuition is that if pyramids are ruled out, the family must come up ex-post with the financing gap of firm B  $((1 - d)r_B - i_B)$  in order to be able to set it up. If pyramids are not ruled out, the family only needs to come up with the funds to pay for the expected financing gap associated with firm B  $((1/2)[(1 - d)r_B - i_B])$ . Thus, an increase in diversion has a smaller effect on the feasibility of firm B when pyramids are not ruled out.

So far we assumed that firm B can only be set up at date 1. The next result endogenizes the timing of this decision.

**Result 12** *Suppose the family has access to both projects at date 0. Under the conditions of result 11, the optimal investment policy is to set up firm A first, and then set up firm B in a pyramid if the cash flows of firm A are high. In other words, the pyramid is always created over time following good performance of the existing group firms.*

Result 11 shows that pyramids can only have ex-ante benefits if the sum of the pledgeable incomes of firms A and B is lower than the sum of the required investments. This follows from the first condition in result 11, because  $\bar{r}_A - i_A + (1 - d)r_B - i_B < -\left(1 - \frac{i_A}{\bar{r}_A}\right)\Delta < 0$ . In this case, the family cannot set up both firms with probability one. Furthermore, as we explained above, the pyramid will be set up when cash flows are high. Thus, pyramids are created following good performance of the existing firms in the group.

## 7 Empirical Implications

Our theory generates a number of empirical implications, which we list and discuss here. We also mention empirical and anecdotal evidence that is consistent with our theory, and suggest some additional implications that can be tested in the future.

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fraction of the cash must always be paid out to firm A's shareholders.

## **1. Family business groups should be more prevalent in countries with poor investor protection**

In the model, families have a financing advantage over potentially more efficient, but less wealthy entrepreneurs, because families can utilize the funds of the firms they already control. In low investor protection countries, this financing advantage is more important because it is more difficult to raise external finance (see result 3). Thus, the fraction of the corporate sector that ends up in business groups should be higher in such countries.

This implication is in the spirit of the arguments in Khanna and Palepu (1997, 1999), who argue that business groups arise in countries with underdeveloped markets. We believe there is no systematic evidence on this implication, but there is some scattered and anecdotal evidence. For example, Claessens, Fan and Lang (2002) show that the incidence of business groups is high in developing Asian countries, where more than 50% of the firms in their sample are affiliated with business groups.

## **2. Business groups are more likely to be organized as pyramids, especially in countries with poor investor protection**

In our model, the conditions that are conducive to the appearance of business groups are also conducive to the choice of pyramids over horizontal structures. Families choose the pyramidal structure if the security benefits associated with the new firm are low (high investment, low revenues and poor investor protection). However, it is precisely in these cases that an outside, talented entrepreneur cannot finance this new venture in the external capital market. As a result, the business group is created and the pyramid is used.

The simple version of the model (section 3) predicts that *all* business groups should be organized as pyramids, irrespective of investor protection. This stark prediction is due to the simplicity of that model. First, notice that our assumption that the entrepreneur is always the most efficient owner of firm B implies that the only reason for the family to own firm B is the family's wealth advantage over the entrepreneur. As we discuss in section 3, if there are other reasons for firm B to be set up in a business group (for example, if the family is also the most efficient owner of firm B), then the business group may be organized horizontally. However, in this case implication 1 also breaks down because the underlying rationale for the existence of the business group does not

necessarily correlate with investor protection. A more general way to state the implication of our model is thus that the *types* of business groups that appear because of poor investor protection (in our model those that appear because of financial market underdevelopment) tend to be organized as pyramids. Thus, while business groups are likely to have pyramidal structures in countries with poor investor protection, they might be organized horizontally in other countries with higher investor protection.

A second assumption that we make in section 3 is that the family can never commit not to set up the pyramid, even when the pyramid is ex-ante inefficient for the family. More generally, some degree of commitment might be possible. As we argue in section 6, however, credible commitment mechanisms are less likely to exist in countries with poor investor protection. Furthermore, the model in that section suggests that in countries with poor investor protection pyramids might not be inefficient for the family. These are additional reasons why pyramidal ownership structures are more likely to appear in poor investor protection countries.

La Porta, Lopez-de-Silanes and Shleifer (1999) show evidence that pyramids are very common in countries with poor investor protection. Another piece of evidence that business groups are typically organized as pyramids is that researchers have treated these two terms as synonymous when analyzing the role of family control in developing countries.<sup>26</sup>

### **3. When a new firm is added to a pyramidal structure, the existing non-family shareholders of the pyramid realize a negative return**

Results 1 and 7 show that pyramids can only be chosen by the family when the pledgeable income of firm B is lower than the required investment, which implies that existing shareholders of firm A realize an ex-post loss when the pyramid is formed. As we show in section 6, this does not mean that shareholders also lose in an “ex-ante” sense because the business group’s shares are priced to reflect future expropriation. We are not aware of any direct evidence for this implication.

### **4. Diversion is higher in firms placed in a pyramid, than in firms controlled directly by the family**

We show that the observed ultimate ownership is lower in pyramidal firms, and thus diversion

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<sup>26</sup>See for example the definition of a business group in Claessens, Fan and Lang (2002): “A group can be described as a corporate organization where a number of firms are linked through stock-pyramids and cross-ownership”.

is higher (result 8). It is important to emphasize that our model does not predict that, for given parameter values, the ultimate ownership is lower in a pyramid. Even though this might seem obvious because of the reduction in ultimate cash flow stake caused by the chain of control in a pyramid, it is not always the case. The reason is that there is an opposing effect: the family has more internal funds when it sets up a pyramidal structure and so, it does not need sell as many shares. Our result is that the *observed* ultimate ownership in a pyramid is lower, because the family chooses pyramids when the investment required is high, or when the revenues of the firm and investor protection are low. In these situations the family needs to sell more shares to finance the firm. As a result, ultimate ownership is lower and diversion higher than in horizontal structures. Thus, even though in our model families do not choose pyramids to facilitate diversion, pyramidal business groups are nonetheless associated with high levels of diversion because of this selection effect. This prediction is consistent with empirical findings that pyramids are associated with high expropriation (Bertrand, Mehta, and Mullanaithan 2002).

**5. It is possible to observe pyramids in which the controlling family has high cash flow stakes in member firms, and thus the separation between ownership and control is not large**

The traditional argument for pyramids considers them simply as a device to separate cash flow and votes. Thus, it cannot explain cases in which the ultimate cash flow concentration in pyramidal firms is high because in such cases pyramids do not substantially separate ownership and control (as discussed in section 2).

In contrast, our model is consistent with these cases. Even though we show that the ultimate ownership concentration in pyramids is lower than that in horizontal structures, it can still be the case that the ultimate ownership in a pyramid is high in an absolute sense (see our discussion following Result 8). In fact, depending on the parameter values, pyramidal firms can have either low or high ultimate ownership concentration (and consequently large or small separation of ownership and control). This is consistent with the evidence that in some pyramids a significant separation is achieved, while in others there is virtually no separation of ownership and control (see section 2 for references). We show in result 9 that pyramids with minor separation from ownership and control are more likely to arise in countries with poor investor protection. We are not aware of any

evidence supporting this claim.

In addition, because we identify a rationale for pyramids over and above the separation of ownership and control, our model can distinguish between pyramids and direct ownership with dual class shares even if there are no legal restrictions to the use of dual class shares. Therefore, according to the model, it should not be surprising to find that pyramids arise even if families did not exhaust the possibility of issuing dual-class shares.

#### **6. Firm value and firm performance will tend to be lower in firms that are owned through pyramids, than in unaffiliated firms or horizontal structures**

Our model predicts that projects of lower profitability will be undertaken inside pyramids (results 2 and 7). Thus, even if the pyramid does not have a direct negative effect on performance, one should observe a negative relationship between measures of firm value such as Tobin's Q and pyramidal membership because of a selection effect. There is evidence that firms in business groups organized as pyramids have lower Tobin's Q than stand-alone firms and firms organized in horizontal groups (Claessens et al. 2002, Volpin 2002) and that this undervaluation is greater if the controlling shareholder has lower ultimate ownership (Holmen and Hogfeldt, 2004). There is also evidence that the separation of ownership and control is detrimental to performance (Claessens et al 2002, Lemmons and Lins 2003, Lins 2003, Mitton 2002, and Joh 2003). Finally, Attig, Fischer and Gadhoun (2003) show that low Tobin's Q predicts membership in a pyramidal group. This last result is particularly consistent with the idea that pyramids undertake lower profitability projects.

#### **7. Firms in pyramids are larger, or they are more likely to belong to capital intensive industries**

As the required investment increases, results 2 and 7 suggest that firms are more likely to belong to pyramidal business groups. Attig, Fischer and Gadhoun (2003) find evidence consistent with this implication, using Canadian data. Claessens, Fan and Lang (2002) also find that in East Asia group firms tend to be larger than unaffiliated firms. Bianchi, Bianco and Enriques (2001) find similar evidence for Italy.

#### **8. Pyramids tend to be created dynamically, following good performance of existing family firms**

The timing of the model is exogenously specified in most of the analysis. However, result 12 partially endogenizes the timing. It shows that pyramids will not be set up at a single point in time, even when the family has access to both firms at date 0. Thus, our model predicts that pyramids evolve over time, as a function of the performance of the existing firms in the pyramid. This is consistent with the claim of Khanna and Palepu (2000) that one of the most important role of groups is to set up new firms in which the family and the member firms acquire equity stakes (p. 869). Aganin and Volpin (2003) describe the evolution of the Pesenti group in Italy, and show that this group was created by adding new subsidiaries to the firms the Pesenti family already owned. One of their conclusions is that, in Italy, business groups expand through acquisitions when they are big and have significant cash resources. Claessens, Fan and Lang (2002) find that firms with the highest separation of votes and ownership (those at the bottom of the pyramid) are younger than those with less separation (those at the top).

## 8 Concluding Remarks

In this paper we propose a theory of pyramidal business groups. The theory explains why families use a pyramidal structure to achieve control of several firms in a business group, as opposed to simply holding shares directly in all these firms (horizontal structure). The theory also explains why families control multiple firms in the first place. We show that pyramids have both a payoff and a financing advantage over horizontal structures when the amount of diversion is expected to be high (e.g., because investor protection is poor). We also show that the cases in which pyramidal structure is optimal for the family are precisely those cases in which the family can actually establish a business group. Thus, our theory explains why many business groups are organized as pyramids, specially in poor investor protection countries.

In addition, the predictions of the model are consistent with a number of empirical findings and anecdotal evidence regarding pyramids and business groups. For example, the model predicts that firm value and performance should be lower in firms that are owned through pyramids, and that pyramids are associated with high levels of cash flow diversion. However, despite this negative association between pyramids and value, we show that pyramids can be an efficient organizational structure for the family in countries in which financial markets are underdeveloped, and thus the value of retained earnings is high. In these cases, pyramids allow the family to set up firms that

could not be established in their absence.

Our theory departs from the traditional argument that pyramids are a device to separate ownership from control, and focuses instead on financing capacity. Our model can generate cases in which pyramidal firms have only minor deviations from one share-one vote. It can also explain why pyramids arise even if the family is free to deviate from one share-one vote with the use of dual class shares. Thus our theory can help understand recent empirical evidence –which is inconsistent with the traditional view– that some pyramidal firms are associated with small separation of ownership and control. The theory also provides an argument for why pyramids and dual class shares are different even when there is no legal restrictions on dual-class shares.

Our model contributes to the understanding of business groups. However, there are some questions that our simplified framework cannot address. In terms of the structure of business groups, there are three questions that need to be answered: 1) why are multiple assets in the hands of a single family, 2) why are these assets grouped into legally independent firms, and 3) what determines the choice of ownership structure of these firms (e.g., pyramidal, horizontal, or more complex structures). Even though we provide answers to questions 1 and 3, we have not dealt with question 2. An important simplifying assumption throughout the paper is that the ‘firms’ in the model are always legally independent entities. Perhaps future work can tackle this question by incorporating elements of the theory of the firm into our framework.

Future theoretical work could also explore the normative implications of the existence of business groups in further depth. We have argued that pyramidal business groups can be efficient for the family, but we do not believe that this is enough to establish efficiency from the perspective of social welfare. Previous authors have argued that family business groups can have deleterious effects on overall efficiency because they foster an inefficient allocation of corporate control through family inheritances (Morck, Strangeland and Yeung, 2000), and because they might hamper the development of external capital markets (Almeida and Wolfenzon, 2004). In addition, even though we have assumed that the level of investor protection is exogenous, this assumption might be unwarranted. In the model, wealthy families benefit from poor investor protection since it acts as a barrier to entry for new entrepreneurs. Thus, these families have incentives to lobby for regulations that impede financial and economic development (Rajan and Zingales, 2003a, 2003b). A model that blends these ideas into our current framework might be able to derive interesting normative

implications regarding pyramidal business groups.

Finally, our model contains a number of empirical predictions regarding pyramidal business groups for which we only have anecdotal evidence. Future empirical work could perhaps test these implications more systematically, and help us better understand these complex organizations.

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# Appendix

## Proof of Result 1

When  $(1-d)r > i$ , the following two inequalities hold:  $U^H > U^P$  (the family prefers the horizontal structure) and also,  $\bar{R}_H \geq i$  (the horizontal structure is feasible). Therefore the horizontal structure is chosen.

When  $(1-d)r < i$ , the following inequality holds  $U^H < U^P$  (the family prefers the pyramidal structure). In this case, we can not guarantee that the pyramidal structure is always feasible. However, because  $\bar{R}_P \geq \bar{R}_H$ , the pyramidal structure is feasible whenever the horizontal structure is. As a result, the family chooses the pyramidal structure whenever it is feasible (i.e., it is never the case that the pyramid is preferred but only the horizontal is feasible). ■

## Proof of Result 2

The condition  $(1-d)r < i$  is more likely to hold when  $r$  is low,  $i$  is high and investor protection is low (i.e.,  $d$  is high). ■

## Proof of Result 3

Whenever the entrepreneur can finance the required investment  $i$ , he will set up firm B and thus business groups will not appear. The condition required for business groups not to appear is that the income that the entrepreneur can pledge to outside investors is enough to finance the investment:

$$\bar{R}_E = (1-d)(1+t)r \geq i \quad (14)$$

Clearly, if  $r$  is high and/or  $i$  is low inequality 14 is more likely to hold. Furthermore, an increase in investor protection  $k$  will decrease diversion  $d(k)$  and facilitate entrepreneurial finance. ■

## Proof of Result 4

By result 1, a horizontal structure can only arise when  $(1-d)r \geq i$ . However, this condition implies that  $\bar{R}_E = (1-d)(1+t)r \geq i$ , and thus the entrepreneur can finance the project and the business group does not appear. ■

## Proof of Result 5

Differentiating  $U_P$  with respect to  $d$  leads to

$$\frac{\partial U_P}{\partial d} = (1 - \alpha - c_d(d, k))r$$

and

$$\frac{\partial^2 U_P}{\partial d^2} = -c_{dd}(d, k)r < 0.$$

From the first equation, the optimal amount of diversion from the viewpoint of date 1 solves  $1 - \alpha = c_d(d, k)$ , which using the definition of  $d(\cdot, \cdot)$  can be expressed as  $d(\alpha, k)$ . However, diversion is decided at date 2, when the family ultimate ownership is  $\alpha\beta_P$ . Thus, actual diversion is given by  $d(\alpha\beta_P, k)$ . Concavity implies that the closer  $d(\alpha\beta_P, k)$  is to the optimal  $d(\alpha, k)$ , the higher is  $U_P$ . Since  $d(\alpha\beta_P, k) > d(\alpha, k)$ , the family gains by reducing diversion. To accomplish this the family needs to increase  $\beta_P$  as much as possible. ■

## Proof of Result 6

By equations 7 and 10,

$$\begin{aligned} R_H(\bar{x}) &= R_P(\bar{x}) \\ \alpha c + (1 - \bar{x})(1 - d(\bar{x}))r &= c + \left(1 - \frac{\bar{x}}{\alpha}\right) (1 - d(\bar{x}))r \\ (1 - d(\bar{x}))r &= \frac{\alpha c}{\bar{x}}. \end{aligned}$$

As explained in the text, the market price and the implied price,  $\frac{\alpha c}{\bar{x}}$ , are the same. Plugging the last equality into equation 7 leads to  $(1 - d(\bar{x}))r = i$ . Now,  $U^P = \alpha c + NPV(\bar{x}) - (1 - \alpha)[(1 - d(\bar{x}))r - i] = \alpha c + NPV(\bar{x}) = U^H$ . ■

## Proof of Result 7

We first show an intermediate result. Let  $i > \bar{i} > c$  and  $x_H(i)$  and  $x_P(i)$  be the optimal ultimate ownership when the horizontal and the pyramidal structures are chosen, respectively. Then, it must be that  $x_H(i) < x_H(\bar{i})$  and  $x_P(i) < x_P(\bar{i})$ . We prove this result only for the horizontal structure (the pyramidal case is identical). Suppose

towards a contradiction that  $i > \bar{i} > c$  and  $x_H(i) \geq x_H(\bar{i})$ .<sup>27</sup> First, because  $R_H(1) = \alpha c < c < \bar{i} < i = R_H(x_H(i))$ , then by the Intermediate Value Theorem ( $R_H$  is continuous), there exists  $\hat{x} \in (x_H(i), 1)$  such that  $R_H(\hat{x}) = \bar{i}$ . Now since by assumption  $x_H(i) \geq x_H(\bar{i})$ , it must be that  $\hat{x} > x_H(\bar{i})$ . But this a contradiction because  $x_H(\bar{i})$  is defined as the highest  $x$  such that  $R_H(x) = \bar{i}$ .

We now prove the result for  $i$ . Proofs for  $r$  and  $k$  are identical. Fix a parameter vector  $(\bar{\alpha}, \bar{c}, \bar{r}, \bar{i}, \bar{k})$  such that, for these parameters,  $x_H(\bar{i}) = x_P(\bar{i}) = \bar{x}$ . We consider the parameter  $(\bar{\alpha}, \bar{c}, \bar{r}, i, \bar{k})$  with  $i > \bar{i}$ . Recall that we are considering only investment levels strictly above  $c$ , that is  $\bar{i} > c$ .

By the intermediate result shown above,  $x_H(i) < x_H(\bar{i}) = \bar{x}$  and  $x_P(i) < x_P(\bar{i}) = \bar{x}$ . Also, because  $(1 - d(x))r - i$  is increasing in  $x$  and decreasing in  $i$ , and  $(1 - d(\bar{x}))r - \bar{i} = 0$ , it must be that  $(1 - d(x_H(i)))r - i < 0$  and  $(1 - d(x_P(i)))r - i < 0$ .

Next, we show that  $x_P(i) > x_H(i)$ . We showed above that  $(1 - d(x_H(i)))r - i < 0$ . Replacing  $i = R_H(x_H(i))$  and rearranging leads to  $\frac{\alpha c}{x_H(i)} > (1 - d(x_H(i)))r$ . Now, evaluating  $R_P$  at  $x_H(i)$ :

$$\begin{aligned} R_P(x_H) &= \alpha c + \left( \frac{x_H}{\alpha} - x_H \right) \left( \frac{\alpha c}{x_H} \right) + \left( 1 - \frac{x_H}{\alpha} \right) (1 - d(x_H))r \\ &> \alpha c + \left( \frac{x_H}{\alpha} - x_H \right) (1 - d(x_H))r + \left( 1 - \frac{x_H}{\alpha} \right) (1 - d(x_H))r \\ &= \alpha c + (1 - x_H)(1 - d(x_H))r = R_H(x_H) = i, \end{aligned}$$

where the inequality follows from  $\frac{\alpha c}{x_H(i)} > (1 - d(x_H(i)))r$ . Since  $R_P(x_H(i)) > i > c = R_P(\alpha)$ , by the Intermediate Value Theorem, there must be a  $\hat{x} \in (x_H(i), \alpha)$  and  $R_P(\hat{x}) = i$ . Because  $x_P(i)$  is defined as the highest  $x$  such that  $R_P(x) = i$ , it must be that  $x_P(i) \geq \hat{x}$ , and consequently  $x_P(i) > x_H(i)$ . This step also shows that whenever the horizontal is feasible so is the pyramidal.

Finally, we compare utilities under both structures

$$U^P = \alpha c + NPV(x_P) - (1 - \alpha)[(1 - d(x_P))r - i] > \alpha c + NPV(x_H) = U^H$$

The inequality follows because 1)  $NPV(x_P) > NPV(x_H)$  since  $x_P > x_H$  and  $NPV(x)$  is increasing, and 2)  $(1 - d(x_P))r - i < 0$ . ■

#### Proof of Result 8

Fix a parameter vector  $(\bar{\alpha}, \bar{c}, \bar{r}, \bar{i}, \bar{k})$  such that, for these parameters,  $x_H = x_P = \bar{x}$ . Suppose that the different structures are chosen due to variation in  $i$  (an identical argument can be made with the other parameters). We know from Result 7 that the pyramidal (horizontal) structure is chosen for  $i > \bar{i}$  ( $i < \bar{i}$ ) and that  $x_P(i > \bar{i}) < \bar{x}$  and  $x_H(i < \bar{i}) > \bar{x}$ . That is, all pyramids we observe have ultimate ownership below  $\bar{x}$  and all horizontal structures have ultimate ownership above  $\bar{x}$ . ■

#### Proof of Result 9

Recall that  $\bar{x}$  is the ultimate ownership concentration at which both the pyramidal and the horizontal structure raise the same amount  $i$ . That is, it is defined by  $R_H(\bar{x}) = i$  and  $R_P(\bar{x}) = i$ . We can re-write this system as

$$\bar{x}(1 - d(\bar{x}, k))r = \alpha c$$

and

$$(1 - d(\bar{x}, k))r = i.$$

Because the system has two equations, for it to hold after a change in  $k$ , at least two parameters need to change. We consider the effect on  $\bar{x}$  and  $i$ . Differentiating the first equation with respect to  $k$  leads to  $r[\bar{x}_k(1 - d) - \bar{x}d_x\bar{x}_k - \bar{x}d_k] = 0$  or  $\bar{x}_k = \bar{x}d_k / (1 - d - \bar{x}d_x) < 0$  because  $1 - d \geq 0$ ,  $d_x < 0$  and  $d_k < 0$ . The solution to  $i$  can be found from the second equation. ■

#### Proof of Result 10

In order to finance firm A, the family sells a fraction  $(1 - \alpha)$  of this firm and raises  $R$ . Because  $(1 - d)r_B < i_B$ , investors at date 0 rationally expect the family to set up a pyramid. Thus, firm A does not pay a dividend at date 1, but rather invests the cash  $c = R - i_A + r_A$  of firm A to set up firm B. Firm A sells a stake of  $(1 - \beta_P)$  of firm B to the market to raise firm B's investment cost. We assume (wlog) that firm A raises just enough cash to set up firm B, that is,  $R - i_A + r_A + (1 - \beta_P)(1 - d)r_B = i_B$ . At date 2, firm A receives dividends of  $\beta_P(1 - d)r_B$ , which by the previous equation equals  $R - i_A + r_A + (1 - d)r_B - i_B$ . For each  $\alpha$ , initial investors of firm A expect to get

<sup>27</sup>Unfortunately, we cannot prove this by simply differentiating  $R_j(x)$ . As we discussed in the text, there are two opposing effects on  $R_j(x)$  when  $x$  changes (the quantity and the price effect) so it is not possible to sign  $R'_j(x)$ .

$(1-\alpha)[R-i_A+r_A+(1-d)r_B-i_B]$ . Thus, at date 0 they are willing to pay  $R = (1-\alpha)[R-i_A+r_A+(1-d)r_B-i_B]$ , or  $R = \frac{1-\alpha}{\alpha}[r_A-i_A+(1-d)r_B-i_B]$ . Note that as long as  $r_A-i_A+(1-d)r_B-i_B > 0$ , the family can raise any amount of money at date 0. In particular, the family can raise enough to fund firm A, and to make the pyramid feasible at date 1. ■

#### Proof of Result 11

Consider first the case in which pyramids are ruled out by contract. At date 0, the family sells a fraction  $1-\alpha$  of firm A to raise the set up cost,  $i_A$  (this is wlog - i.e. can show there is no benefit in raising more). Then  $(1-\alpha)\bar{r}_A = i_A$ . Let  $\alpha^* = 1 - \frac{i_A}{\bar{r}_A}$  denote the stake that the family retains in firm A.

The cash that the family holds at date 1 is  $\alpha^*r_A$  and so setting firm B in a horizontal structure is feasible if and only if:  $\alpha^*r_A + (1-d)r_B > i_B$ . In the low state this inequality becomes:

$$-\alpha^*\Delta + \bar{r}_A - i_A + (1-d)r_B > i_B$$

This inequality never holds since, by assumption,  $\bar{r}_A - i_A + (1-d)r_B < i_B$ . In the high cash flow state the horizontal structure is feasible when

$$\alpha^*\Delta + \bar{r}_A - i_A + (1-d)r_B > i_B \tag{15}$$

or in terms of exogenous parameters:

$$\left(1 - \frac{i_A}{\bar{r}_A}\right) \Delta + \bar{r}_A - i_A + (1-d)r_B > i_B.$$

Consider now the case in which pyramidal structures are not ruled out. In this case, the horizontal structure never arises since  $(1-d)r_B < i_B$ . At date 0, the family sells a fraction  $1-\alpha$  of firm A and raises  $R$ . Suppose that investors expect the family to set up firm B in a pyramid only when the cash flows of firm A are high (we will show below that it will never be an equilibrium to expect that the family sets up the pyramid when cash flows are low). In case of a low cash flow, the family pays the cash in firm A,  $c = R - i_A + \bar{r}_A - \Delta$ , as dividends and does not set the pyramid. In the case of a high cash flow, the family uses all the cash in firm A to set up firm B in a pyramid. The family sells  $1-\beta_P$  shares of firm B to raise cash to set up firm B. We assume (wlog) that the family raises just enough cash to set up firm B, that is:

$$R - i_A + \bar{r}_A + \Delta + (1-\beta_P)(1-d)r_B = i_B$$

At date 2, firm A receives dividends of  $\beta_P(1-d)r_B$ , which by the last equation equal  $R - i_A + \bar{r}_A + \Delta + (1-d)r_B - i_B$ . We now consider the relation between  $R$  and  $\alpha$ . Because  $R$  must equal the expected cash flows to date 0 investors, we have:

$$R = (1-\alpha) \left[ \frac{1}{2}(R - i_A + \bar{r}_A - \Delta) + \frac{1}{2}[R - i_A + \bar{r}_A + \Delta + (1-d)r_B - i_B] \right]$$

or

$$R = \frac{1-\alpha}{\alpha} \left[ \bar{r}_A - i_A + \frac{1}{2}[(1-d)r_B - i_B] \right]$$

As long as  $\bar{r}_A - i_A + \frac{1}{2}[(1-d)r_B - i_B] > 0$ ,  $R$  can be set to any positive value. To sustain the equilibrium  $R$  needs to be large enough so that the pyramid is feasible only when cash flows are high. That is, we need:

$$R - i_A + \bar{r}_A + \Delta + (1-d)r_B \geq i_B$$

and

$$R - i_A + \bar{r}_A - \Delta + (1-d)r_B < i_B$$

Notice that there always is an  $R$  that satisfies these two inequalities. In sum, the only condition needed to sustain this equilibrium is:

$$\bar{r}_A - i_A + \frac{1}{2}[(1-d)r_B - i_B] > 0 \tag{16}$$

Furthermore, notice that the only possible equilibrium when condition 16 holds is the one we consider in which shareholders expect the family to set up the pyramid only when cash flows are high. It might seem that  $R$  can be set sufficiently high so as to finance the pyramid in all states. However, this is not an equilibrium because if  $R$  is sufficiently high to make the pyramid feasible in both states, investors anticipate that the family will always set up firm B in a pyramid, and the expression for  $R$  changes to  $R = \frac{1-\alpha}{\alpha} [\bar{r}_A - i_A + (1-d)r_B - i_B]$ . But the right hand side is always negative. We can also see from this explanation why it is not possible to have a pyramid only when the cash flows are low. If the pyramid is feasible in the low cash flow state, it will also be feasible in the high cash flow state and the family will not be able to raise any money.

Finally, notice that when condition 16 holds, but condition 15 does not, ruling out pyramids eliminates the possibility of setting up firm B, whereas not ruling them out at least allows the family to set up firm B in the high cash flow state. There is a region of the parameter space where it is possible to have both condition 16 holding but not condition 15. This region is defined by:

$$\alpha^* \Delta + \bar{r}_A - i_A + (1-d)r_B < i_B < 2(\bar{r}_A - i_A) + (1-d)r_B.$$

Because  $2(\bar{r}_A - i_A)$  can be greater than  $\alpha^* \Delta + \bar{r}_A - i_A$ , this inequality is possible. ■

Proof of Result 12

If the pyramid is set up at date 0, the maximum pledgeable income of firms A and B is  $\bar{r}_A - i_A + (1-d)r_B - i_B < \left(1 - \frac{i_A}{\bar{r}_A}\right) \Delta + \bar{r}_A - i_A + (1-d)r_B - i_B < 0$ , where the last inequality follow from condition 12. Thus the family cannot set up the pyramid at date 0 even if firm B is available at that date. ■