Dynamic Demand for New and Used Durable Goods without Physical Depreciation: The Case of Japanese Video Games*

Masakazu Ishihara†
Rotman School of Management
University of Toronto

August 23, 2010

*This is work-in-progress. Comments are welcome.

†Ph.D. Candidate in Marketing; Address: Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, ON, CANADA M5S 3E6. Email: mishihara@rotman.utoronto.ca.

Abstract

In product categories such as books, CDs/DVDs, and video games, the competition from used goods markets has been viewed as a serious problem by producers. These products share a common feature: the physical attributes of the products hardly depreciate but the consumption value to owners depreciates quickly due to satiation. The goal of this paper is to examine the substitutability between new and used goods for this type of product, and to study the impact of used goods markets on the profitability of new goods. To achieve this goal, I develop a new structural empirical framework. In my model, the demand for new goods and the demand for and the supply of used goods are generated by a dynamic discrete choice model of forward-looking consumers that incorporates (i) consumers' new and used goods purchasing decisions, (ii) consumers' used goods selling decision, (iii) consumer expectations about future prices of new and used goods and resale values of used goods, and (iv) the depreciation of both owners' and potential buyers' consumption values.

I apply this framework to the Japanese video game software market in which used markets are mostly controlled by used-game retailers. A unique feature of my data set is that in addition to weekly sales and prices of new and used video games by game title, it contains weekly aggregate quantities sold by owners to retailers and the associated resale values, and weekly inventory levels of used games at retailers. The data on both consumers' new and used goods purchasing decisions and used goods selling decisions allow me to separately identify the depreciation rates of consumption values to both owners and potential buyers. In addition, unlike previous studies that impose the market clearing condition in used-good trading, my data set allows me to make use of the excess supply information to control for the impact of the availability of used games on consumer purchasing decisions. To estimate my dynamic model using market-level data, I develop a new Bayesian Markov chain Monte Carlo (MCMC) algorithm that is applicable to a non-stationary dynamic discrete choice model with potential price endogeneity problems.

The estimation results show that (1) the depreciation of consumption values to potential buyers from the release week to the 2nd week is about 5% in one consumer segment and 47% in another segment; and (2) the depreciation of consumption values to owners depends on product characteristics. In particular, I find that positive user ratings have a significant impact in reducing the depreciation rate. Using the parameter estimates, I examine three types of dynamic demand and supply elasticities that account for the inter-temporal substitution. They measure (a) the impact of used-game prices on new-game demand; (b) the impact of used-game inventory levels on new- and used-game demand; and (c) the impact of used-game resale values on used-game supply. The first type of elasticity is a standard measure for the substitutability, but previous works have not taken the inter-temporal substitution into account. The second and third types have never been examined in the previous literature, and they can provide important insights as to how an increase in the availability of used games affects new-game revenues. Furthermore, I quantify the impact of eliminating the used video game market on new-game revenues. I find that the elimination of the used video game market reduces the total revenue for a new game by 4% on average, if video game publishers do not adjust their pricing strategies. This is mainly because the elimination of the used market also eliminates the future selling opportunity for consumers and reduces the demand for new games, especially in the earlier periods. This suggests that the existence of the used video game market in Japan could be beneficial to video game publishers. Finally, I examine the optimality of the conventional flat-pricing strategy used by video game publishers in Japan. I simulate the model by marginally reducing the price of new games over time (as in the pricing-skimming strategy), and compare its profitability to that of the flat-pricing strategy. The result suggests that this marginal price-skimming strategy increases the total revenue for a new game by 0.5% on average.

Keywords: Used Goods Market, Dynamic Consumer Purchasing and Selling Decisions, Consumer Satiation, Bayesian Estimation of Dynamic Discrete Choice Models

1 Introduction

The existence of used goods markets has been viewed as a serious problem by producers in categories such as books, CDs/DVDs, and video games. Producers and their supporters argue that used goods retailing significantly lowers their profits and limits the development of new products. For instance, book publishers and authors have expressed their annoyance to Amazon over used books sold on its websites (Tedeschi 2004). Video game publishers in Japan attempted to eliminate used video game retailing by suing used video game retailers (Hirayama 2006). Their main argument is that products like books and video games physically depreciate negligibly, but the consumption value to owners declines quickly due to satiation. As a result, unlike products that physically depreciate more considerably (such as cars), producers of books, video games, and other digital products may face competition from used goods markets almost immediately after a new product release.

One critical factor that determines the degree of competitive pressure from the used goods markets for these products is how fast owners of a product become satiated with the product. Used goods retailers buy used goods from owners who are satiated, and sell them to consumers who have not yet consumed the product and thus still have high willingness to pay for it. Holding all else constant, the higher the *depreciation of consumption values to owners* due to satiation, the larger the supply of used goods and the lower the resale value retailers pay.[1] Therefore, the competitive pressure from used goods markets could be higher for products with a higher depreciation of consumption values to owners. The depreciation rate of consumption values to owners due to satiation typically depends on product characteristics such as genre and quality. In the case of video games, some games can be played repeatedly and do not have a well-defined ending (e.g., sports games). For these games, the depreciation rate of consumption values to owners should be low. On the other hand, some games lose

[1]In this paper, I define the resale value as the amount of money that used goods retailers pay to consumers when these retailers buy used goods from consumers. Thus, the difference between the price and the resale value of used goods is the profit margin earned by these retailers.

their value to owners after owners have reached an ending (e.g., role-playing games). This suggests that consumer selling decisions may critically depend on product characteristics. In addition, the consumption value of a product may also decline for potential buyers, especially in categories where new products are constantly being introduced and the "freshness" of the product plays a critical role in attracting consumers' attention. I call this the *depreciation of consumption values to potential buyers*. In order to fully understand the competitive pressure that used goods markets impose on new goods markets, it is crucial to incorporate these two types of consumption value depreciation into the analysis.

Despite their importance, no prior research separately incorporated these two types of consumption value depreciation when assessing the impact of used goods markets on new goods markets. This is mainly due to two challenges: (i) when physical depreciation is present, it is difficult to separately measure the decline of consumption values due to satiation from that due to physical depreciation; (ii) even if researchers can control for physical depreciation, they still need to observe both consumer purchasing and selling decisions in order to separately measure these two types of consumption value depreciation. The data sets used in the previous literature include only time-series variation in the quantities sold and price for new and used goods; however, they do not contain crucial information on the quantities sold by consumers to retailers and the associated resale values. Time-series variation in the price of new and used goods alone is not sufficient for disentangling these two types of consumption value depreciation even in the absence of physical depreciation. These challenges have limited previous studies, which have been forced to assume that consumption value depreciation is identical for owners and potential buyers (e.g., Purohit 1992, Esteban and Shum 2007, Schiraldi 2009, Engers et al. 2009, Chen et al. 2010).

The goal of this paper is to examine the substitutability of new and used goods in these types of product categories, and to study the impact of used goods markets on the profitability of new

goods. To achieve this goal, I develop a new structural empirical framework. In my model, the demand for new goods and the demand for and the supply of used goods are generated by a dynamic discrete choice model of forward-looking consumers that incorporates (i) consumers' new and used goods purchasing decisions, (ii) consumers' used goods selling decision, (iii) consumer expectations about future prices of new and used goods and resale values of used goods, and (iv) the depreciation of both owners' and potential buyers' consumption values. To the best of my knowledge, this is the first dynamic model of demand that incorporates these four important features of consumer decision making in durable goods markets. In my model, the expected present discounted value of future payoffs from purchasing a product is determined by the dynamic consumer selling decision problem, which depends on the depreciation rate of consumption values to owners and future resale value. Furthermore, although the previous literature has identified consumer expectations about future prices as a critical element in dynamic durable goods purchase decisions (e.g., Nair 2007, Gordon 2009), this is the first study that uses data on resale values and studies the role of consumer expectations about future resale value in a dynamic durable good purchase decision.

I have assembled a novel data set on the Japanese video game market that allows me to estimate the proposed model. In Japan, used video game markets are mostly controlled by used-game retailers. For 41 video games (software), I have collected weekly data on the sales and prices of new and used games, the quantities sold by consumers to retailers and the associated resale values, and the inventory levels of used games carried by retailers. The data on both consumers' new and used goods purchasing decisions and used goods selling decisions allow me to separately identify the depreciation rates of consumption values to owners and potential buyers. In addition, unlike previous studies that impose the market clearing condition on used durable goods, my data set allows me to make use of the excess supply information to control for the impact of the availability of used games on consumer purchasing decisions.

To estimate my model, I extend the Bayesian Markov chain Monte Carlo (MCMC) algorithm proposed by Imai, Jain and Ching (2009a) (IJC algorithm) to a non-stationary dynamic model. Similar to the IJC algorithm, this new algorithm solves and estimates the model simultaneously. In conventional approaches to estimating a finite-horizon model, value functions need to be computed at all or a subset of pre-determined grid points in all periods by backward induction. However, when a model has multiple continuous state variables, computing value functions at a large enough number of grid points can be computationally very demanding. My new algorithm alleviates the computational burden by partially solving value functions at only one randomly drawn state in each period, storing them, and approximating the expected future values by the weighted average of those partially solved value functions evaluated at different states in past iterations. I then combine the proposed algorithm with the pseudo-policy function approach (Ching 2009; 2010) to control for potential endogeneity problems. The method I propose augments the unobserved shocks based on the joint-likelihood of the demand-side model and the pseudo-detailing policy functions. Thus, unlike the standard GMM approach (Berry et al. 1995), there is no inner loop for inverting the market share to recover the mean utility level. Also, unlike the simulated maximum likelihood method, I do not need to integrate out the unobserved shocks during the estimation.

The estimation results show that (1) the depreciation of consumption values to potential buyers from the release week to the 2nd week is about 5% in one consumer segment and 47% in another segment; and (2) the depreciation rate of consumption values to owners depends on product characteristics. In particular, I find that positive user ratings have a significant impact in reducing the depreciation rate. Using the parameter estimates, I examine three types of dynamic demand and supply elasticities that account for the inter-temporal substitution. They measure (a) the impact of used-game prices on new-game demand; (b) the impact of used-game inventory levels on new- and used-game demand; and (c) the impact of used-game resale values on used-game supply. The

first type of elasticity is a standard measure for the substitutability of new and used goods; however, previous works have not taken the inter-temporal substitution into account. An increase in used-game prices today may induce some consumers to delay purchase, which affects the new- and used-game demand in the future. I take this factor into account by computing the percentage change in the expected present discounted value (PDV) of new-game revenues with respect to one percent increase in used-game prices. The second and third types have never been examined in the previous literature, and they can provide important insights as to how an increase in the availability of used games affects new-game revenues.

Furthermore, I quantify the impact of eliminating the used video game market on new-game revenues. I find that the elimination of the used video game market reduces the total revenue for a new game by 4%, on average, if video game publishers do not adjust their pricing strategies. This is mainly because the elimination of the used market also eliminates the future selling opportunity for consumers, and reduces the demand for new games, especially in the earlier periods. This suggests that the existence of the used video game market in Japan could be beneficial to video game publishers.

Publishers in the Japanese video game industry have historically used a flat-pricing strategy for new games for at least one year after a new game release and have never experimented with a price-skimming strategy. This is in contrast to the US video game industry where price-skimming is a common strategy. It is not clear whether video game publishers in Japan are setting prices optimally. To examine the optimality of flat-pricing strategies, I simulate the total revenues for new games by marginally reducing the price of new games over time and compare its profitability to the one from the conventional flat-pricing strategy. I find that marginally reducing the price of a new game increases its total revenue by 0.5% on average. Although one would need a dynamic equilibrium model to fully account for the effect of competitive pressure from used games on optimal pricing strategies for

new games, my result provides some insights about the optimality of flat-pricing strategies in the Japanese video game market.

The rest of the paper is organized as follows. Section 2 reviews the previous literature. Section 3 describes the Japanese video game data used in this paper and presents some empirical regularities that have not been explored in the previous literature. Section 4 describes the dynamic discrete choice model of consumer purchasing and selling decisions. Section 5 describes the estimation strategy and identification. In Section 6, I discuss the parameter estimates and the counterfactual experiments. Section 7 concludes.

2 Literature Review

My research is closely related to the literature on the dynamic purchase decisions in consumer durable goods markets (e.g., Song and Chintagunta 2003, Chevalier and Goolsbee 2009, Gordon 2009, Gowrisankaran and Rysman 2009). In particular, Nair (2007) studies the intertemporal price discrimination in the US video game industry, and examine the role of consumer price expectation. However, during his sample period, the U.S. used video game market was very small. Thus, he did not account for the impact of used video game markets. Esteban and Shum (2007) build a dynamic equilibrium model of durable goods oligopoly and study the impact of durability and used car markets on equilibrium car manufacturers' behavior. Chen et al. (2010) extend Esteban and Shum (2007) and allow for transaction costs. Schiraldi (2009) estimates a dynamic discrete choice model of automobile replacement decisions, and studies the impact of scrappage subsidies. However, these three papers study car markets in which durability is measured by quality deterioration, and consumer selling decision is mainly driven by the replacement decision.

My research is also closely related to Ghose et al. (2006) who study the cannibalization of new books from used books and find that used books are poor substitute for new books. However, they

do not account for consumers' forward-looking behavior for purchasing and selling decisions. Finally, my research is also related to studies that investigate the relationship between the variations in used goods prices and the depreciation of durable goods. Purohit (1992) examines the depreciation of used cars, measured by used car prices, in response to feature changes incorporated in new model cars in primary markets. Engers et al. (2009) study how much variation in used car prices can be explained by the net flow of benefits to car owners. They provide evidence that the net flow of benefits, which is similar to the consumption value to owners in my research, can explain used car prices. However, these two papers do not study the dynamic consumer purchase decision on new and used goods.

Finally, there is a large theoretical literature on a variety of marketing practices in new and used durable goods markets. These include leasing contracts (e.g., Desai and Purohit 1998, Desai and Purohit 1999), channel coordination (Desai et al. 2004, Shulman and Coughlan 2007), trade-ins (Rao et al. 2009), and retail versus P2P used goods markets (Yin et al. 2010). One common feature of these theoretical models is that the used goods markets clear in every period. In my proposed dynamic structural demand model, I allow for the potential impact of the excess supply of used goods on consumer demand. Although this is done in a reduced-form fashion, it sheds some lights on the possible theoretical explanations for the patterns of the excess supply generated in equilibrium.

3 Data

3.1 Japanese video game industry

Since mid-80s, the Japanese video game market has grown rapidly. The size of the industry in 2009 has reached $5.5 billion on a revenue basis (including sales of hardware, software, other equipments). This is about three times larger than the theatrical movie revenue in Japan, and it has become one of the most important sectors in the Japanese entertainment industry.[2] The existence of the used

[2]In fact, the size of the video game industry in the US has grown to $20.2 billion, which is now higher than the theatrical movie revenues.

video good market has been a serious issue for video game publishers since 90s. In 2009, the sales of used software alone amounts to $1.0 billion on a revenue basis. One reason for the large used video game market in Japan could be that unlike North America, video game renting by third-party companies is prohibited by law in Japan.[3] Another reason argued by Hirayama (2006) is the flat-pricing strategy commonly adopted by video game publishers - the price of new games is maintained at the initial level at least one year after the release.[4] However, it can also be argued that the used market has induced publishers to adopt the flat-pricing strategy. Liang (1999) uses a theoretical model to show that when used goods markets are present, durable goods monopolists may be able to credibly commit to a high price (avoiding the Coarse Conjecture). In this paper, I will conduct a policy experiment to shed some light on whether the flat-pricing puzzle in the Japanese video game market is optimal.[5]

3.2 Japanese video game data

I have collected a data set of 41 video games that were released in Japan between 2003 and 2008. The data come from several sources. For each video game, weekly aggregate sales of new copies and its manufacturer suggested retail price (MSRP) are obtained from the weekly top 30 ranking published in Weekly Famitsu Magazine.[6] The average number of weeks observed across games is 19 weeks. In Japan, the sales of new copies sharply declines after the release week (see Figure 1). In my data set, the median percentage of new game copies sold in the release week (relative to the total sales after one year) is 54%, and the median percentage of new game copies sold within the first month (4 weeks) after release is 82%. Thus, the sales of new copies is concentrated within the

[3]In principle, video game publishers can run the rental business for their own video games. However, only one publisher attempted to operate it in the history and did not succeed and existed.

[4]Note that in Japan, resale price maintenance is illegal for video games while it is still legal for books, magazines, newspapers and music.

[5]Recently, Cho and Rust (2010) investigate the flat rental pricing strategy used by car rental companies, and conclude that actual rental markets are not fully competitive and firms many be behaving suboptimally.

[6]Weekly Famitsu Magazine is a major weekly video game magazine in Japan published by Enterbrain, Inc.

first month in Japan. In addition to the data from the primary market, I collected weekly aggregate trading volumes (both purchasing and selling) and the associated weekly average retail prices and resale values in the used market by game title. These are collected from the Annual Video Game Industry Report published by Media Create Co., Ltd. The average number of weeks observed across games is 36 weeks.

I also collected various video game characteristics from Weekly Famitsu Magazine, including critic rating, user rating, handheld game dummy, sequel game dummy, story-based game dummy,[7] and multi-player game dummy. Finally, the size of market for a video game is measured by the installed base of the platform in which the video game was released. The platforms of the 41 games include four consoles (PlayStation 2, PlayStation 3, Nintendo GameCube, Nintendo Wii) and three handhelds (GameBoyAdvance, Nintendo DS, PlayStation Portable). I collected the weekly sales of all seven platforms above from their release week to calculate the cumulative sales.

Table 1 shows the summary statistics. The average price of used copies across games and time is about two-thirds of the price of a new copy. Also, it can be seen that the profit-margin for used game retailers is large. The average observed sales of used copies is about 10% of that of new copies during the sample period. It should be noted that the sales of used copies typically lasts longer than that of new copies, and my data set covers only up to one year. Thus, the number is expected to be larger if I observe a longer time-series data.

3.3 Some empirical regularities

In this section, I will briefly describe three empirical regularities that have not been documented in the previous studies on used goods markets: (i) quantities demanded and supplied for used goods over time, (ii) inventory level of used goods over time, and (iii) the price of used goods and the resale

[7]Story-based games (e.g., role-playing games) have an ending of the story, while non story-based games (e.g., sports games) have no clear definition for ending.

value.

Figure 2 plots the average quantities demanded and supplied for used goods as well as the average inventory level of used goods over 20 weeks. First, both quantities demanded and supplied for used goods sharply increase in the first few weeks after the opening of used goods markets (second week of game release), reach their peaks, and gradually decrease afterwards. The initial increase is probably because it takes a few weeks for owners of a game to become satiated with their games. As the quantity supplied for used copies increases, the sales of used copies also follows. Second, on average, the inventory level of used games carried by retailers grows in the first 17 weeks, and declines. It is clear that in the Japanese video game market, the used market does not clear in every period. Therefore, unlike previous studies which assume the used goods market clearing in each period, I will make use of this excess supply information when estimating the dynamic demand model.

Figure 3 plots the average price and resale value of used goods over the first 20 weeks. They both gradually decrease over time, and the resale value decreases slightly faster. This is consistent with my assumptions that consumption values to both potential buyers and owners depreciate over time, and their depreciation rates could be different.

4 Model

In this section, I describe the dynamic discrete choice model of consumer purchasing and selling decisions for video games.[8] I assume that consumers make purchasing and selling decisions separately for each game denoted by g.[9] Let $i = 1, 2, \ldots, I$ index discrete consumer type, and let t be the time index ($t = 1$ indicates the release period). In period $t = 1$, no consumers own game g. Thus,

[8]Note that the model can be applied to other markets with similar features as well.

[9]I do not explicitly model the competition among different games since my focus in this paper is the competition between new and used copies of the same game. I control for the impact of the availability of other games on the purchase decision of game g by including the cumulative number of other newly introduced games since game g's release. Note that Nair (2007) finds evidence that the substitutability between two different video games is very low in the US market, and thus does not model the competition among differen games.

consumers' decision problem is to decide whether to buy a new good or not to buy. In period $t > 1$, consumers who have not purchased the game up to $t-1$ observe the prices of new and used copies, the resale value, and the inventory level of used copies at retailers (all the information is available for consumers in the public domain), and decide whether to purchase a new or used good, or not to purchase anything. Let $j = 0, 1, 2$ denote no purchase option, new good purchase, and used good purchase, respectively. If consumers have already bought game g prior to time t and have not sold it yet, then they decide whether or not to keep the game given the resale value in period t. Let $k = 0, 1$ denote keeping and selling options, respectively. If consumers sell their game, they will exit the market.

The state space of the consumer decision model consists of the following variables: (1) price of new and used goods (p_1, p_2), (2) resale value (r), (3) inventory level of used copies at retailers (Y), which controls for the impact of the availability of used copies on consumer purchasing decisions, (4) time since release (t), which characterizes the single-period consumption value to potential buyers, (5) time since purchase (τ), which affects the single-period consumption value to owners, (6) unobserved shocks for the demand for new and used copies (ξ_1, ξ_2), (7) unobserved shocks for the supply of used copies (ξ_s), (8) cumulative number of newly introduced games since the release of the focal game (C). As I will describe later, (1), (6), and (8) appear only in the consumer purchasing decision problem, (5) and (7) appear only in the consumer selling decision problem, and (2)-(4) appear in both consumer purchasing and selling decision problems.

I will first describe the single-period utility functions for purchasing and selling decisions, and then move to the description of the value functions.

4.1 Single-period utility functions

In each period, consumers derive a value from owning game g (consumption values). I assume that once consumers purchase a good, they will derive the same consumption value regardless of whether it is new or used. This is to capture the idea that the goods considered in this research have no physical depreciation. However, the decision to purchase used goods may be influenced by other factors such as the availability of used goods, psychological costs for using pre-owned goods,[10] etc. Let $v_i^g(t,\tau)$ be type-i consumer's single-period consumption value of owning game g at time t if he has owned game g for τ periods prior to time t. If he hasn't purchased game g, he will receive $v_i^g(t,0)$ if he purchases it at time t.

Suppose that a consumer has not bought game g up to time t. Type-i consumer's single-period utility for purchasing decisions at time t is given by:

$$u_{ijt}^g = \begin{cases} v_i^g(t,0) - \alpha p_{1t}^g + \xi_{1t}^g + \zeta_{i1t}^g + \epsilon_{i1t}^g & \text{if purchasing a new copy } (j=1) \\ v_i^g(t,0) - \alpha p_{2t}^g - l_Y(Y_t^g) + \xi_{2t}^g + \zeta_{i1t}^g + \epsilon_{i2t}^g & \text{if purchasing a used copy } (j=2) \\ l_C(C_t^g) + \zeta_{i0t}^g + \epsilon_{i0t}^g & \text{if no purchase } (j=0), \end{cases}$$

where p_{1t}^g and p_{2t}^g are the prices of new and used copies of game g at time t, respectively, and α is the price-sensitivity; ξ_{1t}^g and ξ_{2t}^g are iid unobserved demand shocks to new and used copies, respectively.[11] I assume they are normally distributed with zero mean and the standard deviation σ_{ξ_j}; Y_t^g is the inventory level of used copies for game g at retailers in period t; $l_Y(Y_t^g)$ is the one-time transaction cost that consumers incur when purchasing a used good (search costs, psychological costs for pre-owned games, etc.). I specify $l_Y(Y_t^g) = \lambda_0 + \lambda_1 \ln(1+Y_t^g)$ to capture the idea that search costs may depend on the availability of used copies by this term; C_t^g is the cumulative number of newly introduced games at time t since the introduction of game g (including the games released in the same week as game g); $l_C(C_t^g)$ captures the competitive effect from other newly introduced games

[10] My assumption here is that once consumers overcome this psychological costs when they make a purchase decision, then the consumption value they receive in subsequent periods is not affected by the psychological costs

[11] Unobserved demand shocks may capture variations in publicity about game g, sales of game g-related products (e.g., movies, cartoons, animations, etc.), economic recession, and so on.

and is specified as $l_C(C_t^g) = \pi_0 + \pi_1 \ln(C_t^g)$; $(\zeta_{iht}^g + \epsilon_{ijt}^g)$ represents an idiosyncratic error (h indexes nest), and I assume it is iid extreme value distributed.

Next, consider consumers' selling decision. Suppose that a consumer has bought game g and kept it for τ periods prior to time t. Type-i consumer's single-period utility for selling decisions at time t is given by

$$w_{ikt}^g(\tau) = \begin{cases} \alpha r_t^g - \mu + \xi_{st}^g + e_{i1t}^g & \text{if selling } (k=1) \\ v_i^g(t,\tau) + e_{i0t}^g & \text{if keeping the game } (k=0) \end{cases}$$

where r_t^g is the resale value of game g at time t; μ captures any additional cost of selling (transaction costs, endowment effects, etc.); ξ_{st}^g is an iid unobserved shock to owners for selling decisions at time t.[12] I assume it is normally distributed with zero mean and the standard deviation σ_{ξ_s}; e_{ikt}^g is an idiosyncratic error, and I assume it is iid extreme value distributed.

For the single-period consumption value, $v_i^g(t,\tau)$, I will assume the following parsimonious functional form. In the release period, I set $v_i^g(1,0) = \gamma_g \ \forall i$, where γ_g is the game-specific constant. To capture the depreciation of consumption values to potential buyers due to the aging of a game, I allow $v_i^g(t,0)$ to decay as a function of t.[13] Specifically, I model the depreciation rate as: $v_i^g(t+1,0) = (1-\varphi_{it})v_i^g(t,0)$ where $\varphi_{it} = \frac{\exp(\phi_i+\phi_t \ln(t))}{1+\exp(\phi_i+\phi_t \ln(t))}$. I capture the depreciation of consumption values to owners due to satiation by modeling the depreciation rate as: $v_i^g(t+1,\tau+1) = (1-\kappa_{i\tau}^g)v_i^g(t,\tau)$ where $\kappa_{i\tau}^g = \frac{\exp(X'_{g\tau}\delta_i)}{1+\exp(X'_{g\tau}\delta_i)}$ and $X_{g\tau}$ includes observed product characteristics of game g and the duration of ownership (τ).[14]

4.2 Value functions

Since the dynamic consumer selling decision is nested within the dynamic consumer purchasing decision through the expected future payoff, I start off by describing the dynamic consumer selling

[12] The unobserved shock to owners may capture variations in the economic situation, sales of newly introduced games and their related products, etc.

[13] I do not allow for the appreciation of consumption values, given that the length of my sample is at most one year.

[14] After trying several functional form, this parsimonious specification has so far fit the data best.

decision, and then describe the dynamic purchasing decision. To simplify the notation, I will drop g superscript and t subscript. Also, let $\xi_d = (\xi_1, \xi_2)$ be the unobserved demand shocks (as opposed to ξ_s, which is the unobserved shocks for selling decisions). Also, let β be the discount factor common across consumer types.

Let $W_i(r, Y, \xi_s, t, \tau)$ be the value function of the consumer selling decision problem for type-i consumer. Note that other state variables (p_1, p_2, C, ξ_d) will not enter here. The inventory level, Y, is included since it could affect the distribution of the future resale value. The Bellman equation is given by

$$W_i(r, Y, \xi_s, t, \tau) = E_e \max\{W_{i0}(r, Y, \xi_s, t, \tau) + e_{i0t}, W_{i1}(r, Y, \xi_s, t, \tau) + e_{i1t}\},$$

where W_{ik}'s are type-i consumer's alternative-specific value functions given by

$$W_{ik}(r, Y, \xi_s, t, \tau) = \begin{cases} \alpha r - \mu + \xi_s & \text{selling,} \\ v_i(t, \tau) + \beta E[W_i(r', Y', \xi_s', t+1, \tau+1) | (r, Y, \xi_s, t, \tau)] & \text{keeping.} \end{cases}$$

The expectation in $E[W_i(r', Y', \xi_s', t+1, \tau+1)|(r, Y, t, \tau)]$ is taken with respect to the future resale value (r'), inventory level (Y'), and unobserved shock for selling decision (ξ_s').

Assuming that e_{it} follows the iid extreme value distribution, the probability of selling the game by type-i consumer at (r, Y, ξ_s, t, τ) is given by

$$\Pr(k = 1 | r, Y, \xi_s, t, \tau; i) = \frac{\exp(W_{i1}(r, Y, \xi_s, t, \tau))}{\sum_{k'=0}^{1} \exp(W_{ik'}(r, Y, \xi_s, t, \tau))}.$$

Next, consider the consumer purchasing decision. Let $V_i(p_1, p_2, r, Y, C, \xi_d, t)$ be the value function for type-i consumer who have not bought the game until time t. The Bellman equation is given by

$$\begin{aligned} V_i(p_1, p_2, r, Y, C, \xi_d, t) = E_{\zeta,\epsilon} \max\{&V_{i0}(p_1, p_2, r, Y, C, \xi_d, t) + \zeta_{i0t} + \epsilon_{i0t}, \\ &V_{i1}(p_1, p_2, r, Y, C, \xi_d, t) + \zeta_{i1t} + \epsilon_{i1t}, V_{i2}(p_1, p_2, r, Y, C, \xi_d, t) + \zeta_{i1t} + \epsilon_{i2t}\}, \end{aligned}$$

where V_{ij}'s are type-i consumer's alternative-specific value functions given by

$$V_{ij}(p_1, p_2, r, Y, C, \xi_d, t)$$
$$= \begin{cases} v_i(t,0) - \alpha p_1 + \xi_1 + \beta E[W_i(r', Y', \xi_s', t+1, 1)|(r, Y, \xi_s, t, 0)] & \text{new copy,} \\ v_i(t,0) - \alpha p_2 + \xi_2 - l_Y(Y) + \beta E[W_i(r', Y', xi_s', t+1, 1)|(r, Y, \xi_s, t, 0)] & \text{used copy,} \\ l_C(C) + \beta E[V_i(p_1', p_2', r', Y', C', \xi_d', t+1)|(p_1, p_2, r, Y, C, \xi_d, t)] & \text{no purchase,} \end{cases}$$

where the expectation in $E[V_i(p_1', p_2', r', Y', C', \xi_d', t+1)|(p_1, p_2, r, Y, C, \xi_d, t)]$ is taken with respect to the future prices of new and used goods (p_1', p_2'), resale value (r'), inventory level (Y'), and the unobserved shocks for the purchasing decision (ξ_d'). I assume that consumers have rational expectations about C'. More specifically, I assume that $E[C'|C] = C + N_C$ where N_C is a stationary number of newly introduced games in each period.

Assuming that ϵ_{it} follows the iid extreme value distribution, the choice probability by type-i consumers at $(p_1, p_2, r, Y, C, \xi_d, t)$ is given by

$$\Pr(j|p_1, p_2, r, Y, C, \xi_d, t; i) = \frac{\exp(V_{ij}(p_1, p_2, r, Y, C, \xi_d, t))}{\sum_{j'=0}^{2} \exp(V_{ij'}(p_1, p_2, r, Y, C, \xi_d, t))}.$$

Given a finite time horizon, the value functions for both purchasing and selling decisions can be computed by backward inductions from the terminal period $t = T$.

4.3 Evolution of consumer type and aggregate sales

Let ψ_i be the population proportion of type-i consumers and $\sum_{i=1}^{I} \psi_i = 1$. In order to derive the aggregate market share of new and used goods, and aggregate volume of used goods supplied by owners, I need to derive the evolution of the size of each consumer type. Let M_{it}^d be the size of type-i consumers who have not bought the video game. It evolves according to

$$M_{it+1}^d = M_{it}^d (1 - \sum_{j=1}^{2} \Pr(j|p_1, p_2, r, Y, C, \xi_d, t; i)) + N_{it+1},$$

where N_{it+1} is the size of new type-i consumers who enter the market at time $t+1$.[15] I assume that the proportion of new type-i consumers follows the population proportion, ψ_i.

[15] In my application, this corresponds to the sales of the platform for the game at time $t+1$.

Next, let $M^s_{it}(\tau)$ be the size of type-i consumers who have bought and owned the video game for τ periods at time t. It evolves according to

$$M^s_{it+1}(\tau) = \begin{cases} M^d_{it} \sum_{j=1}^{2} \Pr(j|p_1, p_2, r, Y, \xi_d, t; i) & \text{if } \tau = 1, \\ M^s_{it}(\tau - 1) \cdot \Pr(k = 0|r, Y, \xi_s, t, \tau - 1; i)) & \text{if } \tau > 1. \end{cases}$$

The aggregate sales across consumer types is then given by

$$Q_j(p_1, p_2, r, Y, C, \xi_d, t) = \sum_{i=1}^{I} M^d_{it} \Pr(j|p_1, p_2, r, Y, C, \xi_d, t; i) + \varepsilon_{jt}, \tag{1}$$

where $j = 1$ is new goods, and $j = 2$ is used goods, and ε_{jt} represents a measurement error. The aggregate quantity sold by consumers across consumer types and different durations of ownerships is given by

$$Q_s(r, Y, \xi_s, t) = \sum_{i=1}^{I} \sum_{\tau=1}^{t-1} M^s_{it}(\tau) \Pr(k = 1|r, Y, \xi_s, t, \tau; i) + \varepsilon_{st}, \tag{2}$$

where ε_{st} represents a measurement error.

5 Estimation Strategy

The estimation of consumer preference parameters is carried out in two steps. In the first step, I recover the processes of used game prices, resale values, and inventory levels from the data.[16] These processes will then be used to form consumers' expectation about future price of used goods, resale value, and inventory level in the second step demand estimation, assuming that consumers have rational expectations. I model the processes of the price of used goods and the resale value to be a function of own lagged value, the lagged inventory level, and game characteristics, except for $t = 2$ where I assume that the initial price of used copies and resale value are a function of the price of new copies and game characteristics. I model the processes of the inventory level to be a function of its lagged value and game characteristics. The estimation results based on all 41 video games are reported in Tables 2 and 3.

[16] I assume that the price of new goods is exogenous and consumers expect that it remains constant over time.

In the second step, I estimate the dynamic discrete choice model with a finite time horizon.[17] Notice that the price of used goods and resale value may be correlated with unobserved shocks. I propose a new estimation algorithm that extends the Bayesian MCMC algorithm in Imai et al. (2009a) (IJC algorithm) to a finite-horizon model, and combines it with the pseudo-policy function approach in Ching (2009; 2010) to control for the potential endogeneity problems. I will first describe the modified IJC algorithm.

5.1 Modified IJC algorithm

The IJC algorithm uses the Metropolis-Hastings algorithm to draw a sequence of parameter vectors from their posterior distributions. During the MCMC iterations, instead of fully solving for the value functions at each draw of parameter vectors as proposed in the nested fixed point algorithm (Rust 1987), the IJC algorithm partially solves for the value functions at each draw of parameter vectors (at the minimum, apply the Bellman operator only once), stores those partially solved value functions (they call those value functions *pseudo*-value functions), and uses them to nonparametrically approximate the expected value functions at the current trial parameter vector.[18] Imai et al. (2009b) show that as the MCMC iterations and the number of past pseudo-value functions for approximating the expected value functions increase, pseudo-value functions will converge to the true value functions, and the posterior parameter draws based on the pseudo-value functions will also converge to the true posterior distribution.[19]

One issue in applying the IJC algorithm in my framework is that the dynamic programming (DP) problem is non-stationary. However, the original IJC algorithm in Imai et al. (2009a) applies to stationary DP problems. Their algorithm is essentially an extension of the contraction mapping

[17] In order to model the consumption value as a function of time since release and time since purchase, I use a finite-horizon model.

[18] Ching et al. (2010) provide a step-by-step guide for implementing the IJC algorithm.

[19] Imai et al. (2009b) shows that their algorithm converges under Gaussian kernel. Norets (2009) further shows that the IJC algorithm converges under nearest neighborhood kernel.

procedure for solving the stationary DP problem. In general, the computational advantage of using IJC to estimate a finite horizon DP problem may be limited. However, if the dynamic model has multiple continuous state variables, I argue that their idea for estimating DP models with continuous state variables (see Section 3.2 of Imai et al. 2009a) can be extended to help reduce the computational burden of integrating the continuation value even for finite-horizon non-stationary dynamic models. The main idea behind their algorithm for continuous state variables is to compute pseudo-value functions at one randomly drawn state in each iteration and store them. The set of past pseudo-value functions used in approximating the expected future values will then be evaluated at different state points. Thus, one can simply adjust the weight given to each of the past pseudo-value function by the transition density from the current state to the state at which the past pseudo-value function is evaluated. In my algorithm, the main differences from the original IJC algorithm are (1) pseudo-value functions are computed and stored for each time period, (2) in each MCMC iteration and in each time period, pseudo-value functions are computed only at one randomly drawn vector of continuous state variables, and (3) expected future values at time t are approximated using the set of pseudo-value functions at time $t+1$. Unlike conventional approaches, in which value functions need to be computed at all or a subset of pre-determined grid points in all periods (e.g., Rust 1997), the new algorithm computes pseudo-value functions at only one randomly drawn state point in each period and the integration of the continuation value with respect to continuous state variables can simply be done by the weighted average of past pseudo-value functions. Thus, it has the potential to reduce the computational burden. One drawback of my proposed algorithm is that it is very memory-intensive: past-value functions need to be stored period by period. However, this should not pose a serious problem as the cost of memory has been declining very quickly. I describe the details of the estimation procedure in Appendix A.1.

5.2 Pseudo-policy function approach

To control for the potential endogeneity problems of the price and resale value of used games, I follow the pseudo-policy function approach proposed by Ching (2009; 2010), which proposes to approximate the pricing policy functions as a polynomial of observed and unobserved state variables of the equilibrium model.[20] Note that I assume that the price of new games is exogenous. In my model, the state space of the equilibrium model includes unobserved shocks $(\xi_{1t}, \xi_{2t}, \xi_{st})$, consumption values $(v_i(t,\tau))$, inventory level (Y_t), cumulative number of newly introduced games (C_t), the size of potential buyers for each consumer type (M_{it}^d), and the size of owners for each consumer type and each duration of ownership $(M_{it}^s(\tau))$.

After experimenting several functional forms, instead of using high order polynomials, I decided to use the following specification for the price of used goods (for $t \geq 2$):

$$\begin{aligned} \ln p_{2t} &= \omega_{10} + \omega_{11}\frac{1}{I}\sum_{i=1}^{I} v_i(t,0) + \omega_{12}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} v_i(t,\tau) + \omega_{13}(\xi_{2t} - \xi_{1t}) \\ &\quad + \omega_{14}\xi_{st} + \omega_{15}\frac{1}{I}\sum_{i=1}^{I} M_{it}^d + \omega_{16}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} M_{it}^s(\tau) + \omega_{17}Y_t + \nu_t^p, \end{aligned} \tag{3}$$

where ν_t^p is the prediction error. Also, the pseudo-policy function for the resale value is specified as (for $t \geq 2$):

$$\begin{aligned} \ln r_t &= \omega_{20} + \omega_{21}\frac{1}{I}\sum_{i=1}^{I} v_i(t,0) + \omega_{22}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} v_i(t,\tau) + \omega_{23}(\xi_{2t} - \xi_{1t}) \\ &\quad + \omega_{24}\xi_{st} + \omega_{25}\frac{1}{I}\sum_{i=1}^{I} M_{it}^d + \omega_{26}\frac{1}{I(t-1)}\sum_{i=1}^{I}\sum_{\tau=1}^{t-1} M_{it}^s(\tau) + \omega_{27}Y_t + \nu_t^r, \end{aligned} \tag{4}$$

where ν_t^r is the prediction error.

Note that Y_t in the two equations play a role of an instrument. Y_t is the inventory level of used games at retailers at the beginning of period t. Thus, it is uncorrelated with ξ_t's. However, it is

[20] This approach can also be applied to control for the potential endogeneity of advertising/detailing (e.g., Ching and Ishihara 2010)

reasonable to think that the price and resale value of used games at time t depend on the inventory level. Similarly, C_t could be included as an instrument.[21] C_t is the cumulative number of newly introduced games since the release of the focal game. The introduction timing of games is rarely postponed once the release week has been reached. This is because copies of games are already manufactured before the release week. Thus, I expect C_t to be uncorrelated with ξ_t's. However, C_t would likely affect p_2 and r. On one hand, C_t could influence p_2 because it affects the demand for used copies of the focal game. On the other hand, C_t could affect r because owners of the focal game may be attracted to newly introduced games and choose to sell the focal game, which affect the supply of used copies of the focal game. Inclusion of these two exogenous observed state variables should help reduce the reliance of using functional form restrictions for identification (Ching 2010).

Assuming measurement errors in the sales of new and used copies as well as the quantities sold by consumers to retailers, and prediction errors in the pseudo-pricing policy functions, I derive the joint likelihood of the demand-side model and the pseudo-policy functions. In the IJC algorithm described in the previous subsection, the joint likelihood will be used to compute the acceptance probabilities in the Metropolis-Hastings algorithm. Appendix A.2 describes the construction of the likelihood function.

5.3 Remark

Note that if consumers observe all the state variables and understand how the equilibrium prices are generated, then the consistent way of estimating the model requires me to use the pseudo-policy functions to form the consumers' future price expectations as well. I decided not to take this approach for the following two reasons. First, it is unlikely that consumers are aware of all the state variables of the equilibrium model. In particular, it is difficult for consumers to obtain information about

[21] At this point, I have not included C_t in the estimation.

the size of potential buyers and owners for each consumer type and each duration of ownership at the time consumers make a purchasing or selling decision. Under such a situation, consumers may use a simple Markov process to forecast future prices and resale value (Erdem et al. 2003, Février and Wilner 2009). Second, if one uses the pseudo-policy functions to form the consumer price expectation, one needs to specify the state space for the dynamic consumer purchasing and selling decision model to be that of the equilibrium model (Nair 2007). Given that there are many more continuous state variables in the equilibrium model, including them will increase the computational burden of estimating the dynamic consumer purchasing and selling decision model dramatically.

5.4 Identification

The identification of the parameters are mostly standard. Two new features of my forward-looking consumer model are the two types of consumption value depreciation for owners and potential buyers. The depreciation rate of consumption values to owners is identified by the time-series variations in quantity sold by consumers to retailers across games, controlling for the variation in resale value and the size of owners (both observed). The depreciation rate of consumption values to potential buyers is identified by the average declining rate of new and used game sales. Heterogeneity in the depreciation rate of consumption values to potential buyers is identified by the time-series variations in the relative market share of new and used games, after controlling for the impact of the inventory level and the prices of new and used games on the relative market share. The relative market share over time tells us that after the sales of new games declines, how strong the demand for used games remains. If the demand for used games is still relatively strong after the decline in the sales of new games, this may suggest that the consumption value remains higher for some consumers than for the others. Heterogeneity in the depreciation rate of consumption values to owners is identified by the time-series variations in the quantity sold by consumers to retailers across games. Intuitively

speaking, the decreasing pattern of quantity sold by consumers after its peak (typically in the 5-7th week after release) tells us that some consumers prefer to keep the game even after owning it for a long time while its increasing patter before the peak tells us that games depreciate quickly for some consumers.

Finally, in my framework, the discount factor is also identified. Recall that the resale value does not enter the single-period utility function for purchasing decisions, but enters the expected continuation payoff for purchasing decisions. Thus, the resale value plays a role of an exclusion restriction that allows me to identify the standard discount factor (Rust 1994, Magnac and Thesmar 2002).[22] Note that if I model consumer purchasing and selling decisions independently, I am not able to identify the discount factor.

6 Results

In estimating the dynamic model, I set the terminal period to be 100 (about two years). Also, at this point, I allow for two types of consumers who differ in their depreciation rate of potential buyers' consumption values (i.e., ϕ_i). In the Bayesian estimation algorithm, I set $N = 1,000$ (# past pseudo-value functions used for the approximation of the expected future value) and $h = h_C = 0.01$ (kernel bandwidth).

The estimation results are reported in Table 4. Most of the parameter estimates are statistically significant. The estimated discount factor is 0.89. Remember that the unit of periods in my application is week. Thus, my estimate is lower than the discount factor translated from a weekly interest rate ($\simeq 0.999$), which is typically assumed when a dynamic model does not have any exclusion restriction to help identify the discount factor. However, my result is consistent with previous studies in experimental/behavioral economics which also find that the discount factor is lower than the interest

[22]This should clarify the different roles that the discount factor and the depreciation of consumption values to potential buyers play.

rate (e.g., Frederick et al. 2002). Price-sensitivity (α) is positive. Remember that the price enters the utility function as a negative term. A negative sign of λ_1 indicates that costs for purchasing used video games diminish as the inventory level rises. This is intuitive as the availability of used copies increases, the search costs for consumers may decrease. The parameter for the competitive effect from other games (π_1) is positive, suggesting that the increasing number of new game introduction may reduce the size of potential buyers who still consider purchasing the focal game.

Parameters for the depreciation rate to potential buyers include an type-specific intercept and a time effect. The estimated depreciation rate from the release week to the second week is about 5% for type-1 consumers and 47% for type-2 consumers. After estimating the structural model, I am able to convert the decline in the consumption value to potential buyers from the release week to the second week into a monetary term. Based on game-specific intercepts and the price coefficient, I find that on average, the consumption value to potential buyers declines by JPY 677 ($\simeq$ USD 7) for type-1 consumers and JPY 6,672 ($\simeq$ USD 66) for type-2 consumers from the release week to the second week. Given that the average price of new copies is JPY 6,389 (USD 64), the decline for type-2 consumers is large. However, a negative coefficient for the time effect suggests that the depreciation rate becomes much smaller as time since release increases.

For the depreciation rate of consumption values to owners, I include the following product characteristics in $X_{g\tau}$: an intercept, handheld game dummy, sequel game dummy, story-based game dummy, multi-player game dummy, critic rating and user rating. A positive coefficient of a variable implies that the variable will increase the depreciation rate to owners. My estimates suggest that on average, games released in handheld platforms (GameBoyAdvance, Nintendo DS, PlayStation Portable) and games with a higher user rating exhibit a lower depreciation rate. On the other hand, sequel games, story-based games, multiplayer games, and games with a higher critic rating exhibit a higher depreciation rate. However, by examining the magnitude of the coefficient for each game

characteristic, I find that the user rating plays the largest role in influencing the depreciation rate to owners. For example, the depreciation rate at $\tau = 1$ (for the first week of ownership) for the game with the highest user rating in my sample is 0.71 while that for the game with the lowest user rating is 0.83. The depreciation rates for all other games fall within this range. Finally, the coefficient for the duration of ownership suggests that the per-period depreciation rates become lower as consumers keep the game longer.

The parameters for pseudo-policy functions are reported in Table 5. While the consumption value to potential buyers has a very little impact on both the price and the resale value of used games, the average consumption value to owners across different durations of ownership has a positive impact. The difference in unobserved demand shocks between used and new games ($\xi_{2t} - \xi_{1t}$) has a negative impact, but small for the resale value of used games. The unobserved supply shock to used games has a positive impact on both the price and the resale value. The size of potential buyers has a positive impact on the price of used games and a negative impact on the resale value of used games. The average size of owners across different durations of ownership has a negative impact on the price of used games and a positive impact on the resale value of used games. Finally, the inventory level has a negative impact on both the price and the resale value of used games, but the impacts are small.

6.1 Substitutability between new and used games

6.1.1 The cross-price elasticity of demand

One conventional way of examining the potential competition from used games is to examine the cross-price elasticity of demand/revenue for new goods with respect to the price of used goods (e.g. Ghose et al. 2006). Based on the estimates, I first compute the average cross-price elasticity of single-period revenues for new games for the first 15 weeks (starting from the second week). Figure 4 plots the result. It shows that the cross-price elasticity is low at the beginning and increases to 0.035

after 15 weeks. On average, the cross-price elasticity is low, which suggests that new and used video games in Japan may not be close substitutes. This may be surprising because video games hardly depreciate physically, and thus used games should be almost perfect substitutes for new games. One reason is because of the high costs for purchasing used games. In particular, unlike new copies, used copies are available at a retailer only when other consumers sold their copies to the retailer. Thus, in the earlier periods consumers may think that the search costs are too high for purchasing used games and thus tend to buy new games. This results in a smaller the cross-price elasticity in the earlier periods. However, as the supply of used goods increases over time, the cross-price elasticity increases.

Note that the cross-price elasticity of single-period revenues for new games only accounts for the change in the revenue for new copies in the current period. However, a change in the price of used goods could have a dynamic effect. Some consumers may simply delay their purchase instead of switching to new copies when the price of used goods increases. Thus, there may be an increase in the demand for new and used copies in the subsequent periods. After estimating the dynamic structural model, I am able to examine such a dynamic effect. In order to examine the dynamic effect, I examine the percentage change in the expected present discounted value (PDV) of the current and future revenues (up to the terminal period T) for new games due to an increase in the price of used goods in each of the 14 weeks (week 2-15).[23] In computing the PDV of revenues, I assume that video game publishers' discount factor is 0.999 based on a weekly interest rate. I do not apply the estimated discount factor for consumers to video game publishers because (1) it is likely that they have a different discount factor from consumers as their forward-looking decision making critically affects their business, and (2) the interest rate is used in firms' accounting and thus it is more reasonable to use the discount factor based on a weekly interest rate. In Figure 4, I plot the

[23]Notice that in my application, the price of new games is constant over time. Thus, the percentage change in the PDV of the current and future revenues is numerically equivalent to the PDV of the current and future sales.

percentage changes in the PDV of new-game revenues. As the figure shows, the dynamic effect is even smaller than the cross-price elasticity of the single-period revenue. It may appear puzzling that the dynamic effect is smaller than the static effect. To understand it, notice that the change in the price of used games today lowers the demand for used games today,[24] leading to a larger inventory level of used games tomorrow. Consumers who delay purchase today then may find it more attractive to buy a used copy since the search costs for finding a used copy is now lower compared to the baseline case. Therefore, an increase in the price of used games today could lower the sales of new games in the future, resulting in a smaller cross-price elasticity of the PDV of new-game revenues. However, the main results in this section show that regardless of whether I examine the static or PDV of new-game revenues, the substitutability between new and used games with respect to the price of used games is very low.

6.1.2 The inventory elasticity of demand

The comparison of the static and dynamic effects in the previous section points out the potential importance of examining the impact of the inventory level of used goods on the substitutability between new and used goods. Previous studies on the substitutability between new and used goods do not incorporate the excess supply information about used goods, thus no prior research has investigated the importance of the inventory level of used goods. However, after incorporating the excess supply information and estimating its impact on the utility function for used game purchase (λ_1), I am able to quantify its impact on the substitutability between new and used games.

To quantify the economic significance of λ_1, I first examine the inventory elasticity of used-game sales, i.e., the percentage change in used-game sales due to one percent change in the inventory level, from week 3 to 15 (note that the inventory level is zero for the first two weeks by construction). I find that the average inventory elasticity of used-game sales is 0.132. Compared to the own-price elasticity

[24]The own-price elasticity of used-game sales averaged across weeks 2-15 is 2.21.

of used-game sales (2.21), the inventory elasticity of used-game sales turns out to be smaller. Next, I examine the inventory elasticity of new-game revenues (both static and PDV). This analysis provides me with the insights for how an increase in the availability of used games induces consumers to switch from new games to used games. In Figure 5, I plot the percentage changes in both static and PDV of new-game revenues from week 3 to 15. The figure shows that both elasticities are increasing over time in absolute value, but small. The results further support the results in the previous section that the substitutability between new and used games in Japan is small.

6.1.3 The price elasticity of used-game supply

An important factor that determines the inventory level of used games is the resale value. While a higher resale value will lower retailers' profit-margin, it will induce more consumers to sell their games and increase the inventory level of used games. This may then increase the sales of used games. Therefore, it is of retailers' interest to examine the price elasticity of used-game supply with respect to the resale value. Figure 6 shows the results. The average price elasticity of used-game supply is initially 1.58 and declines over time. This number is comparable but smaller than the average own-price elasticity of used-game demand, which is 2.21.

6.2 Elimination of used markets

Video game publishers often claim that the existence of used game markets lowers the sales of new games. The claim is often based on the conjecture that if there were no used game market, most of the consumers who used to purchase a used copy would buy a new copy. In the previous section, I examine this conjecture and find that consumers may not actually switch to new games. Rather, consumers are more likely to delay the purchase and buy a used copy in the future since the search costs for finding a used copy will decrease in the future due to an increase in the price of used games today.

However, the role of used game markets for consumers is not to simply offer a cheaper alternative. It also serves as a place for them to sell games that they no longer want to keep. If the used game market shuts down, it is possible that consumers demand even less new games because the future value from purchasing a new game could be lowered due to the lack of selling opportunities. The analysis on the elasticity commonly used to examine the substitutability between new and used games cannot take this important factor into account. After estimating the dynamic structural model, I am able to quantify the overall impact for new-game revenues due to the existence of the used game market.

To examine the impact of eliminating used video game markets, I simulate the model by setting the parameters that capture the transition costs of purchasing and selling used games (λ_0 and μ) to be large so that the used game market shuts down in effect. In the current experiment, I simply take the observed price of new games as given. In general, the optimal price of new games in the absence of the used game market could be very different from the flat-pricing strategy I observe in the data. However, it is still of video game publishers' interest to examine how the elimination of the used game market changes the total revenue for new games given the current flat-pricing scheme.[25] Alternatively, one can think of the results presented here as the lower bound (i.e., in general, video game publishers could do better by optimally setting the price).

Based on the simulation, I compute the statistics on the percentage change in the total revenue for new games due to the elimination of the used game market. Here, the total revenue is the summation of the revenues in T periods. Table 6 shows the statistics. Surprisingly, on average across games, the new-game revenue declines by 4%, and there are only two games that exhibit an increase in the total revenues. This suggests that the future selling opportunity plays an important role in determining their purchasing decisions. To further investigate this conjecture, I plot the

[25]However, I do plan to include the analysis where video game publishers optimally adjust their price over time in the absence of the used game market.

average percentage change in new-game demand across games for the first 30 weeks in Figure 7. It shows that the demand for new games actually declines right after the game release. However, the percentage changes in demand increase and become positive after 22 weeks. The initial decline is due to the lowered future value from purchasing: when the resale value is still high, consumers' purchasing decision is critically influenced by the future selling opportunity. The increasing trend of the percentage change in demand reflects the fact that after the resale value of used games drops, it is no longer important for consumers to have the future selling opportunity as they probably keep the game even in the presence of the used game market. Thus, the elimination of the used game market will be profitable for video game publishers in the later periods.

Remember that the results here are based on the observed flat-pricing strategy. If I allow video game publishers to choose the price of new games optimally over time, the results could reverse. However, if the conventional flat-pricing strategy is due to some institutional constraints in the industry and it is too costly to change the price of new games after release,[26] the result here could be a good approximation for the actual impact of the used game market on new-game revenues.

6.3 Is the flat-pricing strategy optimal?

As mentioned repeatedly in this paper, one interesting fact in the Japanese video game industry is that video game publishers typically keep the price of new copies constant over time. This is in contrast to the price-skimming strategy employed in the US video game industry or in typical durable goods markets. Although investigating why they employ a flat-pricing strategy is beyond the scope of this paper, it is interesting to investigate whether the observed flat-pricing strategy is optimal.

To investigate the optimality of the flat-pricing strategy, I run a counterfactual experiment in

[26]It is possible that the complex structure of the distribution system in Japan (many middlemen between manufacturers and retailers) makes retailers' profit-margin from new games very thin, which makes it difficult for manufacturers to provide incentives for retailers to lower the price over time.

which video game publishers marginally reduce the price of new games over time, fixing the price and resale value of used at observed values. More specifically, I reduce the price of new copies by 0.1% in each period, simulate the model, and compute the change in total revenue for new copies.

Table 7 reports the summary statistics on the percentage change in the total revenue for new games. First, I find that the "marginal price-skimming" strategy is on average more beneficial than the flat-pricing strategy: it increases the total revenue by 0.5%. Given that the median revenues of the games in my data set is about 65 million US dollars, this amounts to an increase in total revenues by 0.33 million US dollars.

However, of 41 games in my sample, there are several games that exhibit a decline in total revenues after switching to the marginal price-skimming. In the future research, it is interesting to find out if this is the case for some video games in the Japanese video game industry by developing an equilibrium model and examining the optimal pricing strategy for video game publishers and retailers.

7 Conclusion

My estimation results show that (1) the depreciation of consumption values to potential buyers from the release week to the 2nd week is about 5% in one consumer segment and 47% in another segment; and (2) the depreciation of consumption values to owners depends on product characteristics. In particular, I find that user ratings have a significant impact in reducing the depreciation rate. Using the parameter estimates, I examine three types of elasticities that account for the inter-temporal substitution: (a) cross-price elasticity of new-game demand with respect to used-game price, (b) inventory elasticity of new-game and used-game demand, (c) price elasticity of used-game supply with respect to resale value of used games. The first type of elasticity is a standard measure for the substitutability. The second and third types have never been examined in the previous literature,

but could provide important insights for how an increase in the availability of used games could affect new-game revenues.

Furthermore, I quantify the impact of eliminating the used video game market on new-game revenues. The result shows that on average, the elimination of the used video game market reduces the total revenue for new games by 4% if video game publishers do not adjust their pricing strategy. This is mainly because the elimination of the used market also eliminates the future selling opportunity for consumers and reduces the demand for new games especially in the earlier periods. These two results suggest that the existence of the used video game market in Japan could be beneficial to video game publishers.

Finally, I examine the optimality of the conventional flat-pricing strategy used by video game publishers in Japan. I simulate the model by marginally reducing the price of new games, and compare its profitability to that of the flat-pricing strategy. The result suggests that on average this marginal price-skimming strategy increases the total revenues for new games by 0.5%.

Like any other research, this research also has limitations. First, for the current results, I do not control for other marketing variables such as advertising. It is possible that high sales in the release week could be due to a high amount of pre-release advertising. Without controlling for advertising, it is possible that my estimate for the depreciation of consumption values to potential buyers could be biased upwards. I have obtained weekly TV advertising data and plan to control for its impact on demand.

Second, I do not explicitly model the competition among different video games. In my model, the competition is captured in a reduced-form fashion through the cumulative number of newly introduced games since the release of the focal game. However, if the substitutability between two games is mainly determined through product characteristics, my approach suffers from mis-specification bias. There are two reasons why I take the reduced-form approach. First, Nair (2007)

finds in the US video game market data that the substitutability between different games is generally low. Second, the focus of this paper is to examine the substitutability between new and used versions of the same game. Thus, I believe that the substitutability across different games is of second-order importance.

Third, and related, I do not allow for the impact of newly introduced games on consumer selling decisions. It is possible that when a new game is released, some consumers may sell their own games to finance the new game. I plan to model this effect in the new estimation result. Finally, in the counterfactual experiment for the elimination of used goods markets, I plan to compute the optimal pricing strategy by video game publishers and show the change in new-game revenues.

This paper focuses on developing and estimating the dynamic structural demand model. In the future, I plan to combine the proposed demand-side model with the dynamic supply-side model where video game publishers compete with used game retailers. Such an extension allows me to investigate the optimal pricing strategy of video game publishers in the presence of the used game market.

References

Berry, Steven, James Levinsohn, Ariel Pakes. 1995. Automobile Prices in Market Equilibrium. *Econometrica* **63**(4) 841–890.

Chen, Jiawei, Susanna Esteban, Matthew Shum. 2010. How Much Competition is a Secondary Market. Working Paper, Department of Economics, University of California, Irvine.

Chevalier, Judith, Austan Goolsbee. 2009. Are Durable Goods Consumers Forward Looking? Evidence from the College Textbook Market. *Quarterly Journal of Economics* **124**(4) 1853–1884.

Ching, Andrew, Susumu Imai, Masakazu Ishihara, Neelam Jain. 2010. A Guide to Bayesian Estimation of Dynamic Discrete Choice Models with an Application to a Store-Level Reward Program. Working Paper, Rotman School of Management, University of Toronto.

Ching, Andrew, Masakazu Ishihara. 2010. The Effects of Detailing on Prescribing Decisions under Quality Uncertainty. *Quantitative Marketing and Economics* **8**(2) 123–165.

Ching, Andrew T. 2009. A Dynamic Oligopoly Structural Model for the Prescription Drug Market After Patent Expiration. Forthcoming in *International Economic Review*.

Ching, Andrew T. 2010. Consumer Learning and Heterogeneity: Dynamics of Demand for Prescription Drugs After Patent Expiration. Forthcoming in *International Journal of Industrial Organization*.

Cho, Sungjin, John Rust. 2010. The Flat Rental Puzzle. *Review of Economic Studies* **77**(2) 560–594.

Desai, Preyas, Oded Koenigsberg, Devavrat Purohit. 2004. Strategic Decentralization and Channel Coordination. *Quantitative Marketing and Economics* **2**(1) 5–22.

Desai, Preyas, Devavrat Purohit. 1998. Leasing and Selling: Optimal Marketing Strategies for a Durable Goods Firm. *Management Science* **44**(11) S19–S34.

Desai, Preyas, Devavrat Purohit. 1999. Competition in Durable Goods Markets: The Strategic Consequences of Leasing and Selling. *Marketing Science* **18**(1) 42–58.

Engers, Maxim, Monica Hartmann, Steven Stern. 2009. Annual Miles Drive Used Car Prices. *Journal of Applied Econometrics* **24**(1) 1–33.

Erdem, Tulin, Susumu Imai, Michael P. Keane. 2003. Brand and Quantity Choice Dynamics Under Price Uncertainty. *Quantitative Marketing and Economics* **1**(1) 5–64.

Esteban, Susanna, Matthew Shum. 2007. Durable-Goods Oligopoly with Secondary Markets: The Case of Automobiles. *RAND Journal of Economics* **38**(2) 332–354.

Fevrier, Philippe, Lionel Wilner. 2009. Do Consumers Correctly Expect Price Reductions? Working paper, CREST.

Frederick, Shane, George Loewenstein, Ted O'Donoghue. 2002. Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature* **40**(2) 351–401.

Ghose, Anindya, Michael D. Smith, Rahul Telang. 2006. Internet Exchange for Used Books: An Empirical Analysis of Product Cannibalization and Welfare Impact. *Information System Research* **17**(1) 3–19.

Gordon, Brett. 2009. A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry. *Marketing Science* **28**(5) 846–867.

Gowrisankaran, Gautam, Marc Rysman. 2009. Dynamics of Consumer Demand for New Durable Goods. Working Paper, Department of Economics, Boston University.

Hirayama, Taro. 2006. A Sequel to the Used Game Lawsuit. *Research Center for the Legal System of Intellectual Property Column* .

Imai, Susumu, Neelam Jain, Andrew Ching. 2009a. Bayesian Estimation of Dynamic Discrete Choice Models. *Econometrica* **77**(6) 1865–1899.

Imai, Susumu, Neelam Jain, Andrew Ching. 2009b. Supplement to 'Bayesian Estimation of Dynamic Discrete Choice Models'. *Econometrica Supplementary Material* **77**. Http://www.econometricsociety.org/ecta/Supmat/5658 proofs.pdf.

Liang, Meng-Yu. 1999. Does a Second-Hand Market Limit a Durable Goods Monopolist's Market Power? Working Paper, University of Wisconsin-Madison.

Magnac, Thierry, David Thesmar. 2002. Identifying Dynamic Discrete Decision Processes. *Econometrica* **70**(2) 801–816.

Nair, Harikesh. 2007. Intertemporal Price Discrimination with Forward-Looking Consumers: An Application to the US Market for Console Video-Games. *Quantitative Marketing and Economics* **5**(3) 239–292.

Norets, Andriy. 2009. Inference in Dynamic Discrete Choice Models with Serially Correlated Unobserved State Variables. *Econometrica* **77**(5) 1665–1682.

Purohit, Devavrat. 1992. Exploring the Relationship between the Markets for New and Used Durable Goods: The Case of Automobiles. *Marketing Science* **11**(2) 154–167.

Rao, Raghunath Singh, Om Narasimhan, George john. 2009. Understanding the Role of Trade-Ins in Durable Goods Markets: Theory and Evidence. *Marketing Science* **28**(5) 950–967.

Rust, John. 1987. Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher. *Econometrica* **55**(5) 999–1033.

Rust, John. 1994. Structural Estimation of Markov Decision Processes. R. Engle, D. McFadden, eds., *Handbook of Econometrics*, vol. 4. North Holland, Amsterdam, 3081–3143.

Rust, John. 1997. Using Randomization to Break the Curse of Dimensionality. *Econometrica* **65**(3) 487–516.

Schiraldi, Pasquale. 2009. Automobile Replacement: a Dynamic Structural Approach. Working Paper, Department of Economics, London School of Economics.

Shulman, Jeffrey D., Anne T. Coughlan. 2007. Used Goods, Not Used Bads: Profitable Secondary Market Sales for a Durable Goods Channel. *Quantitative Marketing and Economics* **5**(2) 191–210.

Song, Inseong, Pradeep K. Chintagunta. 2003. A Micromodel of New Product Adoption with Heterogeneous and Forward-Looking Consumers: Application to the Digital Camera Category. *Quantitative Marketing and Economics* **1**(4) 371–407.

Tedeschi, Bob. 2004. Online Battle of Low-Cost Books. *The New York Times* (July 12).

Yin, Shuya, Saibal Ray, Haresh Gurnani, Animesh Animesh. 2010. Durable Products with Multiple Used Goods Markets: Product Upgrade and Retail Pricing Implications. *Marketing Science* **29**(3) 540–560.

Table 1: Summary statistics

Variables	Average	S.D.	Max	Min
Price of new copies (in JPY)	6,388.6	1,381.7	9,240	3,800
Price of used copies (in JPY)	4,154.0	931.3	7,433	1,461
Resale value (in JPY)	2,724.5	963.2	6,547	813
Sales of new copies	68,312.1	165,143.5	2,236,881	2,772
Sales of used copies	7,135.6	5,497.3	62,734	18
Quantity sold by owners	8,486.0	7,314.0	55,830	105
Inventory level of used copies	38,096.2	42,260.0	208,776	31
Market size (installed base)	12,418,879.6	7,051,686.4	25,391,982	247,429
Weekly # new game introduction	6.91	3.94	17	0
Dummy for handheld games	0.39	0.49	1	0
Dummy for sequel games	0.93	0.26	1	0
Dummy for story-based games	0.63	0.49	1	0
Dummy for multiplayer games	0.56	0.50	1	0
Critic rating (in 10-point scale)	8.77	0.68	10	7
User rating*	54.6	10.6	72	18

Notes: USD 1 ≈ JPY 100

* user rating is a standardized score against a set of video games released in the same year (by Enterbrain, Inc.).

Table 2: Regressions for the price of used games and resale value ($t = 2$)

	price of used copies		resale value	
variable	estimate	s.e.	estimate	s.e.
price of new copy	0.737*	0.046	0.638*	0.077
dummy for handheld games	43.6	116.7	22.5	197.5
dummy for sequel games	-128.7	116.7	166.1	197.5
dummy for story-based games	8.25	71.3	72.9	120.5
dummy for multi-player games	51.3	69.2	200.6	117.1
critic rating	132.8*	48.8	210.3*	82.5
user rating	-8.27*	2.89	-8.22	4.89
constant	-156.1	406	-1491.4*	686.7
Adjusted R-squared	0.974		0.915	
# observations	41		41	

Notes: * p < 0.05

Table 3: Regressions for the price of used games, resale value, and inventory level ($t > 2$)

	price of used copies		resale value		inventory level	
variable	estimate	s.e.	estimate	s.e.	estimate	s.e.
lagged value	0.964*	0.003	0.930*	0.004	0.996*	0.003
dummy for handheld games	36.6*	6.61	51.6*	7.23	1813.4*	219.7
dummy for sequel games	-23.0*	10.0	-44.1*	11.4	442.7	348.6
dummy for story-based games	-11.6	7.44	-32.3*	8.49	609.1*	258.0
dummy for multi-player games	31.4*	7.59	34.9*	8.68	309.6	262.8
critic rating	9.21	4.89	14.6*	5.59	617.0*	168.9
user rating	0.898*	0.306	1.22*	0.349	-1.63	10.6
lagged inventory level	-7.29e-4*	7.89e-5	8.30e-4*	9.40e-5	-	-
constant	-8.79	42.9	-20.5	48.0	-5496.4*	1459.3
Adjusted R-squared	0.988		0.985		0.993	
# observations	1343		1343		1384	

Notes: * p < 0.05

Table 4: Demand estimates

	mean	s.d.
preference parameters		
discount factor (β)	0.885	0.006
price sensitivity (α)	4.03E-04	2.71E-06
costs for buying used goods		
intercept (λ_0)	3.890	0.049
inventory level (logged) (λ_1)	-0.112	0.008
costs for selling used goods (μ)	6.117	0.072
no-purchase option		
intercept (π_0)	0.069	0.025
competitive effect from other games (π_1)	0.236	0.066
depreciation rates		
potential buyers (φ)		
intercept for type-1 consumer	-2.998	0.057
intercept for type-2 consumer	-0.126	0.046
time since release	-14.710	0.014
proportion of type-1 consumers	1.192	0.064
game owners (κ)		
intercept	0.123	0.009
handheld	0.520	0.037
sequel	1.299	0.011
story-based	0.224	0.015
multi-player	0.262	0.035
critic rating	0.037	0.001
user rating	-0.022	0.001
duration of ownership (logged)	-0.007	7.92E-05
parameters for error terms		
s.d. (ζ)	1	
s.d. (ϵ)	0.929	0.027
s.d. of measurement error for Q_1^d	68.2	2.94
s.d. of measurement error for Q_2^d	125.0	3.26
s.d. of measurement error for Q^s	101.0	5.07
s.d. (ξ_1)	0.362	0.020
s.d. (ξ_2)	1.67	0.14
s.d. (ξ_s)	0.553	0.035

- 41 game specific intercepts are not reported here.

Table 5: Estimates for pseudo-policy functions

pseudo-pricing policy function parameters			
price of used goods			
	ω_{10}	8.472	0.025
	ω_{11}	0.005	0.005
	ω_{12}	0.015	0.047
	ω_{13}	-0.178	0.094
	ω_{14}	0.016	0.066
	ω_{15}	3.50E-09	1.25E-24
	ω_{16}	-1.98E-07	4.00E-07
	ω_{17}	-1.06E-06	5.55E-07
	s.d. (v^p)	0.205	0.019
resale value of used goods			
	ω_{20}	8.236	0.013
	ω_{21}	-0.012	0.004
	ω_{22}	0.081	0.012
	ω_{23}	-0.035	0.027
	ω_{24}	0.168	0.143
	ω_{25}	-1.29E-08	2.79E-11
	ω_{26}	1.21E-06	2.69E-07
	ω_{27}	-2.78E-06	1.73E-07
	s.d. (v^r)	0.276	0.004

Table 6: The percentage change in the total revenue of new games due to the elimination of used games

	Average	S.D.	Min	Max
% change in total revenues	-3.85%	2.33%	4.62%	-7.61%

Table 7: The percentage change in the total revenue of new games from switching to the marginal price-skimming strategy

	Average	S.D.	Min	Max
% change in total revenues	0.54%	1.15%	-3.99%	3.29%

Figure 1: Average quantities demanded for new video games

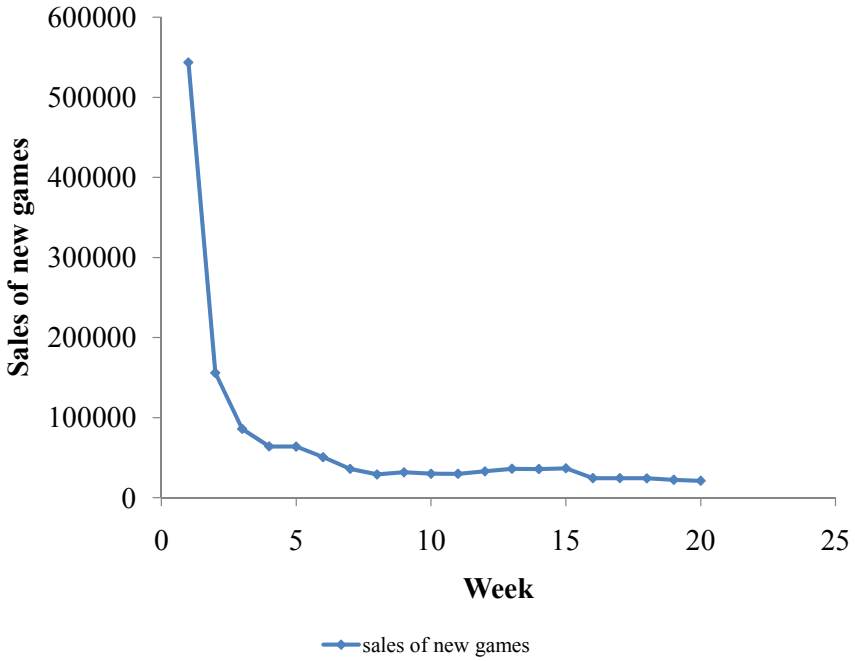


Figure 2: Average quantities demanded and supplied and inventory level for used video games

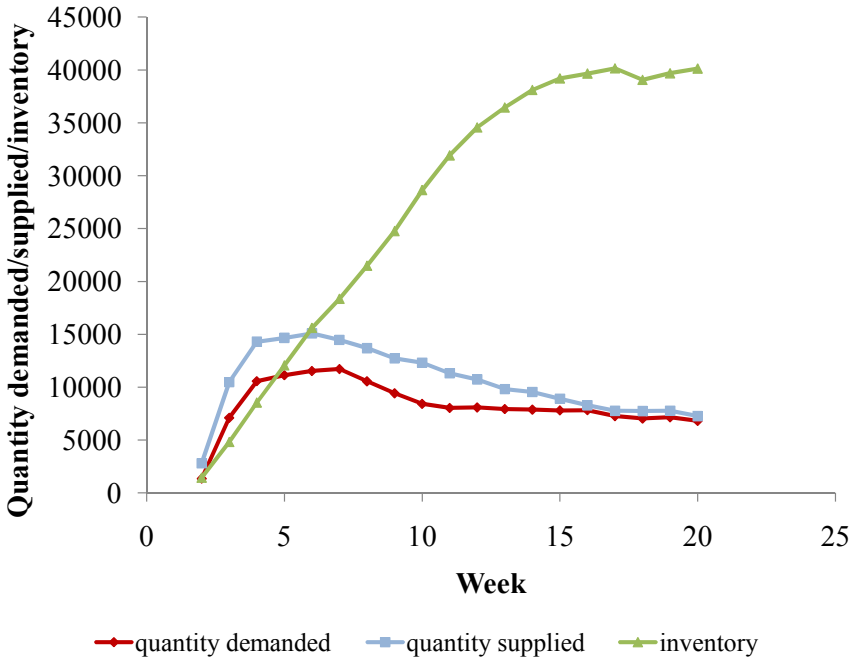


Figure 3: Average price and resale value of used video games

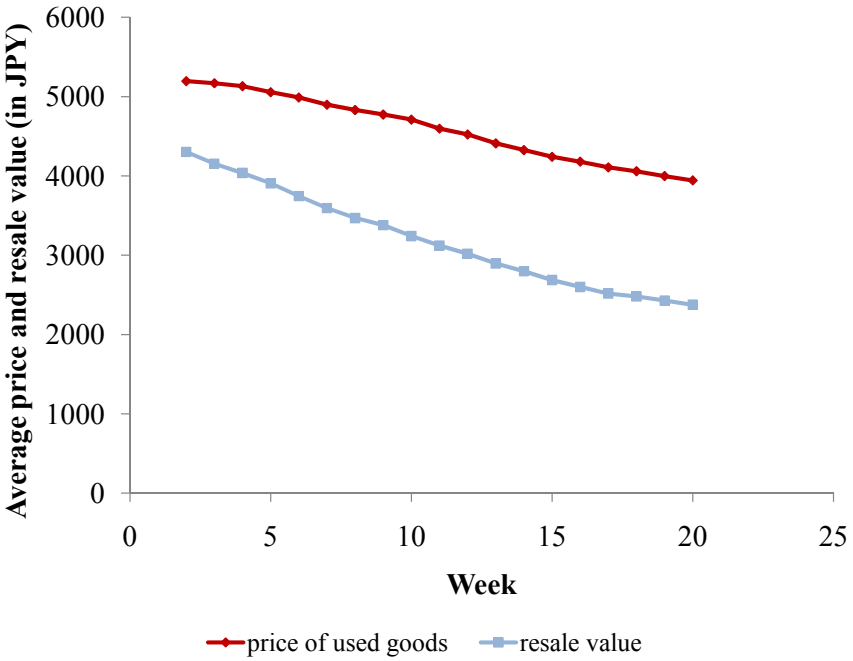


Figure 4: Cross-price elasticity of new-game revenues

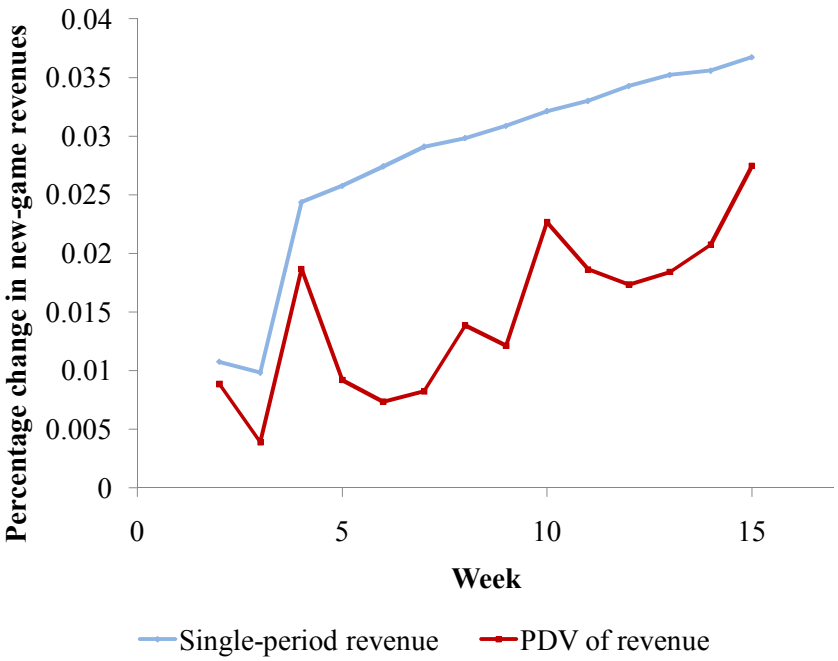


Figure 5: Inventory elasticity

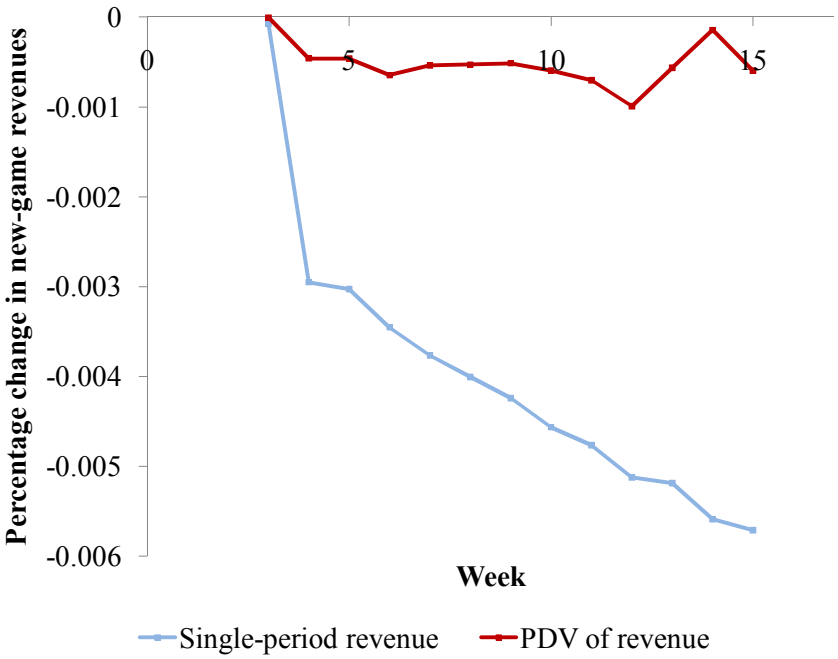


Figure 6: Elasticity of used-game supply

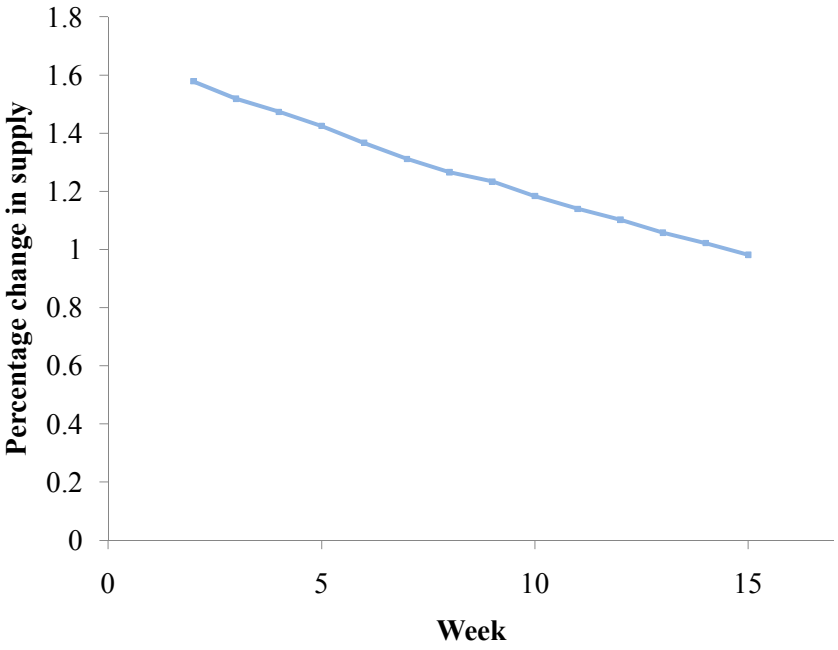
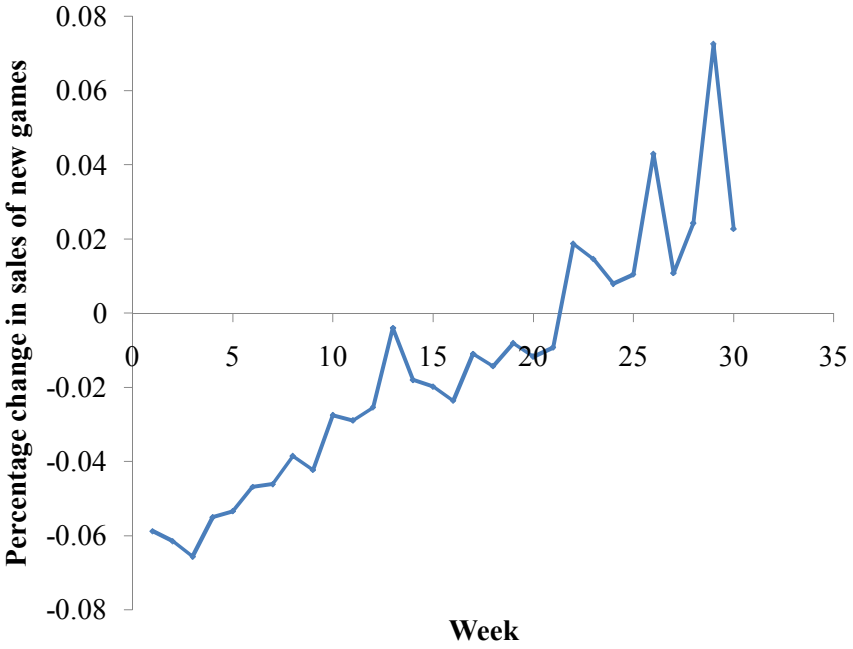


Figure 7: Percentage changes in new-game demand due to the elimination of used games



A Appendix

A.1 The procedure for the estimation algorithm

This appendix describes the details for the estimation algorithm described in section 5.1.

Let θ_d and θ_s be the vectors of demand-side parameters and pseudo-policy function parameters, respectively. In the context of the present model, the output of the new algorithm in iteration m is

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_i^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l})\ \forall i\}_{t=2}^T, \{\{\tilde{W}_i^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l})\ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

where $\tilde{V}_i^l$ and $\tilde{W}_i^l$ are type-i consumers' pseudo-value functions for purchasing and selling decisions in iteration l, respectively; N is the number of past pseudo-value functions used for approximating the expected future value functions; θ_d^l and θ_s^l are the accepted parameter vectors of the demand-side model and the pseudo-policy functions in iteration l, respectively; θ_d^{*l} is the candidate parameter vector for the demand-side model in iteration l; $(\tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l)$ are a draw of (serially correlated) state vector at time t in iteration l (e.g., drawn from uniform distribution); $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are drawn from the corresponding normal distributions. At this point, I assume $(\tilde{\xi}_d^l, \tilde{\xi}_s^l)$ are iid across time, and thus I can (1) use the same draws for all periods, and (2) the integration of unobserved shocks can be done by the simple average of the past pseudo-value functions.

The pseudo-value functions for selling decision at time t in iteration m are defined as follows:

$$\begin{aligned}
&\tilde{W}_i^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) \\
&\quad = E_e \max\{\tilde{W}_{i0}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) + e_{i0t}, \tilde{W}_{i1}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) + e_{i1t}\},
\end{aligned}$$

where $\tilde{W}_{ik}^m$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\begin{aligned}
&\tilde{W}_{ik}^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m}) \\
&\quad = \begin{cases} \alpha \tilde{r}_t^m - \mu + \tilde{\xi}_s^m & \text{selling}, \\ v_i(t, \tau) + \beta \hat{E}^m[W_i(r', Y', \xi_s', t+1, \tau+1; \theta_d^{*m}) | (\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau)] & \text{keeping}. \end{cases}
\end{aligned}$$

The *pseudo*-expected future value function for selling decision, $\hat{E}^m[W_i(.;\theta_d^{*m})|.]$, is defined as the weighted average of the past pseudo-value functions for selling decision in period $t+1$. It is constructed as follows:

$$\begin{aligned}&\hat{E}^m[W_i(r',Y',\xi_s',t+1,\tau+1;\theta_d^{*m})|(\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{\xi}_s^m,t,\tau)] \\ &\quad=\sum_{l=m-N}^{m-1}\tilde{W}_i^s(\tilde{r}_{t+1}^l,\tilde{Y}_{t+1}^l,\tilde{\xi}_s^l,t+1,\tau+1;\theta_d^{*l})\frac{K_h(\theta_d^{*m}-\theta_d^{*l})f(\tilde{r}_{t+1}^l,\tilde{Y}_{t+1}^l|\tilde{r}_t^m,\tilde{Y}_t^m)}{\sum_{q=m-N}^{m-1}K_h(\theta_d^{*m}-\theta_d^{*q})f(\tilde{r}_{t+1}^q,\tilde{Y}_{t+1}^q|\tilde{r}_t^m,\tilde{Y}_t^m)},\end{aligned} \tag{5}$$

where $K_h(.)$ is a Gaussian kernel with bandwidth h, and $f(.|.)$ is the transition density estimated in the first step. Note that the kernel captures the idea that one assigns higher weights to the past pseudo-value functions which are evaluated at parameter vectors that are closer to θ_d^{*m}. Also, this weighted average integrates out both ξ_s' and r'. In particular, ξ_s' is integrated out by the simple average since ξ_s^l's are drawn from its distribution. In contrast, r' is integrated out by the weighted average, where weights are given by the transition probability.

The pseudo-value functions for the purchasing decision at time t in iteration m are defined as follows:

$$\begin{aligned}\tilde{V}_i(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t;\theta_d^{*m}) = E_{\zeta,\epsilon}\max\{&\tilde{V}_{i0}^m(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t;\theta_d^{*m})+\zeta_{i0t}+\epsilon_{i0t},\\ &\tilde{V}_{i1}^m(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t;\theta_d^{*m})+\zeta_{i1t}+\epsilon_{i1t},\tilde{V}_{i2}^m(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t;\theta_d^{*m})+\zeta_{i1t}+\epsilon_{i2t}\},\end{aligned}$$

where $\tilde{V}_{ij}^m$'s are type-i consumer's pseudo alternative-specific value functions in iteration m, which are given by

$$\begin{aligned}&\tilde{V}_{ij}^m(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t;\theta_d^{*m}) \\ &\quad=\begin{cases}v_i(t,0)-\alpha p_1+\tilde{\xi}_1^m+\beta\hat{E}^m[W_i(r',Y',\xi_s',t+1,1;\theta_d^{*m})|(\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{\xi}_s^m,t,0)] & \text{new copy,}\\ v_i(t,0)-\alpha\tilde{p}_{2t}^m+\tilde{\xi}_2^m-l_Y(\tilde{Y}_t^m)+\beta\hat{E}^m[W_i(r',Y',\xi_s',t+1,1;\theta_d^{*m})|(\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{\xi}_s^m,t,0)] & \text{used copy,}\\ l_C(\tilde{C}_t^m)+\beta\hat{E}^m[V_i(p_1,p_2',r',Y',C',\xi_d',t+1;\theta_d^{*m})|(p_1,\tilde{p}_{2t}^m,\tilde{r}_t^m,\tilde{Y}_t^m,\tilde{C}_t^m,\tilde{\xi}_d^m,t)] & \text{no purchase.}\end{cases}\end{aligned}$$

The *pseudo*-expected future value function for the purchasing decision, $\hat{E}^m[V_i(,;\theta_d^{*m})|.]$, is defined as the weighted average of the past pseudo-value functions for the purchasing decision in period $t+1$,

and is constructed as follows:

$$\begin{aligned}
&\hat{E}^m[V_i(p_1, p_2', r', Y', C', \xi_d', t+1; \theta_d^{*m})|(p_1, \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t)] \qquad (6)\\
&= \sum_{l=m-N}^{m-1} \tilde{V}_i(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\\
&\times \frac{K_h(\theta_d^{*m} - \theta_d^{*l}) K_{h_C}(\tilde{C}_t^m + N_C - \tilde{C}_{t+1}^l) f(\tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m)}{\sum_{q=m-N}^{m-1} K_h(\theta_d^{*m} - \theta_d^{*q}) K_{h_C}(\tilde{C}_t^m + N_C - \tilde{C}_{t+1}^q) f(\tilde{p}_{2t+1}^q, \tilde{r}_{t+1}^q, \tilde{Y}_{t+1}^q | \tilde{p}_{2t}^m, \tilde{r}_t^m, \tilde{Y}_t^m)}.
\end{aligned}$$

Note again that this weighted average integrates out p_2', r', Y', and ξ_d'. The assumption that consumers know the evolution of C implies that the transition of C is deterministic ($E[C'|C] = C + N_C$). I handle this by a kernel-based local interpolation approach by Imai et al. (2009a), where K_{h_C} is the kernel function for C with bandwidth h_C.

Each MCMC iteration in the proposed algorithm consists of five blocks:

1. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions conditional on $\xi_t^{m-1} = (\xi_{1t}^{m-1}, \xi_{2t}^{m-1}, \xi_{st}^{m-1})$ for all observed t and g.

2. Draw ξ_t^m for all observed t and g conditional on the data, σ_ξ^m, θ_d^{m-1} and θ_s^{m-1}. In the Metropolis-Hastings algorithm, the joint-likelihood of the demand-side model and the pseudo-policy functions will be used to compute the acceptance probability.

3. Draw θ_d^m conditional on the data, $\{\xi_t^m\}$ and θ_s^{m-1} using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, the joint-likelihood will be used.

4. Draw θ_s^m conditional on the data, $\{\xi_t^m\}$ and θ_d^m using the random-walk Metropolis-Hastings algorithm. In the Metropolis-Hastings algorithm, only the likelihood of the pseudo-policy functions will be used since θ_s^m does not enter the demand-side model.

5. Compute the pseudo-value functions for purchasing and selling decision problems. Starting from the terminal period, I sequentially compute the pseudo-value functions backwards at only one randomly drawn state point in each period. I store them and update H^m to H^{m+1}.

In deriving the posterior distribution of parameters, I use an inverted gamma prior on σ_ξ, and a flat prior on θ_d and θ_s. Also, note that the likelihood used in the IJC algorithm is called *pseudo*-likelihood as it is a function of *pseudo* alternative-specific value functions. Below, I provide a step-by-step procedure for the five blocks described above.

1. Suppose that we are at iteration m. We start with

$$H^m = \{\theta_d^l, \theta_s^l, \theta_d^{*l}, \{\tilde{V}_i^l(p_1, \tilde{p}_{2t}^l, \tilde{r}_t^l, \tilde{Y}_t^l, \tilde{C}_t^l, \tilde{\xi}_d^l, t; \theta_d^{*l})\ \forall i\}_{t=2}^T, \{\{\tilde{W}_i^l(\tilde{r}_t^l, \tilde{Y}_t^l, \tilde{\xi}_s^l, t, \tau; \theta_d^{*l})\ \forall i\}_{\tau=1}^{t-1}\}_{t=2}^T\}_{l=m-N}^{m-1},$$

   where N is the number of past iterations used for the expected future value approximation.

2. Draw $\sigma_\xi^m = (\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_{\xi_s})$ directly from their posterior distributions (inverted gamma) conditional on $\xi_t^{g,m-1} = (\xi_{1t}^{g,m-1}, \xi_{2t}^{g,m-1}, \xi_{st}^{g,m-1})$ for all observed t and g.

3. For each observed t and g, draw $\xi_t^{g,m}$ from its posterior distribution conditional on $\sigma_{\xi_1}^m$, θ_d^{m-1}, θ_s^{m-1}, $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, and $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$. I will draw $\xi_{1t}^{g,m}$, $\xi_{2t}^{g,m}$, and $\xi_{st}^{g,m}$ separately. Below, I will describe how to draw $\xi_{1t}^{g,m}$, but the procedure can be applied for drawing $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$.

   (a) Draw $\xi_{1t}^{g,*m}$ (candidate parameter value).

   (b) We compute the pseudo-joint likelihood at $\xi_{1t}^{g,*m}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, $\xi_{2t}^{g,m-1}$, $\xi_{st}^{g,m-1}$, $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}. Note that conditional on $\sigma_{\xi_1}^m$, the pseudo-joint likelihood prior to time t does not depend on $\xi_{1t}^{g,*m}$. Thus, we only need to compute the pseudo-joint likelihood at time t and later. To compute the pseudo-joint likelihood, we need to obtain the pseudo-alternative specific value functions for both purchasing and selling decisions at time t and later: $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{m-1})$ and $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$. To obtain $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{m-1})$, we need to calculate both $\hat{E}^m V_i(\cdot, t+1; \theta_d^{m-1})$ (pseudo-expected future value when consumers choose no option) and $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{m-1})$ (pseudo-expected future value when

consumers choose to buy new or used game), which are computed as the weighted average of past-pseudo value functions evaluated at time $t+1$:

i. For $\hat{E}^m V_i(\cdot, t+1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{V}_i^l(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 6.

ii. For $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{m-1})$, we take the weighted average of $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 5. Note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here only include those evaluated at $\tau = 1$.

To obtain $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{m-1})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{m-1})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 5.[27]

(c) Similarly, we compute the pseudo-joint likelihood at $\xi_{1t}^{g,m-1}$ conditional on $\{\xi_k^{g,m}\}_{k=1}^{t-1}$, $\xi_{2t}^{g,m-1}$, $\xi_{st}^{g,m-1}$, $\{\xi_k^{g,m-1}\}_{k=t+1}^{T^g}$, θ_d^{m-1} and θ_s^{m-1}.[28]

(d) Based on the pseudo-joint likelihoods at $\xi_{1t}^{g,*m}$ and $\xi_{2t}^{g,m-1}$, we compute the acceptance probability for $\xi_{1t}^{g,*m}$ and decide whether to accept (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,*m}$) or reject (i.e., set $\xi_{1t}^{g,m} = \xi_{1t}^{g,m-1}$).

(e) Using a similar procedure, draw $\xi_{2t}^{g,m}$ and $\xi_{st}^{g,m}$. One difference in drawing $\xi_{st}^{g,m}$ is that conditional on $\sigma_{\xi_s}^m$, $\xi_{st}^{g,*m}$ does not influence the likelihood function for purchasing decisions.

4. Use the Metropolis-Hastings algorithm to draw θ_d^m conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

(a) Draw θ_d^{*m} (candidate parameter vector).

[27] Conditional on σ_ξ^m, pseudo alternative-specific value functions do not depend on $\xi_{1t}^{g,*m}$. This is also true for $\xi_{2t}^{g,*m}$ and $\xi_{st}^{g,*m}$. Thus, pseudo alternative-specific value functions can be pre-computed right after step 2.

[28] In a standard Metropolis-Hastings algorithm, this step is not necessary as this value has been computed in the previous iteration. However, the IJC algorithm updates the set of past pseudo-value functions in each iteration. Thus, the pseudo-likelihood at $\xi_{1t}^{g,m-1}$ in iteration $m-1$ will be different from that at $\xi_{1t}^{g,m-1}$ in iteration m.

(b) We compute the pseudo-joint likelihood at θ_d^{*m} conditional on $\{\xi_t^m\}$ and θ_s^{m-1} based on the pseudo-alternative specific value functions for both purchasing and selling decisions at θ_d^{*m}: $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all observed t and g. To obtain $\tilde{V}_{ij}^m(\cdot, t; \theta_d^{*m})$, we need to calculate both $\hat{E}^m V_i(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_i(\cdot, t, 1; \theta_d^{*m})$, which are computed as the weighted average of past-pseudo value functions evaluated at time $t+1$:

i. For $\hat{E}^m V_i(\cdot, t; \theta_d^{*m})$, we take the weighted average of $\{\tilde{V}_i^l(p_1, \tilde{p}_{2t+1}^l, \tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{C}_{t+1}^l, \tilde{\xi}_d^l, t+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 6.

ii. For $\hat{E}^m W_i(\cdot, t, 1; \theta_d^{*m})$, we take the weighted average of $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, 1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 5. Again, note that since potential buyers at time t will have owned the game for one period when they reach $t+1$, the set of past pseudo-value functions used here are all evaluated at $\tau = 1$.

To obtain $\{\tilde{W}_{ik}^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we only need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ by the weighted average of the past pseudo-value functions $\{\tilde{W}_i^l(\tilde{r}_{t+1}^l, \tilde{Y}_{t+1}^l, \tilde{\xi}_s^l, t+1, \tau+1; \theta_d^{*l})\ \forall i\}_{l=m-N}^{m-1}$ as in Equation 5.

(c) Similarly, we compute the pseudo-joint likelihood at θ_d^{m-1} conditional on $\{\xi_t^m\}$ and θ_s^{m-1}.

(d) Based on the pseudo-joint likelihoods at θ_d^{*m} and θ_d^{m-1}, we compute the acceptance probability for θ_d^{*m} and decide whether to accept (i.e., set $\theta_d^m = \theta_d^{*m}$) or reject (i.e., set $\theta_d^m = \theta_d^{m-1}$).

5. Use the Metropolis-Hastings algorithm to draw θ_s^m conditional on $\{\xi_t^m\}$ and θ_d^m.

(a) Draw θ_s^{*m} (candidate parameter vector).

(b) We compute the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that the pseudo-alternative specific value functions do not depend on θ_s^{*m},

but are required to compute the pseudo-likelihood at θ_s^{*m} since they influence the evolution of equilibrium state variables. However, they have already been computed in step 4(b) (if θ_d^{*m} has been accepted) or 4(c) (if θ_d^{*m} has been rejected), there is no need to re-compute them here to form the pseudo-likelihood for pseudo-policy functions.

(c) To form the acceptance probability of θ_s^{*m}, we need the pseudo-likelihood for pseudo-policy functions at θ_s^{m-1} conditional on $\{\xi_t^m\}$ and θ_d^m. Note that this value has been computed in step 4 and needs not be re-computed here.

(d) Based on the pseudo-likelihood for pseudo-policy functions at θ_s^{*m} and θ_s^{m-1}, we compute the acceptance probability for θ_s^{*m} and decide whether to accept (i.e., set $\theta_s^m = \theta_s^{*m}$) or reject (i.e., set $\theta_s^m = \theta_s^{m-1}$).

6. Compute the pseudo-value functions for purchasing and selling decision problems.

(a) For each $t = 2, \ldots, T$, make a draw of used-game price ($\tilde{p}_{2t}^m$), resale value ($\tilde{r}_t^m$), inventory level ($\tilde{Y}_t^m$), and cumulative number of newly introduced games ($\tilde{C}_t^m$) from uniform distributions with appropriate upper- and lower-bound (e.g., upper- and lower-bound of observed values).

(b) Make a draw of $\tilde{\xi}_1^m$, $\tilde{\xi}_2^m$, and $\tilde{\xi}_s^m$ from the corresponding distribution based on $\sigma_{\xi_1}^m$, $\sigma_{\xi_2}^m$, and $\sigma_{\xi_s}^m$.

(c) Start from the terminal period T.

i. Compute the value functions $\tilde{V}_i^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_T^m, \tilde{Y}_T^m, \tilde{C}_T^m, \tilde{\xi}_d^m, T; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\tilde{r}_T^m, \tilde{Y}_T^m, \tilde{\xi}_s^m, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$ for all i. Note that at time T, there is no need to compute the pseudo-expected future value. Thus, the value functions computed at time T are not pseudo-value functions.

ii. Store $\tilde{V}_i^m(\cdot, T; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\cdot, T, \tau; \theta_d^{*m})\}_{\tau=1}^{T-1}$.

(d) For $t = T-1, \ldots, 2$, compute the pseudo-value function $\tilde{V}_i^m(p_1, \tilde{p}_{2T}^m, \tilde{r}_t^m, \tilde{Y}_t^m, \tilde{C}_t^m, \tilde{\xi}_d^m, t; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\tilde{r}_t^m, \tilde{Y}_t^m, \tilde{\xi}_s^m, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$ for all i backwards.

i. To compute $\tilde{V}_i^m(\cdot, t; \theta_d^{*m})$, we need to calculate $\hat{E}^m V_i(\cdot, t+1; \theta_d^{*m})$ and $\hat{E}^m W_i(\cdot, t+1, 1; \theta_d^{*m})$ based on Equations 6 and 5, respectively.

ii. To compute $\{\tilde{W}_i^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$, we need to calculate $\{\hat{E}^m W_i(\cdot, t+1, \tau+1; \theta_d^{*m})\}_{\tau=1}^{t-1}$ based on Equation 5.

iii. Store $\tilde{V}_i^m(\cdot, t; \theta_d^{*m})$ and $\{\tilde{W}_i^m(\cdot, t, \tau; \theta_d^{*m})\}_{\tau=1}^{t-1}$.

7. Go to iteration $m+1$.

A.2 The likelihood function

Assuming that the prediction errors, ν_t^p and ν_t^r, in Equations 3 and 4 are normally distributed, I obtain the conditional likelihood of observing (p_{2t}^g, r_t^g),

$$f_s(p_{2t}^g, r_t^g | \{M_{it}^{d,g}, v_i^g(t,0), \{M_{it}^{s,g}(\tau), v_i^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^I, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, Y_t^g; \theta_s)$$

where θ_s is the parameter vector of pseudo-policy functions. Note that (i) $v_i^g(t,\tau)$ depends on product characteristics, X_g; (ii) $M_{it}^{d,g}$ (size of potential buyers) and $M_{it}^{s,g}(\tau)$ (size of owners) are a function of X_g, p_1^g, $\{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}$, $\{C_m^g\}_{m=1}^{t-1}$, $\{\xi_{1m}^g\}_{m=1}^{t-1}$, $\{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^{t-1}$, $M_1^{d,g}$ (initial size of potential buyers), and $\{N_m^g\}_{m=2}^t$ (potential buyers who entered at time m). Thus, I can rewrite f_s as

$$f_s(p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, Z_t^g; \theta_s).$$

where $Z_t^g = \{X_g, p_1^g, \{p_{2m}^g, r_m^g, Y_m^g\}_{m=2}^{t-1}, \{C_m^g\}_{m=1}^{t-1}, M_1^{d,g}, \{N_m^g\}_{m=2}^t\}$ is a vector of observed variables.

Assume further that the measurement errors, ε_{1t}, ε_{2t}, ε_{st}, in Equations 1 and 2 are normally distributed. Then, the conditional likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g)$ is written as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{M_{it}^{d,g}, v_i^g(t,0), \{M_{it}^{s,g}(\tau), v_i^g(t,\tau)\}_{\tau=1}^{t-1}\}_{i=1}^I, \xi_{1t}^g, \xi_{2t}^g, \xi_{st}^g, p_1^g, p_{2t}^g, r_t^g, C_t^g, Y_t^g; \theta_d),$$

where θ_d is the vector of demand-side parameters. Similar to f_s, f_d can be rewritten as

$$f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, C_t^g, Y_t^g, Z_t^g; \theta_d).$$

The joint likelihood of observing $(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g)$ is the product of f_s and f_d:

$$\begin{aligned} &l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, C_t^g, Y_t^g, Z_t^g; \theta_d, \theta_s) = \\ &\quad f_d(Q_{1t}^g, Q_{2t}^g, Q_{st}^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, p_{2t}^g, r_t^g, C_t^g, Y_t^g, Z_t^g; \theta_d) \times \\ &\quad f_s(p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, Y_t^g, Z_t^g; \theta_s). \end{aligned}$$

The likelihood of observing $\mathbf{D} = \{\{Q_{1t}^g\}_{t=1}^{T^g}, \{Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g\}_{t=2}^{T^g}\}_{g=1}^G$ is

$$\begin{aligned} &L(\mathbf{D} | \xi, C, Y, Z; \theta_d, \theta_s) = \\ &\quad \prod_{g=1}^G \left[f_d(Q_{11}^g | \xi_{11}^g, C_1^g, Z_1^g; \theta_d) \prod_{t=2}^{T^g} l(Q_{1t}^g, Q_{2t}^g, Q_{st}^g, p_{2t}^g, r_t^g | \{\xi_{1m}^g\}_{m=1}^t, \{\xi_{2m}^g, \xi_{sm}^g\}_{m=2}^t, C_t^g, Y_t^g, Z_t^g; \theta_d, \theta_s) \right] \end{aligned}$$

where G is the total number of games, T^g is the length of observations for game g, $C = \{\{C_t^g\}_{t=1}^{T^g}\}_{g=1}^G$, $Y = \{\{Y_t^g\}_{t=1}^{T^g}\}_{g=1}^G$, and $Z = \{\{Z_t^g\}_{t=1}^{T^g}\}_{g=1}^G$.

Note that $\{\xi_{1t}^g\}_{t=1}^{T^g}, \{\xi_{2t}^g, \xi_{st}^g\}_{t=2}^{T^g}$ are unobserved to the econometricians. In the proposed Bayesian framework, these variables are augmented from the corresponding distributions to form the likelihood $L(\mathbf{D}|\cdot)$.