

Private Equity Fund Returns: Do Managers Actually Leave Money on the Table?*

Robert Marquez
Boston University

Vikram Nanda
Georgia Tech

M. Deniz Yavuz
Purdue University

August 16, 2010

*Contact: Deniz Yavuz (myavuz@purdue.edu). We would like to thank Philip H. Dybvig, Alex Edmans, Paolo Fulghieri, Thomas F. Hellman, Steven N. Kaplan, Laura Lindsey, Umit Ozmel, Merih Sevilir and seminar participants at Arizona State University, the 6th Annual Corporate Finance Conference (2009) at Washington University in St. Louis, and the Entrepreneurial Finance and Innovation Conference (2010).

Private Equity Fund Returns: Do Managers Actually Leave Money on the Table?

Abstract

Evidence indicates that private equity funds, unlike mutual funds, deliver persistent abnormal returns and that top performing funds are often oversubscribed. Why do private equity funds appear to leave money on the table, rather than, say, increasing fund size and/or fees? We argue that private equity funds are fundamentally different from mutual funds because their success is contingent on matching with high quality entrepreneurial firms, and these firms are looking to match with high ability managers. In the presence of asymmetric information about managerial actions or fund attributes, we show that fund managers limit fund size and fees and deliver persistent excess returns to investors. They do this in order to manipulate entrepreneurs' beliefs about managerial ability to add value, even though firms are not fooled in equilibrium. The model provides several novel time series and cross sectional predictions about performance persistence, fees and size, in addition to addressing the questions raised above.

Keywords: venture capital, private equity, performance persistence, signal jamming, leveraged buyout, fund size and fees.

1 Introduction

Anecdotal evidence abounds that some successful private equity funds do not accept all the money that investors are willing to invest.¹ At the same time, these funds appear to generate persistent abnormal returns for their investors (see, e.g., Kaplan and Schoar, 2005, or Phalippou and Gottschalg, 2008),² in stark contrast to case of mutual funds (e.g., Jensen, 1968).³ Kaplan and Schoar (2005) conjecture that top-performing funds voluntarily restrict their size which, given diseconomies of scale, can enhance the returns delivered to investors. The performance persistence of successful private equity (PE) funds is puzzling and raises an obvious question: Why do these funds not capture higher rents by increasing fund size or fees?

To address this question we focus on two fundamental differences between private equity and mutual funds. The first is that, unlike mutual funds that invest in public securities, investments by private equity funds are subject to a two-sided matching problem: Private equity funds want to match with high quality entrepreneurial firms; on the other side, entrepreneurs want to match with talented fund managers that are more likely to add value. There are a variety of ways in which private equity managers can add value: through providing strategic advice, helping to professionalize firm management, and by attracting better resources, business partners and human capital.⁴ Hsu (2004) provides evidence that firms are both more likely to accept offers from and sell their shares at a discount to VCs that are more reputable and presumably have a higher ability to add value. It is argued that positive assortative matching between PE funds and entrepreneurs, with high ability fund managers

¹Several cases in which VC funds were oversubscribed are noted in the article “Oversubscribed,” *European Venture Capital Journal*, November 2006. Also see Kaplan and Schoar (2005).

²See also Quigley and Woodward (2002), Jones and Rhodes-Kropf (2003), Ljungqvist and Richardson (2003), Cochrane (2005), and Korteweg and Sorensen (2008), which explore the risk, cash flow and performance of private equity funds.

³For successful funds, Carhart (1997) shows that “common factors in stock returns and investment expenses almost completely explain persistence in equity mutual funds’ mean and risk-adjusted returns” (p. 57). The worst performing mutual funds do seem to exhibit some performance persistence (see Carhart, 1997), possibly resulting from inattention by investors in these funds.

⁴See Gorman and Sahlman (1989), Megginson and Weiss (1991), Hellmann and Puri (2000, 2002), Baum and Silverman (2004).

matching with good firms, is likely very important for the success of private equity funds.⁵ Indeed, Sorensen (2007) provides evidence that VC fund success can be largely explained by experienced VCs matching with better firms.

A second critical difference is that PE funds, unlike mutual funds, are largely exempt from public disclosure requirements and, consequently, are far less transparent to outsiders with regard to managerial decisions and fund performance. Transparency is important since entrepreneurs, in assessing the ability of fund managers to add value, need to rely on the performance of funds operated by the manager in the past. Though information about past performance is available, it is typically self-reported either by general or limited partners. Even with past return data, it may be difficult to estimate a manager's ability because sufficient information is unlikely to be available about various factors that could affect fund returns such as the quality of the firms in the manager's portfolio, the precise fees charged, etc., especially if the fund has not fully exited from all of its investments (Phalippou and Gottschalg, 2008; Cumming and Walz, 2009).⁶ This creates an information asymmetry which provides incentives for fund managers to manipulate entrepreneurs' beliefs about their ability to add value. This can account for the difference in return patterns between mutual and private equity funds.

We present a simple model that incorporates these features. Fund managers raise capital from competitive investors, and try to invest in target firms where they can use their talent to add some value. This talent is unknown to all parties and must be inferred from past fund returns. Managers with higher perceived ability have a higher probability of matching with firms where they can add value. We first show that when all information about managers' past actions is available to entrepreneurs, the model collapses to a variant of the model in

⁵Matching is important also in financial intermediation (Chemmanur and Fulghieri, 1994; Fernando, Gatchev and Spindt, 2005; Bao and Edmans, 2009).

⁶Venture funds usually invest in a number of portfolio companies, with the returns to the investments being realized upon exit – whether through IPOs, acquisition by other firms or liquidation. For investments that result in exit through IPO, which only about a third of portfolio firms undergo (see e.g., Giot and Schwiabacher, 2007), the value added by the manager is not any easier to estimate than in the case of other exits. The reason is that outsiders may have little information about the stage of development of the firm's products and organizations when it matched with the VC, the market conditions that tend to affect IPO activity and firms' qualities. Even less information is available in the case of non-IPO exits.

Berk and Green (2004). Since the fund manager is unable to manipulate the beliefs of firms, he increases fund size until investors' expected excess return is zero.

We then move to a more realistic setting in which entrepreneurs are not perfectly informed about various aspects of a manager's actions or fund characteristics. We start with a simple case where the performance of the manager's earlier funds is observable but there is asymmetric information with regard to the overall sizes of the funds run by the manager in the past.⁷ In this case, we show that the manager chooses a lower fund size compared to the case where all information is symmetric and delivers persistent excess returns (i.e., "alpha") to investors. Entrepreneurs are not, however, fooled in equilibrium since they recognize the manager's incentives and can rationally anticipate his actions. Nevertheless, the manager cannot avoid the attempt to manipulate entrepreneurs' beliefs since entrepreneurs, in assessing the manager's ability, assume that the manager is indeed attempting to manipulate their beliefs. As a result, if the manager were to not take the manipulative actions, the fund would deliver lower returns and these would be regarded by entrepreneurs as being driven by a lower managerial ability to add value.⁸

We next show that asymmetric information about virtually any choice variable for the manager which affects gross returns can provide similar incentives for the manager to manipulate entrepreneurs' beliefs. To illustrate this point, we introduce unobserved managerial effort which increases fund returns. We show that, in his attempt to manipulate entrepreneurs' beliefs, the manager exerts greater effort, chooses a smaller fund size, and provides higher excess returns than when effort is observed. Investors' expected excess return is again persistent.

Finally, we endogenize the fee structure and investigate how these are affected by information asymmetries. We consider two specific scenarios: first, when the fees are not fully

⁷While total committed capital is often observable by outsiders ex post, it may not be possible to observe this quantity if, for instance, a subsequent fund is established before the previous fund is fully invested or exited. The size can also be viewed as capturing information asymmetries about the workload of the manager or about the extent of any diseconomies of scale.

⁸This is a "signal jamming" equilibrium in which an agent tries to affect the principal's perception of his ability by manipulating the signal. See, e.g., Holmstrom (1999) or Stein (1988).

transparent to outsiders and, second, when fees are transparent but the fund needs to commit to a small fund size in order to attract entrepreneurial firms. In the context of the first scenario, there is ample evidence that estimating the actual fees charged can be complicated even if most funds ostensibly announce a 1.5-3 percent management fee and a 20-25 percent carry interest (Gompers and Lerner, 1999; Phalippou and Gottschalg, 2008; Litvak, 2009). For example, distributions may generate an interest-free loan to VCs from limited partners (Litvak, 2009). Some fees may not be visible to outsiders and, as a result, Phalippou (2007) concludes that fee contracts are opaque. We show that, when the precise fees charged by the manager are not observed by the entrepreneurs, the manager not only charges lower fees compared to when fees are observable, but he also limits fund size. The fund's expected return to investors is therefore positive and persistent, even when fees are set optimally to maximize the fund manager's return.

For the second scenario, there are various reasons why entrepreneurs may be concerned about the size of the fund. One reason is that the value received by entrepreneurs may decrease as fund size increases and the fund manager has insufficient time to devote to various ventures, as in Fulghieri and Sevilir (2009). If the entrepreneurs do not observe the final size of the fund at the time they decide with which fund to go, the manager may need to reassure entrepreneurs that the final fund size is not going to be excessive. In this setting, we show that the fund's fees can act as a means of committing to a small size and can once again lead to performance persistence. However, this only happens when there is an incentive to manipulate the beliefs of investors, thus reinforcing our previous results.

Our model delivers a number of novel predictions in addition to explaining why private equity fund managers may limit fund size and provide persistent positive returns to investors. An interesting implication of the model is that, unlike VC funds, managers of LBO funds have little incentive to either keep fund size small or to deliver abnormal returns to their investors. The reason is that since target shareholders sell their stake and exit the company, there is no reason to manipulate their view of the fund manager beyond the actual offer made to them. This distinction between the different kinds of funds is, to the best of our knowledge, unique

to our framework. This prediction is generally consistent with the empirical evidence that buyout firms scale their size much faster in reaction to past positive returns (Metrick and Yasuda, 2010) and that their performance persistence is lower (Kaplan and Schoar, 2005) compared to venture capital funds.

There are also cross-sectional differences among venture capital funds in terms of their focus on early versus later investment stage (Gompers and Lerner, 1999), on lead versus non-lead position, and on investing in firms versus other funds. The two-sided matching process we study should be more important for venture capital funds that invest in early stage companies, given that the VC's added value could be more pronounced in the first two rounds of financing (Chemmanur, Krishnan and Nandy, 2009), for lead funds, and for funds that directly invest in portfolio firms. To the best of our knowledge whether performance persistence changes with the focus of venture capital funds has not been tested yet.

Although we do not explicitly model how competition affects the matching process, managers' incentives to manipulate entrepreneurs' beliefs should depend on the degree of competition among VC funds. With no competition, a manager has no incentive to manipulate beliefs, while too much competition decreases the marginal impact of manipulation. Therefore, we expect most of the effect to be present for intermediate levels of competition. To the extent that individual markets may exhibit different degrees of competition - either because of geographic separation (Chen, Gompers, Kovner and Lerner, 2009), specialization (Gompers, Kovner, Lerner, Scharfstein, 2008; Fulghieri and Sevilir, 2009), or over time - this presents an as yet untested prediction.

Over time a fund manager's incentive to manipulate entrepreneurs' beliefs should decrease as entrepreneurs obtain better information about his ability to add value. We therefore expect a positive relationship between past fund returns and changes in fund size, consistent with the evidence in Kaplan and Schoar (2005). Learning should also lead to larger fees by more experienced managers (see, e.g., Gompers and Lerner, 1999).

Our paper is related to Glode and Green (2008) and Hochberg, Ljungqvist and Vissing-Jorgensen (2008), both of which offer alternative explanations based on the observation

that private equity fund investors are mainly institutions, as compared to individuals in mutual funds. In Glode and Green (2008), fund managers limit their size and deliver excess returns to investors to induce them to not divulge information about the fund's strategy. Hochberg, Ljungqvist and Vissing-Jorgensen (2008) argue that past fund investors have private information concerning the fund manager's skill which allows them to extract rents by threatening to hold up the fund manager when he next starts a fund. This is because other potential investors interpret failure to reinvest by incumbent investors as a negative signal. The relation between institutional/large investors and fund performance has been investigated in some recent papers. Busse, Goyal and Wahal (2009), for instance, do not find performance persistence in mutual funds that cater to institutional investors and Phalippou (2009) finds that venture capital funds that are expected to be backed by more skilled investors show no performance persistence. These findings suggest that other fundamental differences between private equity and mutual funds may also be important in accounting for performance differences. Our work complements the existing literature by providing an asset-side explanation for why private equity and mutual funds exhibit different performance persistence.

Several recent papers analyze the optimal size of venture capital firms. Fulghieri and Sevilir (2009) show that a venture capitalist may limit fund size when it is important to provide entrepreneurial incentives to firms. A small portfolio increases the value-added to each firm by the VC and encourages entrepreneurs to exert higher effort, while a large portfolio allows the VC to reallocate resources in the case of startup failure and to extract greater surplus from entrepreneurs. Inderst, Mueller and Muennich (2007) argue that limiting the size benefits the venture capitalist by decreasing the bargaining position of portfolio firms. We provide an additional reason for why VC funds may limit size, which is to enhance the beliefs of outsiders with regard to his ability and, thereby, to improve the probability of matching with high quality firms in the future.

2 Model

At the start of any period t , a VC fund manager establishes a single new fund, raises cash from investors, and invests in entrepreneurial firms. The payoff to these investments, distributions to investors, and fund liquidation occur at the end of the period, and we describe these in more detail below. The manager is expected to start new funds for the indefinite future.⁹

In any period, there is a continuum of firms/entrepreneurs¹⁰ which can either be “good” or “bad.” A good firm is defined as one for which a fund’s investment, coupled with the fund manager’s ability, leads to value creation. For bad firms, however, investment and managerial ability creates no additional value. Fund managers are not always able to identify firm type perfectly, resulting in some bad firms receiving investment offers and some good firms being rejected. Firms do not know their types.

Firms need to raise a fixed amount of financing, normalized to \$1, in return for giving a fraction of the company to the investing fund. The quantity of shares and the price at which a firm’s shares are sold to a fund affect the manner in which any value created is shared between the fund and the entrepreneur. We abstract from the details of the bargaining game between the entrepreneurs and the fund manager and assume that both the entrepreneurs and the fund emerge with an equity stake in the firms. Hence, both parties receive a strictly positive share of the value created.

We denote a fund manager’s ability or talent by X , which is distributed according to $N(\bar{X}, \sigma_X^2)$. For good firms, the fund’s investment and managerial ability results in a value added of $X + \epsilon_t$ percent in period t , where ϵ_t is a manager-specific random shock. The shock ϵ_t is i.i.d. over time and is distributed as $N(0, \sigma_\epsilon^2)$. The actual realization of X is unknown to the fund manager, fund investors and firms. However, market participants update their expectation of the manager’s ability, $E_t[X]$, over time, using the information available at

⁹The assumption that the number of periods is infinite simplifies the exposition since it makes the value function stationary. Assuming either a finite number of periods or that there is a probability each period of the manager exiting permanently will not qualitatively affect our results.

¹⁰We will use the terms “firm” and “entrepreneur” interchangeably to refer to the party receiving an investment from a VC fund.

that time. Since managerial ability is associated with a higher value that is shared by firms, the firms always want to match with the manager that has the highest expected value added, $E_t[X]$, among available funds.

The return to the fund from investing in any period depends on the fraction of good firms in the portfolio and on the fund manager's ability to add value. We assume that matching between entrepreneurs and managers is positive assortative: managers with higher perceived ability are more likely to match with good firms. Positive assortative matching is a common prediction of various models (Titman and Trueman, 1986, Chemmanur and Fulghieri, 1994, Fernando, Gatchev and Spindt, 2005), and we simply take it as given here. However, it is straightforward to see how positive assortative matching could arise in our setting. Since entrepreneurs want to match with high ability managers, let us suppose that they approach available fund managers sequentially, in the order of their perceived ability. Managers, in turn, screen for firm type, albeit with some error, and choose from among the firms that approach them, until they are fully invested. Assuming that managers do not observe each others' decisions, the proportion of good firms among the set of firms that still seek financing will decrease after each fund manager invests. The reason is that a firm that is declined financing will continue to search and this firm is more likely to be bad given that managers have some ability to screen. As a result, managers that are lower ranked face a pool that becomes progressively worse.

In light of this discussion, we make the (reduced form) assumption that managers that have higher expected ability to add value, $E_t[X]$, end up investing in a relatively larger fraction of good firms, denoted by P'_t . Using β as the share of the surplus that goes to the fund, we can now define $P_t = \beta P'_t$ as the fraction of a fund's investment that is in good firms and to which the fund can lay claim. Our matching assumption implies that P_t is increasing in $E_t[X]$.

We assume that the per unit cost of adding value to good firms is $c(Q_t)$, where Q_t is the amount of funds under management. This cost is independent of ability and is increasing and convex in Q_t (this is as in Berk and Green, 2004), and is borne privately by the fund,

representing, for instance, the cost of identifying and monitoring investments. However, it does not affect returns to entrepreneurs, who only care about $E_t[X]$. This assumption implies decreasing returns to scale in private equity, as documented by Lopez-de-Silanes, Phalippou and Gottschalg (2009). The gross percentage return from investing in good firms is equal to $W_t = X + \epsilon_t - c(Q_t)$, while that from investing in bad firms is zero.¹¹

The fund manager raises capital from investors and charges a fixed fee f_t , which is a percentage of the committed funds Q_t , as well as a variable fee $0 \leq v_t \leq 1$, which is a percentage of the fund's return net of the fixed fee.¹² Given the total return $P_t W_t$ to the fund in period t , the net return to investors can be written as $\alpha_t = (1 - v_t)(P_t W_t - f_t)$. Investors are competitive and risk-neutral and, hence, are willing to provide capital as long as their expected net return, $E_t[\alpha_t]$, is nonnegative.

We assume that there is symmetric information between fund investors (limited partners) and managers (general partners). Entrepreneurs, by contrast, observe only the fund's history h_t , which includes all realized returns α_i for $i < t$ as well as possibly other fund characteristics depending on our assumptions about observability. We will consider a variety of different cases depending on whether the fees (f_t, v_t) , the fund's size (Q_t) , and/or managerial effort, which will be introduced later, are observable by the target firms.

The manager chooses his actions to maximize his expected payoff subject to the participation constraint of the investors and the impact of his actions on firms' beliefs about his ability X . We can write the total expected future payoff for the fund manager as of time t

¹¹Note that, in principle, the return from investing in "bad" firms could actually be negative if the fraction of shares the fund obtains on its investment is low relative to the size of the investment being made. It is straightforward to show that even assuming that bad firms have a value of zero, so that the return to the fund is -100%, does not qualitatively affect our results. We therefore assume a zero return for ease of exposition.

¹²Throughout our analysis we impose limited liability for the fund manager ex ante, but not ex post. This dramatically simplifies the analysis, since with a normally distributed noise term ϵ_t , one can never rule out the possibility that the realized return after the fixed fee, $P_t W_t - f_t$, could be negative. In such cases, the manager's payoff from the variable fee would be negative. Imposing limited liability ex post requires that the variable fee be non-linear, since it would have to have a flat (i.e., zero) portion whenever $P_t W_t - f_t < 0$. We believe such an extension, while considerably more complicated, would not qualitatively change our arguments.

as

$$V_t = E_t \left[\sum_{i=t}^{\infty} \delta_t (Q_i v_i (P_i W_i - f_i) + Q_i f_i) \right],$$

where $\delta_t < 1$ is the discount factor, which we ignore in the rest of the paper for brevity.

3 The Symmetric Information Case

As a starting point, we analyze the case where there are no information asymmetries, so that entrepreneurs observe all relevant fund characteristics as well as all managerial decisions. We start with the case where the variable fee v_t is equal to zero, and all the compensation the fund manager receives comes from the fixed fee f_t , which (for now) is set exogenously at some time-invariant level $f > 0$. This corresponds to the case studied in Berk and Green (2004), who show that when managers receive a fee that is a fraction of assets under management, there is no performance persistence in mutual fund returns. This occurs because positive returns attract fund flows, which in turn eliminates a manager's ability to provide excess returns. As we show below, the same result obtains in our setting.

Specifically, in each period entrepreneurs observe managers' choices of fund size Q_i for $i < t$ before deciding whether to accept an offer. The fixed fee f is common knowledge. Entrepreneurs can then use the information available to them to estimate a manager's ability to add value, X . Bayesian updating after observing α_t gives us the conditional expectation of X at time $t + 1$ as:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) \left(\frac{\alpha_t}{P_t} + \frac{f}{P_t} + c(Q_t) \right), \quad (1)$$

since $\alpha_t = (P_t W_t - f)$ and $W_t = X + \epsilon_t - c(Q_t)$. (1) simplifies to

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) (X + \epsilon_t).$$

The weights w_t reflect the importance of the most recent realization of fund returns, α_t ,

relative to the past history, h_{t-1} , in updating entrepreneurs' expectations concerning X . The weight assigned to new information, $1 - w_t$, decreases over time through simple Bayesian updating.¹³ Since Q_t is observed, entrepreneurs' expectations about the manager's ability at time $t + 1$ should not be affected by the manager's choice of fund size:

$$\frac{\partial}{\partial Q_t} E_{t+1} [X|h_t] = 0. \quad (2)$$

Writing the Lagrangean of the optimization problem as

$$\max_{Q_t} E_t \left[\sum_{i=t}^{\infty} Q_i f - \lambda_i (P_i W_i - f) \right],$$

where λ_i is the Lagrange multiplier on the investors' participation constraint, that $E_t [\alpha_t] = E_t [P_t W_t - f] \geq 0$, we can now state the following result.

Proposition 1 *For any fixed fee $f > 0$, when there is no information asymmetry between the fund manager and entrepreneurs, the manager chooses a fund size Q_t such that his expected payoff in period t is maximized. Fund investors' expected excess return $E_t[\alpha_t]$ is zero.*

Since, as highlighted by (2), the manager's policy decisions today do not affect entrepreneurs' expectations, they also do not affect his payoffs in the future. As a consequence, in any given period the manager will choose the size of the fund to maximize his expected payoff in that period, subject to the constraint that investors earn a nonnegative expected return. Since the market for investors is competitive, this is achieved by choosing a fund size that leaves no excess returns for investors. We therefore obtain the same result as Berk and Green (2004).

We next endogenize the fees charged by the fund. The manager not only chooses fund

¹³Given that the underlying distributions are normal, the weight placed on the new information in Bayesian updating is determined by the precision of the new information relative to the precision of the prior. Specifically, if the conditional distribution of X at $t - 1$ is denoted by $N(E(X|h_{t-1}), \sigma_{X,t-1}^2)$, we have $1 - w_t = \frac{1/\sigma_\epsilon^2}{1/\sigma_\epsilon^2 + 1/\sigma_{X,t-1}^2}$. It is apparent that as the precision of the conditional distribution of X increases through time, the weight placed on the new information will correspondingly decrease.

size Q_t but also all fees, v_t and f_t , every period, and entrepreneurs observe these choices. Again, since the manager's decision variables are all observed, entrepreneurs' expectations about the manager's ability at time $t + 1$ should not be affected by the manager's choices:

$$\frac{\partial}{\partial Q_t} E_{t+1} [X|h_t] = 0, \quad \frac{\partial}{\partial f_t} E_{t+1} [X|h_t] = 0, \quad \frac{\partial}{\partial v_t} E_{t+1} [X|h_t] = 0. \quad (3)$$

We can write the Lagrangean of the optimization problem as

$$L = \max_{Q_t, f_t, v_t} E_t \left[\sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_i (1 - v_i) (P_i W_i - f_i) + \gamma_i (1 - v_i) + \beta_i (P_i W_i - f_i) \right].$$

Denoting by Q_t^* the optimal fund size in period t , we can now establish the following result.

Proposition 2 *When all the fund manager's decision variables are observable, the manager chooses Q_t , v_t and f_t such that the expected total value generated by the fund in period t is maximized. In equilibrium, the fund manager sets either $v_t = 1$ or $f_t = E[P_t W_t^*]$, where W_t^* is the gross return at the optimal fund size Q_t^* , and captures all the expected surplus he generates, so that $E_t[\alpha_t]$ is zero.*

The proposition establishes that when the manager can choose what fees to charge, he will choose them in a way that extracts all the expected surplus. In the proof of Proposition 2 we show that whether $v_t = 1$ or $f_t = E[P_t W_t]$ is immaterial in terms of the choice of fund size. In other words, whether the manager is compensated through the variable or the fixed does not affect his choice of fund size (or his expected compensation). Note as well that the solution where $f_t = E[P_t W_t^*]$ is exactly the optimal compensation scheme analyzed in Berk and Green (2004).

4 The Role of Asymmetric Information

Asymmetric information is a widely accepted characteristic of financial markets. Especially in the context of private equity funds, it is likely that certain characteristics of the fund that directly affect realized returns to fund investors are not observed by outsiders (see, e.g., Phalippou and Gottschalg, 2008, or Cumming and Walz, 2009). In particular, information asymmetries are likely to exist about variables under the manager’s control. These may include the quality of the firms in the fund’s portfolio, the risk of the portfolio, the exact fees charged, the fund size, and managerial effort, among others. The information asymmetry problem is even more severe for funds that have not fully exited all of their investments.

Here, we study the effect of asymmetric information between fund managers and entrepreneurs regarding certain key managerial decisions. To highlight the role of information asymmetry, throughout we will compare the results under asymmetric information against what would obtain if the decisions were observable by the target entrepreneurs.

4.1 Unobserved Fund Size

We start with a simple case, which is where entrepreneurs cannot directly observe the actual quantity placed by a manager, so that the overall size of the previous fund is not observed at the time entrepreneurs decide to match with a manager. This is likely to be true when a manager establishes a new fund before fully investing the funds of the previous one. For example, a fund manager may return some of the committed capital back to investors if he cannot find good investment opportunities. However, one could also consider this case as capturing information asymmetries about the workload of the manager or about the extent of any diseconomies of scale (Lopez-de-Silanes, Phalippou and Gottschalg, 2009) because, in our model, fund size affects the estimation of ability through the cost of identifying and monitoring the investments, $c(Q_t)$.

To establish a benchmark, we begin by considering the case where there are no information asymmetries, so that fund size is observed by the entrepreneurs. For now we will take

the fees to be given in order to isolate the effect of information asymmetry regarding fund size. Later we endogenize a manager's choice of fees under asymmetric information (the case with symmetric information has already been studied in Section 3). For any given fee structure (v_t, f_t) , Bayesian updating after observing α_t gives us the conditional expectation of X at time $t + 1$ as:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) \left(\frac{\alpha_t}{(1 - v_t)P_t} + \frac{f_t}{P_t} + c(Q_t) \right), \quad (4)$$

since $\alpha_t = (1 - v_t)(P_t W_t - f_t)$ and $W_t = X + \epsilon_t - c(Q_t)$. (4) simplifies to

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t)(X + \epsilon_t).$$

As in the previous section, since the manager's decision variable Q_t is observed by everyone, entrepreneurs' expectations about the manager's ability at time $t + 1$ should not be affected by the manager's choice at time t : $\frac{\partial}{\partial Q_t} E_{t+1}[X|h_t] = 0$. We can write the Lagrangean of the optimization problem as

$$\max_{Q_t} E_t \left[\sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_i (1 - v_i) (P_i W_i - f_i) \right],$$

where λ_i is the Lagrange multiplier on the investors' participation constraint. The Kuhn-Tucker conditions for the fund manager's maximization problem at time t are

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t (P_t E_t[W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t (1 - v_t) P_t c'(Q_t) = 0, \quad (5)$$

$$\lambda_t (1 - v_t) (P_t E_t[W_t(Q_t^O)] - f_t) = 0, \quad (6)$$

where (6) is the complementary slackness condition for the investors' participation. Q_t^O is simply the equilibrium fund size when fund size is observable.

We can now compare the manager's maximization problem under symmetric information

with the equivalent problem under asymmetric information about fund size. Specifically, in each period, entrepreneurs do not observe managers' past choices of quantity Q_i for $i < t$ before deciding whether to accept an offer.¹⁴ Since firms are unaware of the fund's size, they must now form a conjecture Q_t^C at time $t + 1$ about the fund's size, and use that in updating their estimate of the manager's talent. Firms update their beliefs in a Bayesian fashion as follows:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) (X + \epsilon_t - c(Q_t) + c(Q_t^C)), \quad (7)$$

where the term $X + \epsilon_t - c(Q_t) + c(Q_t^C)$ represents the inference drawn from observing fund returns, α_t , about managerial talent X , given entrepreneurs' conjectured fund size Q_t^C . From (7) it is clear that increasing fund size negatively affects entrepreneurs' estimation of the managers' ability since

$$\frac{\partial}{\partial Q_t} E_{t+1}[X|h_t] = -(1 - w_t) \frac{\partial c(Q_t)}{\partial Q_t} < 0. \quad (8)$$

Given that the manager's actions affect entrepreneurs' future expectations concerning his talent, the manager's maximization problem now becomes dynamic. In this dynamic optimization problem the state variable is entrepreneurs' beliefs about the ability of the manager. The choice variable is fund size. We can write the Bellman equation as

$$V_t(E_t[X|h_{t-1}]) = \max_{Q_t} [Q_t v_t(P_t W_t - f_t) + Q_t f_t + V_{t+1}(E_{t+1}[X|h_t])],$$

subject to the constraint that the state variable, entrepreneurs' beliefs, evolves according to (7), as well as investors' participation constraint, $E_t[(1 - v_t)(P_t W_t - f_t)] \geq 0$. Given that (7) is an equality, we can plug in the expected value $E_{t+1}[X|h_t]$ in V_{t+1} and write the

¹⁴Note that all our results go through if only Q_{t-1} is not observed. This is very likely to be the case if a subsequent fund is established before fully investing the previous fund.

Kuhn-Tucker conditions as follows:

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_t E_t[W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t(1 - v_t)P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0, \quad (9)$$

$$\lambda_t(1 - v_t)(P_t E[W_t(Q_t^U)] - f_t) = 0, \quad (10)$$

where λ_t is again the Lagrange multiplier on the investors' participation constraint at time t and Q_t^U denotes the equilibrium fund size when fund size is unobserved. We can now state the following result.

Proposition 3 *For any fee structure (v_t, f_t) with $v_t \in [0, 1)$ and $f_t \in [0, P_t E[W_t(Q_t^U)]]$, when fund size is not observed managers choose a lower fund size Q_t compared to the case when fund size is observed. Fund investors' expected return $E_t[\alpha_t]$ is always strictly positive and persistent when fund size is not observed, and is strictly higher than what obtains when fund size is observed.*

The proposition highlights that asymmetric information about the fund's size leads fund managers to always offer a positive return to investors, even in situations where no such return would arise if fund size were observed by entrepreneurs. Managers do this precisely by restricting the size of the fund, hoping that by doing so and offering a larger than expected return to investors they will be seen by entrepreneurs as having greater talent and, therefore, a greater ability to add value.

The key to Proposition 3 is (8), which highlights the fact that since fund size Q_t is not observed, a fund manager believes he can influence entrepreneurs' beliefs upward by raising a smaller fund. The effect of fund size can be seen directly by comparison of the first order conditions for the symmetric information case, given by (5), and the asymmetric information case, given by (9). These conditions differ by the single term, $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t}$, which reflects precisely the incentives the fund manager has to manipulate entrepreneurs' beliefs about his ability. Given that this term is negative, the equilibrium fund size will be lower when fund size is not observed. As a result, the manager provides positive and persistent returns to

fund investors. In equilibrium, of course, entrepreneurs correctly conjecture the manager's incentives and anticipate the actual fund size. That is, the manager is unable to fool entrepreneurs, and his ability is inferred in an unbiased way. Nevertheless, the asymmetry of information leads the manager to leave money for investors, given his incentive to manipulate entrepreneurs' beliefs. This incentive and investors' expected excess return decreases over time as entrepreneurs obtain more precise information about a manager's ability to add value. Nevertheless, there is performance persistence.

It is worth noting that Proposition 3 holds for all possible fee structures (v_t, f_t) that are not strictly at the boundary, where the entire return goes to the fund manager independently of the fund size that is chosen. This would be the case if the variable fee v_t were equal to 1, so that 100% of the fund's return accrued to the manager. Likewise, if the fixed fee f_t were equal to $P_t E[W_t(Q_t^U)]$, the entire equilibrium return of the fund would be appropriated by the manager through the fixed fee, leaving nothing to investors. Later we show that, if we allow the fund manager to also choose the fund's fee structure, he will always choose a structure such that $v_t < 1$ and $f_t < E[P_t W_t^*]$ under asymmetric information about fees, so that the constraint on fees is without loss of generality. By contrast, under symmetric information the manager will choose either a variable fee v_t or a fixed fee f_t at the boundary so that he always extracts all rents from investors as shown in Section 3.

4.2 Unobserved Effort

While the results above are presented in the context of information asymmetry about fund size, our argument is more general. Information asymmetry about any choice variable the manager controls and which affects gross returns can provide similar incentives to the manager to manipulate the beliefs of entrepreneurs. To illustrate this point, we introduce unobserved managerial effort that improves the value of the investments by mitigating the cost of allocating funds. For instance, this would be the case if effort is used to identify good firms but not necessarily to add value. The gross return delivered by a fund manager in any period t is now given by $W_t = X + \epsilon_t - (c(Q_t) - e_t)$, where e_t denotes the manager's effort.

Effort is privately costly to the manager, with the cost equal to $Q_t C(e_t)$, which is increasing and convex in effort, and is increasing with the size of the fund because the marginal effect of effort on fund returns would be expected to be lower for larger funds. For simplicity, we assume that all managers have the same cost of effort.

Since effort is not observed, entrepreneurs will need to conjecture the actual level of managerial effort in order to assess the managers' ability. We denote such conjecture as e_t^C . As before, entrepreneurs' form their expectations about the manager's ability to add value based on the history of realization of net returns for investors as well as the current returns:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t)(X + \epsilon_t + e_t - e_t^C). \quad (11)$$

Given entrepreneurs' fixed beliefs about the manager's effort choice, e_t^C , the manager's actual decision clearly affects the perceived talent next period since

$$\frac{\partial}{\partial e_t} E_{t+1}[X|h_t] = (1 - w_t) > 0.$$

We can write the Bellman equation as

$$V_t(E_t[X|h_{t-1}]) = \max_{Q_t, e_t} [Q_t v_t (P_t W_t - f_t) + Q_t f_t - Q_t C(e_t) + V_{t+1}(E_{t+1}[X|h_t])],$$

subject to the usual participation constraint, $E_t[(1 - v_t)(P_t W_t - f_t)] \geq 0$, as well as (11).

The Kuhn-Tucker conditions are as follows:

$$v_t(P_t E[W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1 - v_t) - C(e_t) = 0, \quad (12)$$

$$\lambda_t E_t[(1 - v_t)(P_t W_t - f_t)] = 0, \quad (13)$$

$$Q_t v_t P_t - Q_t \frac{\partial C(e_t)}{\partial e_t} + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t} = 0. \quad (14)$$

As argued above, asymmetric information about managerial effort implies that $\frac{\partial E_{t+1}[X]}{\partial e_t} >$

0. Likewise, $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} > 0$ since it must be that a greater perceived managerial talent translates into a higher expected continuation value. When effort is unobserved, the multiplication of these two terms, seen in (14), affects the level of effort. This effect is not present when information is symmetric, and it increases the incentive for the manager to exert higher effort because he considers not only the contribution to his payoff in the current period but also the effect of his effort on his future payoff through entrepreneurs' expectations of his ability to add value. As in the case with unobserved fund size, entrepreneurs will, of course, recognize the manager's incentive to mislead them. In equilibrium, entrepreneurs' conjectures must be correct and they will not be fooled by the manager's effort choice. Nevertheless, the manager attempts to manipulate entrepreneurs' beliefs by increasing his effort because otherwise he would be perceived as having low ability. In addition, the manager raises a smaller fund than what is optimal when effort is observed, despite the fact that his choice of fund size is observed by the entrepreneurs. This occurs because managing a larger fund size makes it more costly to provide higher returns to investors through higher effort. We can now summarize this discussion in the following result.

Proposition 4 *For any fee structure (v_t, f_t) with $v_t \in [0, 1)$ and $f_t \in [0, E[P_t W_t^*])$, when effort is not observed, managers exert higher effort and choose a lower fund size Q_t compared to the case when effort is observed. Fund investors' expected return $E_t[\alpha_t]$ is always positive, persistent and larger than the case when effort is observed.*

As firms obtain more information about a manager's ability to add value, they give less weight to new information, i.e., $(1 - w_t)$ decreases over time, which reduces a manager's gains from attempting to manipulate entrepreneurs' beliefs. As a result, excess effort decreases and quantity (i.e., fund size) approaches over time the level obtained under symmetric information. However, there will be performance persistence because investors' excess returns are positive and only gradually approach zero.

5 Endogenizing the Fee Structure

So far we have treated the fees charged by the fund as a parameter (other than in Section 3 with symmetric information). In this section we endogenize the fee structure in the presence of asymmetric information. We consider two specific scenarios: (i) when the fees are not transparent to outsiders; and (ii) when fees are transparent, but the fund needs to commit to a small fund size in order to attract entrepreneurial firms.

5.1 Opaque Fees

We start with a setting in which fees are chosen by the fund manager but are not perfectly transparent to outsiders - in particular to the entrepreneurs. That the exact structure of fees may not be fully visible to all outsiders is realistic and conforms to many of the recent findings concerning the fee structures for private equity funds. Although most funds announce a 1.5-3 percent management (i.e., “fixed”) fee and a 20-25 percent carry interest (i.e., “variable” fee) (Gompers and Lerner, 1999; Phalippou and Gottschalg, 2008; Litvak, 2009), in many instances it appears that there is some uncertainty regarding the exact fees that are paid and the discretion the manager has in determining the level of these fees. For instance, there may be hidden fees such as distributions that generate an interest free loan to VCs from limited partners or kickbacks that rewards investors. Litvak (2009) argues that the timing of distributions might be as important a part of managerial compensation as the management fee or carry interest. Some fees are not visible to outsiders such as the fees charged to portfolio companies by buyout funds (Phalippou, 2007; Metrick and Yasuda, 2010). There are also a number of extra fees or costs that can be imposed, leading some observers, such as Phalippou (2007), to conclude that fee contracts are opaque. These findings suggest that contracts may not be very transparent even to the investors in the fund, making it even less likely that they are transparent to the target entrepreneurs.

As we have done so far, we first establish a basis for comparison by analyzing the extreme case where the fees chosen by the fund manager are perfectly observed by entrepreneurs. We

do assume, however, that fund size is not observed by the entrepreneurs, although similar results obtain if, instead of fund size, we assume that there is managerial effort that is unobservable. (The case where fund size is observable has already been studied in Section 3.) Entrepreneurs conjecture the size of the fund Q_t^C in trying to estimate the talent of the manager. Beliefs are updated as follows:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) (X - c(Q_t) + c(Q_t^C)) \quad (15)$$

From (15) it is clear that decreasing fund size would be expected to positively affect entrepreneurs' estimates of the fund manager's talent. However, the manager's choice of fees does not affect the inference entrepreneurs make about his ability since these fees are perfectly observed, allowing entrepreneurs to filter out the choice of fees in forming their expectations of X . From this, we can now establish the following.

Proposition 5 *When fees are perfectly observed but fund size is unobserved, the manager limits fund size by choosing a size lower than what he would choose if fund size were observed. However, the manager captures all the surplus he generates through the fee structure (v_t, f_t) , so that investors' expected excess return is zero: $E_t[\alpha_t] = 0$.*

This result establishes that under asymmetric information (about fund size, in this case), fund managers find it optimal to limit the size of the fund in an attempt to manipulate entrepreneurs' beliefs, even when fees are set endogenously and are designed to maximize the manager's return. As the proof in the appendix shows, this is achieved either by setting a maximal variable fee, $v_t = 1$, or by setting the fixed fee f_t equal to the expected gross return of the portfolio, $P_t E[W_t^*]$, where W_t^* represents the realized return at the equilibrium choice of fund size Q_t^* . In this case, the fund manager loses some surplus by choosing a fund size that is lower than in the symmetric information case. However, the fund delivers neither abnormal nor persistent returns.

We next endogenize the fee structure for the more realistic case where there is asymmetric information regarding the fund's fees. We will do this for two different scenarios: first, for

the case in which there is no information asymmetry regarding any variable other than fees; second, for the case where fund size is unobservable in addition to fees. The first case can be compared against the full-information case described in Proposition 2. The second is analogous to the case studied above in Proposition 5.

In the first case, entrepreneurs conjecture the fee structure v_t^C and f_t^C and use the fund's past history to estimate the manager's talent:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) \left(\frac{(1 - v_t)(P_t(X + \epsilon_t - c(Q_t)) - f_t)}{(1 - v_t^C)P_t} + \frac{f_t^C}{P_t} + c(Q_t) \right). \quad (16)$$

The manager's choice of fees today clearly affects the talent perceived by entrepreneurs next period. By charging a lower fixed or variable fee than conjectured by the entrepreneurs, the manager delivers a higher return to investors at time t (i.e., α_t) and hopes to convince entrepreneurs that he has superior talent, which increases the fraction of good firms in the manager's portfolio in the future. This effect can be seen clearly from the derivative of the conditional expectation of talent with respect to the manager's choice variables,

$$\frac{\partial}{\partial f_t} E_{t+1}[X|h_t] < 0, \quad \frac{\partial}{\partial v_t} E_{t+1}[X|h_t] < 0.$$

We can now write the Bellman equation for the manager:

$$V_t(E_t[X|h_{t-1}]) = \max_{Q_t, f_t, v_t} [Q_t v_t (P_t E_t[W_t] - f_t) + Q_t f_t + V_{t+1}(E_{t+1}[X|h_t])],$$

subject to the investors' participation constraint, (16), and the feasibility constraints that $v_t \leq 1$ and $f_t \leq P_t E[W_t^*]$ for all t .

Proposition 6 *When fund fees are not observed by entrepreneurs, the manager chooses fees such that $v_t < 1$ and $f_t < P_t E[W_t^*]$. In addition, he does not accept all the funds investors are willing to invest. Investors' expected return, $E_t[\alpha_t]$, is always strictly positive and persistent.*

The proposition establishes that the manager limits fees and provides positive expected

returns to investors, thus validating the parametric restriction on fees that was maintained throughout Section 4, namely that $v_t < 1$ and $f_t < P_t E[W_t^*]$, so that the fund's fees are not at the boundary (where all returns accrue to the manager). The intuition for this is the same as that discussed in prior sections: when fees are not observed, the manager has an incentive at the margin to reduce his fees so as to increase the return to investors, with the hope of improving entrepreneurs' perceptions of his ability. In this case, the manager provides positive expected return $E_t[\alpha_t]$ to the fund's investors and thus does not capture all the surplus. Given that the fees are lower, investors would actually be willing to invest more than in the case where fees are perfectly observable. However, the manager limits the size of the fund and does not accept all funds offered by investors. As a result, the fund's size could be larger or smaller than in the case where fees are observed, such as in Proposition 2.

We now characterize the optimal fee structure for the second case indicated above, where both fees and fund size are unobservable to entrepreneurs. Entrepreneurs must now form conjectures v_t^C , f_t^C and Q_t^C , about fees and fund size, respectively, in order to form expectations of the fund manager's ability:

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) \left(\frac{(1 - v_t)(P_t(X + \epsilon_t - c(Q_t)) - f_t)}{(1 - v_t^C)P_t} + \frac{f_t^C}{P_t} + c(Q_t^C) \right). \quad (17)$$

The manager's choice of fees and fund size at time t clearly affect the talent perceived by entrepreneurs at $t + 1$. We then obtain the following result.

Proposition 7 *When both fund size and fees are not observed by entrepreneurs, the manager again limits fees ($v_t < 1$ and $f_t < P_t E[W_t^*]$) and fund size. The fund's excess expected return to investors, $E_t[\alpha_t]$, is always strictly positive and persistent.*

The results from this proposition can be contrasted with those of Proposition 5, which analyzes the case where fees are perfectly observable by entrepreneurs. With asymmetric information about fees, Proposition 7 once again establishes that the fund manager finds it optimal to set a fee structure that is strictly lower than the maximum values he could choose (i.e., $v_t = 1$ or $f_t = P_t E[W_t^*]$) in order to attempt to manipulate the beliefs of the target

entrepreneurs. In other words, even when fees are set optimally and the manager can choose any size fund as long as it satisfies investors' participation constraints, he will prefer to leave money on the table along both dimensions, fees and fund size, when entrepreneurs are not informed about his choices.

It is important to note that while we believe some degree of unobservability of fees on the part of entrepreneurs is realistic, the results above suggest that it can also be an important ingredient in accounting for positive fund returns received by investors and performance persistence. In the scenario above, when fees are perfectly observable, investors do not receive excess return. However, as we discuss below, a simple extension to the model can deliver positive and persistent performance even when fees are fully observable and freely determined by the manager.

5.2 Using Fees in Committing to a Small Fund Size

In our model the only factor that affects the likelihood of a manager matching with high quality firms is his perceived ability. This simple setup allows us to focus our attention on the effect of information asymmetries on the matching process and its role in explaining return patterns in private equity funds. It is, however, possible to extend the model to one in which entrepreneurs also care about the size of the fund. One reason for caring about fund size is that the value received by entrepreneurs may decrease as fund size increases, as in Fulghieri and Sevilir (2009). This may be the case, for instance, if running a large fund leaves a manager relatively little time to dedicate to individual firms, so that their value added is likely to be low when the fund gets too large (Lopez-de-Silanes, Phalippou and Gottschalg, 2009; Cumming and Dai, 2010). If firms can match with other funds that are smaller or remain independent in hopes of getting a better match at a later date, this would imply a maximum fund size for a given level of perceived ability. If the entrepreneurs do not observe the final size of the fund at the time they decide with which fund to go, the manager may need to reassure entrepreneurs that the final fund size is not going to be excessive. In this setting, we can show that the fund's fees can act as a means of committing to a small

size and lead to performance persistence even when fees are observed. In other words, small variations of the basic model yield the result that the fund manager may optimally choose to provide investors with a positive excess expected return, even when fees are perfectly observable.

Specifically, assume that fees are determined first and fund size Q_t second, and that fund size Q_t is not observed and cannot be contracted upon at the time entrepreneurs decide to accept a manager's offer. If entrepreneurs believe that the fund's size will exceed some cutoff or maximum size Q^M , they reject the offer. Hence, it is important for the fund manager to reassure entrepreneurs that the fund's size will remain below this threshold.

We only analyze the most interesting case when the size threshold Q^M is binding in the sense that, absent other concerns, the fund manager would optimally choose a fund size larger than the threshold, at least for situations where the fees are (exogenously) set to close to their maximal levels, i.e., for $v_t \approx 1$ and $f_t \approx E_t[P_t W_t^*]$. We have the following result.

Proposition 8 *When the fund manager needs to commit to a maximum fund size Q^M , there exist parameter values such that: (1) the equilibrium scale is $Q_t \leq Q^M$; (2) $v_t < 1$ and $f_t < E_t[P_t W_t]$; and (3) $E_t[\alpha_t] > 0$, even when all fees are perfectly and publicly observed.*

The perfect observability of fees makes the manipulation of entrepreneurs' beliefs impossible using only fees, since entrepreneurs can easily back out the total fund returns from observing the actual fees that are paid to the manager. This proposition demonstrates that the finding of positive and persistent fund returns is robust to the possibility that fees are observed if fund managers have other considerations in determining fees. In the case studied here, setting a relatively low fee structure, and one that fails to fully extract all surplus from investors, reduces the incentives of the fund manager to raise too much capital from investors and thus provides a commitment (to the entrepreneurs) to run a relatively small fund.

In the proof of Proposition 8 we also show that the optimality of providing positive excess returns (i.e., $E_t[\alpha_t] > 0$) relies crucially on the incentive the fund manager has to manipulate entrepreneurs' beliefs concerning his talent. Absent such incentive for the fund manager, the

optimal fee structure will always fully extract all surplus from investors, even if the manager needs to commit to maintain a relatively small fund size. This occurs because, in the absence of an incentive to manipulate firms' beliefs, any fee structure that achieves commitment can be replaced by another structure that achieves the same commitment but that fully extracts all the rents from investors. This can be seen from the derivatives of optimal size with respect to fees. This derivative is always negative for the variable fee and positive for the fixed fee when there is no incentive to manipulate beliefs of firms. Therefore one can keep fund size the same while slightly increasing both the variable and the fixed fee until the manager captures all the surplus. This, however, cannot always be done when there is an incentive to manipulate entrepreneurs' beliefs. Put differently, the signal jamming argument used throughout our analysis is a crucial requirement for performance persistence.

6 Empirical Predictions

In addition to explaining why private equity fund managers may limit the size of their funds and provide persistent positive returns to investors, we provide several novel cross sectional and time series predictions about the performance of private equity funds.

In most instances of venture capital financing, the entrepreneur retains a significant stake in the firm, especially when his input (e.g., effort) is important for the firm. Therefore, entrepreneurs have an incentive to match with fund managers that can add value to their firms. By contrast, in leveraged buyout (LBO) deals existing investors are paid ex-ante and typically sell all of their stake in the company. Therefore, firms that are targets of a buyout are not concerned about inferring the value-adding ability of the manager beyond the actual offer that is being made to them. As a result, managers of LBO funds should have little incentive to either keep fund size small or to charge lower fees to their investors as a way of manipulating the beliefs of shareholders at the target firms. An exception could be LBO funds that keep the management of the target firm in place. Although management turnover increases after the LBOs, some managers stay with the firm (Muscarella and Vetsuypens,

1990; Cornelli and Karakas, 2010) and management often has substantial ownership after the LBO (Kaplan 1989; Kaplan and Stromberg, 2009). Therefore, management may be more supportive of a buy-out offer if the buyout fund has a higher ability to add value. However, this becomes important primarily when there are no other higher competing bids as shareholders should have little interest in the continuation value of the firm. Our model, therefore, distinguishes between the returns of venture capital versus LBO funds. This prediction, to the best of our knowledge, is unique to our framework, and is consistent with recent empirical findings. For example, Kaplan and Schoar (2005) find that the coefficient of past returns in estimating future returns (performance persistence) is much lower for LBO funds compared to venture capital funds. Consistent with this, Metrick and Yasuda (2010) find that LBO firms scale their size much faster in reaction to past positive returns compared to venture capital firms.

There are also cross-sectional differences among venture capital funds in terms of their focus on early versus later investment stage (Gompers and Lerner, 1999), on lead versus non-lead position, and on investing in firms versus other funds. Empirical evidence finds that VC's value added is more pronounced in the first two rounds of financing (Chemmanur, Krishnan and Nandy, 2009). Therefore, the matching process should be more important for VC funds that specialize in earlier round investing, with the implication that our findings concerning fund returns and persistence should be more applicable to these funds. Similarly, one could also argue that lead VCs spend more time with their portfolio firm and may potentially add more value compared to non-lead VCs. This would make our arguments more applicable to VCs that undertake lead investments. In addition, we expect our arguments to apply more for funds that directly invest in portfolio companies rather than funds that invest in other funds because matching with entrepreneurs is only important for the former. To the best of our knowledge, these are also novel predictions of our framework.

Although we do not explicitly model how competition affects the matching process between entrepreneurs and funds, a manager's incentive to manipulate entrepreneurs' beliefs could depend on the degree of competition among venture capital funds to attract good

targets. With no (or little) competition, a manager has little incentive to manipulate beliefs: since there are no (or few) other funds, the manager does not expect to improve the odds of matching with good firms by manipulating beliefs. On the other hand, too much competition also decreases the marginal impact of manipulation on the probability of matching with good firms. Therefore, we expect the relationship between competition and performance persistence to be nonlinear, with most of the effect being present for intermediate levels of competition. If individual markets exhibit different degrees of competition - either because of geographic separation (Chen, Gompers, Kovner and Lerner, 2009), specialization (Fulghieri and Sevilir, 2009; Gompers, Kovner, Lerner, Scharfstein, 2008), or fund availability over time, one could test whether performance persistence varies within these dimensions. These predictions we believe have not been tested so far.

In the time series, as a manager gets more experienced, firms should have better information about the ability of this manager. Therefore, according to our model, over time a fund manager's incentive to manipulate firms' beliefs should decrease. As a result we expect a positive relationship between past fund returns and changes in fund size. This is consistent with recent evidence in Kaplan and Schoar (2005). Our model also predicts that the learning that takes place over time about a fund manager's ability implies larger fees by more experienced managers, as found by Gompers and Lerner (1999).

7 Conclusion

Anecdotal evidence suggests that many successful private equity funds are oversubscribed. On the other hand, private equity funds appear to generate persistent abnormal returns for their investors, in contrast to mutual funds, which exhibit little or no performance persistence. We argue that private equity funds are fundamentally different from mutual funds because of two reasons: First, private equity funds need to match with good firms, which want to match with managers who have higher ability to add value. Second, there is greater asymmetry of information regarding private equity fund managers' ability to add value be-

cause many fund characteristics are not fully observed by firms. Therefore, there is a high incentive for private equity fund managers to attempt to manipulate the beliefs of firms about their ability. In particular, by charging lower fees and/or limiting the fund's size, a manager tries to improve firms' beliefs about his ability to add value.

In the signal jamming equilibrium we develop, firms are not fooled and they correctly form an unbiased expectation. Nevertheless, managers limit the sizes of their fund and/or the fees they charge, and they provide persistent returns because otherwise their probability of matching with good firms would decrease. Our model not only explains differences in performance persistence between mutual and private equity funds but also provides new predictions about how our results would vary with the types of funds, managers' experience, competition across markets and over time.

There are a number of additional issues that one could explore using modified versions of our model. One such example is how performance of funds changes around technology cycles, which are times when a manager's value added is also likely to change. Such an analysis would require introducing a dynamic component to the way in which value is added to the portfolio firms, and could constitute a fruitful avenue for future research.

Appendix

Proof of Proposition 1: The Kuhn-Tucker conditions of the fund manager's maximization problem are

$$\frac{\partial E[V_t]}{\partial Q_t} = f - \lambda_t P_t c'(Q_t) = 0, \quad (18)$$

$$\lambda_t (E[P_t W_t] - f) = 0. \quad (19)$$

If $\lambda_t = 0$, then (18) cannot be satisfied. If $\lambda_t > 0$ then it must be the case that $f = E[P_t W_t]$ from (19). Therefore $E_t[\alpha_t] = 0$ and there is no performance persistence. $\square$

Proof of Proposition 2: The objective function is maximized with respect to the following constraints:

$$(1 - v_t)(E_t[P_i W_i] - f_t) \geq 0,$$

$$0 \leq v_i \leq 1,$$

$$0 \leq f_i \leq E_t[P_i W_i].$$

This gives us the following Lagrangean

$$L = \max_{Q_t, f_t, v_t} E_t \left[\sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_i (1 - v_i) (P_i W_i - f_i) + \gamma_i (1 - v_i) + \beta_i (P_i W_i - f_i) \right],$$

where λ , γ and β are Lagrange multipliers. The Kuhn-Tucker conditions for each t are as follows:

$$\frac{\partial L}{\partial Q_t} = v_t (E[P_t W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t) (1 - v_t) - \beta_t P_t c'(Q_t) = 0 \quad (20)$$

$$\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t) (1 - v_t) - \beta_t = 0 \quad (21)$$

$$\frac{\partial L}{\partial v_t} = (Q_t - \lambda_t) (E[P_t W_t] - f_t) - \gamma_t = 0 \quad (22)$$

$$\lambda_t (1 - v_t) (E[P_t W_t] - f_t) = 0 \quad (23)$$

$$\gamma_t(1 - v_t) = 0$$

$$\beta_t(E[P_t W_t] - f_t) = 0$$

Note that the multipliers, γ_t and β_t , cannot both be positive. To see this, suppose that both are positive. This would imply that $v_t = 1$ and $f_t = E[P_t W_t]$. Plugging this back into (21) and (22) would then imply that γ_t and β_t are both zero, a contradiction to them both being (strictly) positive. We next show that at least one of the two constraints, $v_t \leq 1$ and $f_t \leq E_t[P_t W_t]$, must bind. Suppose not, so that the multipliers γ_t and β_t are both zero. Then the only way to satisfy (21) and (22) is if $\lambda_t = Q_t$. But this would then imply that (23) could not be satisfied with both $v_t < 1$ and $f_t < E_t[P_t W_t]$, contradicting the assumption. This further implies that in equilibrium $E_t[\alpha_t] = E_t[(1 - v_t)(P_t W_t^* - f_t)] = 0$, where W_t^* is the gross return at the optimal fund size Q_t^* .

In addition, the solution for Q_t^* , whether for $v_t = 1$ or $f_t = E_t[P_t W_t^*]$, is derived from (20). After simplifying, the equation that Q_t^* must satisfy is given by:

$$\frac{\partial E[V_t]}{\partial Q_t} \Big|_{Q_t=Q_t^*} = -Q_t^* c'(Q_t^*) + X_t - c(Q_t^*) = 0. \quad (24)$$

This completes the characterization. $\square$

Proof of Proposition 3: Comparing (5) to (9), we see that (9) has the additional term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t}$, which we know to be negative from (8). As a result, the equilibrium fund size when it is unobservable, Q_t^U , is lower than in the case when the size is observed, Q_t^O . In addition, given that $v_t < 1$ and $f_t < P_t E[W_t(Q_t^U)]$, $\lambda_t = 0$ and $E_t[\alpha_t] > 0$ for all t when fund size is unobservable.

Observe that $\frac{\partial E_{t+1}[X]}{\partial Q_t}$ approaches zero as the weight assigned to new information, $1 - w_t$, approaches zero over time. $E_t[\alpha_t] = (1 - v_t)(P_t E[W_t(Q_t^U)] - f_t)$ is positive and decreases over time in a non random way. Therefore, there is performance persistence over time.

Note finally that our restriction on the (exogenously given) fees is that $f_t < P_t E[W_t^*]$, the equilibrium gross return under asymmetric information. Therefore, we require that

$f_t < P_t E [W_t (Q_t^U)]$. Now, as argued above, $Q_t^U < Q_t^O$, implying that $W_t (Q_t^U) > W_t (Q_t^O)$. Therefore, for $f_t \in (P_t E [W_t (Q_t^O)], P_t E [W_t (Q_t^U)])$, investors' expected return (i.e., $E_t [\alpha_t] = E_t [(1 - v_t)(P_t W_t - f_t)]$) will be zero when fund size is observed, but will be strictly positive when there is asymmetric information concerning the fund's size. $\square$

Proof of Proposition 4: The Kuhn-Tucker conditions are as follows:

$$v_t(P_t E [W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1 - v_t) - C(e_t) = 0 \quad (25)$$

$$\lambda_t E_t [(1 - v_t)(P_t W_t - f_t)] = 0 \quad (26)$$

$$Q_t v_t P_t - Q_t \frac{\partial C(e_t)}{\partial e_t} + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t} = 0 \quad (27)$$

The term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial e_t} > 0$, the third term in (27), is not present in the case when effort is observed by entrepreneurs. As a result, the equilibrium level of effort is larger when effort is not observed. In addition, given that $v_t < 1$ and $f_t < P_t E [W_t (Q_t^U)]$, $\lambda_t = 0$ and $E_t[\alpha_t] > 0$ for all t . $E_t[\alpha_t]$ is larger compared to the case when effort is observed because larger effort translates into a higher $E[W_t]$. Note that $\frac{\partial E_{t+1}[X]}{\partial e_t}$ approaches zero as the weight assigned to new information, $1 - w_t$, approaches zero over time. Therefore, $E_t [\alpha_t] = (1 - v_t)(P_t E [W_t] - f_t)$ decreases over time. Investors' expected excess return is positive and decreases over time in a non random way. Hence, there is performance persistence over time. $\square$

Proof of Proposition 5: The Bellman equation is:

$$V_t(E_t [X|h_{t-1}]) = \max_{Q_t, f_t, v_t} [Q_t v_t (P_t W_t - f_t) + Q_t f_t + V_{t+1}(E_{t+1} [X|h_t])],$$

subject to the constraints

$$E_t [(1 - v_t)(P_t W_t - f_t)] \geq 0,$$

$$E_{t+1} [X|h_t] = w_t E_t [X|h_{t-1}] + (1 - w_t) \left(\frac{\alpha_t}{(1 - v_t)P_t} + \frac{f_t}{P_t} + c(Q_t^C) \right).$$

$$0 \leq v_i \leq 1, \quad 0 \leq f_i \leq E_t [P_i W_i]$$

This gives us the following Lagrangean

$$L = \max_{Q_t, f_t, v_t} E_t \left[\sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_i (1 - v_i) (P_i W_i - f_i) + \gamma_i (1 - v_i) + \beta_i (P_i W_i - f_i) \right],$$

where λ , γ and β are Lagrange multipliers. The Kuhn-Tucker conditions are as follows:

$$\frac{\partial L}{\partial Q_t} = v_t (E[P_t W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t) (1 - v_t) - \beta_t P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (28)$$

$$\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t) (1 - v_t) - \beta_t = 0 \quad (29)$$

$$\frac{\partial L}{\partial v_t} = (Q_t - \lambda_t) (E[P_t W_t] - f_t) - \gamma_t = 0 \quad (30)$$

$$\lambda_t (1 - v_t) (E[P_t W_t] - f_t) = 0 \quad (31)$$

$$\gamma_t (1 - v_t) = 0$$

$$\beta_t (E[P_t W_t] - f_t) = 0.$$

The rest of the proof is similar to the proof of Proposition 2. Note that the multipliers, γ_t and β_t , cannot both be positive, as this would imply that $v_t = 1$ and $f_t = E[P_t W_t]$. Plugging this back into (29) and (30) would then imply that γ_t and β_t are both zero, a contradiction to them being both positive.

We next show that at least one of the two constraints, $v_t \leq 1$ and $f_t \leq E_t[P_t W_t]$, must bind. Suppose not, so that the multipliers γ_t and β_t are both zero. Then the only way to satisfy (29) and (30) is if $\lambda_t = Q_t$. But this would then imply that (31) could not be satisfied with both $v_t < 1$ and $f_t < E_t[P_t W_t]$, contradicting the assumption. This further implies that in equilibrium $E_t[\alpha_t] = E_t[(1 - v_t)(P_t W_t - f_t)] = 0$, as desired.

We next establish that in this setting either $v_t = 1$ or $f_t = E_t[P_t W_t]$ are equivalent in the sense of providing the same return and choice of fund size Q_t . To see this, first suppose that $v_t < 1$. Then $\gamma_t = 0$. From the argument above, we know that it must be that $f_t = E[P_t W_t]$. (This can also be checked independently equation by equation to rule out the possibility that

$f_t \neq E[P_t W_t]$). This leaves us with the following conditions:

$$\frac{\partial L}{\partial Q_t} = -Q_t v_t P_t c'(Q_t) + E[P_t W_t] - \lambda_t P_t c'(Q_t)(1 - v_t) - \beta_t P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (32)$$

$$\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) - \beta_t = 0, \quad (33)$$

Now, if $\lambda_t = Q_t$ then from $\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) - \beta_t = 0$ we must have $\beta_t = 0$. We would now have from (32):

$$\begin{aligned} \frac{\partial L}{\partial Q_t} &= -Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} \\ &= E[P_t W_t] - Q_t P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \end{aligned} \quad (34)$$

Suppose instead that $\lambda_t < Q_t$. Then from $\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) - \beta_t = 0$ we must have $\beta_t = (Q_t - \lambda_t)(1 - v_t) > 0$. Plugging this into (32) above yields:

$$\begin{aligned} \frac{\partial L}{\partial Q_t} &= -Q_t v_t P_t c'(Q_t) + E[P_t W_t] - \lambda_t P_t c'(Q_t)(1 - v_t) - (Q_t - \lambda_t)(1 - v_t) P_t c'(Q_t) \\ &\quad + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} \\ &= E[P_t W_t] - Q_t P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \end{aligned} \quad (35)$$

These are precisely the same conditions as for the case where $\lambda_t = Q_t$, and in neither case does λ_t show up in the condition describing the optimal Q_t .

Now instead suppose that $f_t < E[P_t W_t]$. Then $\beta_t = 0$. From the argument above, we must have $v_t = 1$. (Again, this can be checked equation by equation to rule out that $v_t \neq 1$). With this, we are left with the following conditions:

$$\frac{\partial L}{\partial Q_t} = E[P_t W_t] - Q_t P_t c'(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (36)$$

$$\frac{\partial L}{\partial v_t} = (Q_t - \lambda_t)(E[P_t W_t] - f_t) - \gamma_t = 0, \quad (37)$$

Note that neither λ_t nor γ_t show up in FOC for Q_t , so that (36) describes the solution for the optimal Q_t . More importantly, this are precisely the same conditions as for the case in which $f_t = E[P_t W_t]$. We can therefore conclude that the solution to the problem is given by (36), and is the same whether the fund manager chooses $v_t = 1$ or $f_t = E[P_t W_t]$.

Finally, since $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} < 0$ because $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} > 0$ but $\frac{\partial E_{t+1}[X]}{\partial Q_t} < 0$, inspection of (36) makes it clear that the optimal choice of Q_t when fund size is not observed is lower than when fund size is observed, since the FOC in that case would not contain this additional negative term. $\square$

Proof of Proposition 6: From the Bellman equation provided in the text, we can write the Lagrangean for the manager's optimization problem:

$$L = \max_{Q_t, f_t, v_t} E_t \left[\sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_i (1 - v_i) (P_i W_i - f_i) + \gamma_i (1 - v_i) + \beta_i (P_i W_i - f_i) \right], \quad (38)$$

with the additional constraint that the expectation over managerial talent evolve according to (16). In what follows we ignore the constraints that $v_i \leq 1$ and $f_i \leq E_t[P_i W_i]$, and show after that these conditions are satisfied for the unconstrained problem. We now obtain the Kuhn-Tucker conditions, which are:

$$\frac{\partial L}{\partial Q_t} = v_t (P_t E[W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t) (1 - v_t) = 0 \quad (39)$$

$$\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial f_t} = 0 \quad (40)$$

$$\frac{\partial L}{\partial v_t} = (Q_t - \lambda_t)(P_t E[W_t] - f_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial v_t} = 0 \quad (41)$$

$$\lambda_t (1 - v_t) (P_t E[W_t] - f_t) = 0 \quad (42)$$

Assume that $\lambda_t > 0$, so that the investors' participation constraint binds. In this case,

either $1 - v_t = 0$ or $P_t E[W_t] - f_t = 0$ or both. However, this cannot be true because we could not then satisfy equations (40) and (41). Therefore, λ_t must be equal to zero. In this case again $1 - v_t$ or $P_t E[W_t] - f_t$ cannot be equal to zero from (40) and (41). As a result investors' expected return α_t must be positive. We can find the solutions for f_t and v_t from (40) and (41) and plug this into (39) to solve for the manager's choice of Q_t . The manager accepts a lower quantity than investors are willing to invest because $E_t[\alpha_t]$ is positive and investors are willing to invest until their expected return is zero.

Note as well that, from

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t) \left(\frac{\alpha_t}{(1 - v_t^C)P_t} + \frac{f_t^C}{P_t} + c(Q_t) \right) \quad (43)$$

both $\frac{\partial E_{t+1}[X]}{\partial f_t}$ and $\frac{\partial E_{t+1}[X]}{\partial v_t}$ are negative and approach zero as the weight assigned to new information, $1 - w_t$, approaches zero over time. As $\frac{\partial E_{t+1}[X]}{\partial f_t}$ approaches zero, v_t approaches 1, and as $\frac{\partial E_{t+1}[X]}{\partial v_t}$ approaches zero, f_t approaches $P_t E[W_t]$. Therefore, $E_t[\alpha_t] = E_t[(1 - v_t)(P_t W_t - f_t)]$ also approaches zero over time. We also know that $E_t[\alpha_t]$ is positive for all t . We can now conclude that investors' expected return is positive and not purely random, implying performance persistence over time. $\square$

Proof of Proposition 7: The Lagrangean for the manager's optimization problem is the same as in (38), but this time with the constraint that the expectation over managerial talent evolve according to (17). As in the proof of Proposition 6, we ignore the constraints that $v_i \leq 1$ and $f_i \leq E_t[P_i W_i]$, and show after that these conditions are satisfied for the unconstrained problem. The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial Q_t} = v_t(P_t E[W_t] - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0, \quad (44)$$

$$\frac{\partial L}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial f_t} = 0, \quad (45)$$

$$\frac{\partial L}{\partial v_t} = (Q_t - \lambda_t)(P_t E[W_t] - f_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial v_t} = 0, \quad (46)$$

$$\lambda_t(1 - v_t)(P_t E[W_t] - f_t) = 0. \quad (47)$$

Assume that $\lambda_t > 0$. In this case there are three possibilities: $1 - v_t = 0$ or $P_t E[W_t] - f_t = 0$ or both. If $P_t E[W_t] - f_t = 0$, then (46) cannot be satisfied and if $1 - v_t = 0$ then (45) cannot be satisfied. This yields a contradiction, and as a result it must be that λ_t is zero. When $\lambda_t = 0$ both $1 - v_t > 0$ and $P_t E[W_t] - f_t > 0$, as otherwise (45) and (46) cannot be satisfied, respectively. Therefore, $E_t[\alpha_t]$ is positive, and this is true for all t .

For all possible cases discussed, (44) has an additional negative term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t}$ compared to case when fund size observed, which implies that the optimal fund size that satisfies (44) is lower than the size in the case when fund size is observed.

This also implies performance persistence given that $E_t[\alpha_t]$ is not random and slowly decreases over time (both because of slowly increasing Q_t as shown in the proof of Proposition 3 and fees as shown in the proof of Proposition 6). $\square$

Proof of Proposition 8: Denote by Q'_t the solution to the fund manager's maximization problem in the absence of the constraint. Assuming that $\frac{\partial^2 E[V_t]}{\partial Q_t^2} < 0$, which is a necessary condition for the fund manager's maximization problem to be well defined, we can invoke the Implicit Function Theorem to determine that $\frac{\partial Q'_t}{\partial v_t}$ is proportional to

$$\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = P_t (E[W_t] - Q_t c'(Q_t)) - f_t = (P_t W_t - f_t) - P_t Q_t c'(Q_t).$$

For $f_t \approx P_t E[W_t^*]$, this reduces to $\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = -P_t Q_t c'(Q_t) < 0$ for $Q_t > 0$. By contrast, $\frac{\partial Q'_t}{\partial f_t}$ is proportional to

$$\frac{\partial^2 E[V_t]}{\partial f_t \partial Q_t} = 1 - v_t,$$

which is weakly positive, and strictly positive for $v_t < 1$.

Since by assumption $Q'_t > Q^M$ for $v_t \approx 1$ and $f_t \approx P_t E[W_t^*]$, the only way to commit to choose $Q_t \leq Q^M$ is by reducing f_t , since reducing v_t would actually increase Q_t . The impact of such reductions, however, is minimal for $v_t = 1 - \epsilon$. Note, however, that as f_t becomes smaller, $\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = (P_t E[W_t] - f_t) - P_t Q_t c'(Q_t)$ may be either positive or negative,

and will be strictly positive for small enough Q_t . Such Q_t will be optimal when the signal jamming incentive is sufficiently strong: $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} \ll 0$. This can be seen by noting that if the incentive to signal jam is sufficiently large in absolute magnitude, either because $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \gg 0$ or $\frac{\partial E_{t+1}[X]}{\partial Q_t} \ll 0$, or both, then the first order condition can only be satisfied with $W_t - Q_t c'(Q_t) > 0$. From this, it is now straightforward to see that the only way to credibly commit to choose $Q_t \leq Q^M$ is by choosing some $v_t < 1$ and some $f_t < P_t E[W_t^*]$, as desired.

We also establish here the necessity of a signal jamming incentive for this result. Suppose to the contrary, and that the term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0$, so that there is either no incentive or no ability to influence entrepreneurs' beliefs. In this case, the fund manager's first order condition for the case where $0 < v_t < 1$ and $0 < f_t < P_t W_t$ reduces to

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t P_t (E[W_t] - Q_t c'(Q_t)) + f_t (1 - v_t) = 0. \quad (48)$$

For any $v_t < 1$, note that this FOC can only be satisfied if $E[W_t] - Q_t c'(Q_t) < 0$. Therefore, it is straightforward to see from this FOC that $\frac{\partial Q_t}{\partial v_t} < 0$ and $\frac{\partial Q_t}{\partial f_t} > 0$.

Suppose now that there is some combination $0 < v_t < 1$ and $0 < f_t < P_t E[W_t^*]$ that deliver $Q_t \leq Q^M$. Since $\frac{\partial Q_t}{\partial v_t} < 0$ and $\frac{\partial Q_t}{\partial f_t} > 0$, the fund manager should be able to increase v_t by some ϵ and at the same time increase f_t by some δ such that Q_t remains constant at or below Q^M , but now the manager makes higher profits since both fees have been increased. Adjusting the fees in this way will always be possible until either $v_t = 1$ or $f_t = P_t E[W_t^*]$. Therefore, any fee structure that achieves commitment can be replaced by another structure that achieves the same commitment but that fully extracts all the rents from investors. We can now conclude that absent the signal jamming incentive it is never optimal to leave money on the table. $\square$

Further Characterizing the Fee Structure in Section 5.2

We now turn to the issue of whether an optimal fee structure can be further characterized. We look for fees that maximize expected returns for the fund manager, when fees are such

as to commit the fund to a maximum size Q^M and money is left on the table in equilibrium. As before, we denote the quantity that the manager will choose (given a fee structure) by Q' and note, as we have seen above, the manager does not extract all the surplus only when increasing v_t or f_t will induce a larger quantity choice, i.e., $\frac{\partial Q'_t}{\partial v_t}|_{Q'_t=Q^M} > 0$ and $\frac{\partial Q'_t}{\partial f_t}|_{Q'_t=Q^M} > 0$.

Let us consider a fee structure v, f that induces $Q'_t = Q^M$. To examine the range of possible fee structures that would commit the fund to a quantity choice of Q^M , let us consider a small change to v_t , say Δv . From the characterization of $\frac{\partial Q'_t}{\partial v_t}$ and $\frac{\partial Q'_t}{\partial f_t}$, we have that, to keep the commitment to Q^M , the corresponding change in f should be such that:

$$\Delta v \approx -\Delta f \frac{(1 - v_t)}{(P_t W_t - f_t) - P_t Q_t c'(Q_t)}. \quad (49)$$

We turn now to the effect of such changes of fee terms on expected manager profits, keeping the size commitment fixed at Q^M . The change in profits on account of the changes to fees can be expressed as:

$$\Delta V_t|_{Q'_t=Q^M} = Q^M [\Delta v(P_t W_t - f_t) + \Delta f(1 - v_t)]. \quad (50)$$

Substituting for Δf we have that:

$$\begin{aligned} \Delta E(V_t)|_{Q'_t=Q^M} &= Q^M [\Delta v(P_t W_t - f_t) - \Delta v((P_t W_t - f_t) - P_t Q^M c'(Q^M))] \\ &= \Delta v [P_t Q^M c'(Q^M)] \geq 0. \end{aligned} \quad (51)$$

This indicates that the expected profits are increasing in v_t , at least as long as f_t can be reduced to offset the effect of increasing v_t on fund size. Hence, the optimal contract is one that minimizes f_t , implying $f_t = 0$ in absence of other considerations, while setting v_t to the highest value consistent with the size commitment of Q^M . $\square$

References

Bao, Jack and Alex Edmans, 2009, “Do Investment Banks Have Skill? Performance Persistence of M&A Advisors,” Working Paper.

Baum, J. A., and B. S. Silverman, 2004, “Picking Winners or Building Them? Alliance, Intellectual, and Human Capital as Selection Criteria in Venture Financing and Performance of Biotechnology Startups,” *Journal of Business Venturing*, 19, 411–436.

Berk, J., and R. Green, 2004, “Mutual Fund Flows and Performance in Rational Markets,” *Journal of Political Economy* 112, 1269-1295.

Busse, Jeff, Amit Goyal and Sunil Wahal, 2009, “Performance Persistence in Institutional Investment Management,” forthcoming in *Journal of Finance*.

Carhart, Mark M., 1997, “On Persistence in Mutual Fund Performance,” *Journal of Finance*, Vol. 52 No. 1, 57-82.

Chen, Henry, Paul Gompers, Anna Kovner and Josh Lerner, 2009, “Buy Local? The Geography of Successful and Unsuccessful Venture Capital Expansion,” *NBER Working Papers 15102*.

Chemmanur, Thomas J and Paolo Fulghieri, 1994, “Reputation, Renegotiation, and the Choice between Bank Loans and Publicly Traded Debt,” *Review of Financial Studies*, vol. 7(3), pages 475-506.

Chemmanur, Thomas J., Karthik Krishnan and Debarshi Nandy, 2009, “How Does Venture Capital Financing Improve Efficiency in Private Firms? A Look Beneath the Surface,” Working paper.

Cochrane, J., 2005, “The Risk and Return of Venture Capital”, *Journal of Financial Economics* 75, 3-52.

Cumming, Douglas J. and Na Dai, 2010, “Fund Size, Limited Attention and Valuation of Venture Capital Backed Firms ” *Working paper*.

Cumming, Douglas J. and Uwe Walz, 2009, “Private Equity Returns and Disclosure Around the World,” forthcoming at *Journal of International Business Studies*.

Fernando, Chitru S., Vladimir A. Gatchev and Paul A. Spindt, 2005, “Wanna Dance?

How Firms and Underwriters Choose Each Other,” *Journal of Finance*, Vol. 60, pp. 2437-2469.

Fulghieri, Paolo and Merih Sevilir, 2009, “Size and Focus of a Venture Capitalist’s Portfolio,” forthcoming at *Review of Financial Studies*.

Glode, V., and R. Green, 2008, “Information Spill-Overs and Performance Persistence in Private Equity Partnerships,” Unpublished working paper, Carnegie-Mellon University.

Gompers, P.A., and J. Lerner, 1999, “An Analysis of Compensation in the U.S. Venture Capital Partnership,” *Journal of Financial Economics* 51, 3-44.

Gompers, P.A., and J. Lerner, 2001, *The Money of Invention: How Venture Capital Creates New Wealth*, Harvard Business School Publishing, Boston.

Gompers, P. A., Anna Kovner and Josh Lerner, 2009, “Specialization and Success: Evidence from Venture Capital,” *Journal of Economics & Management Strategy*, Vol. 18, Issue 3, pp. 817-844.

Gorman, M., and W. A. Sahlman, 1989, “What Do Venture Capitalists Do?,” *Journal of Business Venturing*, 4, 231–248.

Hellmann, T., and M. Puri, 2000, “The Interaction between Product Market and Financing Strategy: The Role of Venture Capital,” *Review of Financial Studies*, 13, NO. 4, 959–984.

Hellmann, T., and M. Puri, 2002, “Venture Capital and the Professionalization of Start-Up Firms: Empirical Evidence,” *The Journal of Finance*, LVII, NO. 1, 169–197.

Hochberg, Y., A. Ljungqvist, A. Vissing-Jorgensen, 2008, “Informational Hold-up and Performance Persistence in Venture Capital,” working paper.

Hochberg, Y.V., A. Ljungqvist, and Y. Lu, 2007, “Whom You Know Matters: Venture Capital Networks and Investment Performance,” *Journal of Finance* 62, 251-301.

Holmstrom, B., 1999, “Managerial Incentive Problems: A Dynamic Perspective,” *Review of Economic Studies*, 66, 169-182.

Hsu, D. H., 2004, “What Do Entrepreneurs Pay for Venture Capital Affiliation?,” *The Journal of Finance*, LIX, NO. 4, 1805–1844.

Inderst, R., H. Mueller, and F. Muennich, 2007, "Financing a Portfolio of Projects," *Review of Financial Studies* 20: 1289-1325.

Jensen, Michael C., (1968), "The Performance of Mutual Funds in the Period 1945-1964," *The Journal of Finance*, 23, No. 2, 389-416

Jones, C., and M. Rhodes-Kropf, 2003, "The Price of Diversifiable Risk in Venture Capital and Private Equity," Unpublished working paper, Columbia University.

Kaplan, S.N., and A. Schoar, 2005, "Private Equity Performance: Returns, Persistence and Capital Flows," *Journal of Finance* 60, 1791-1823.

Kaplan, S.N., B. Sensoy, and P. Stromberg, 2002, "How Well Do Venture Capital Databases Reflect Actual Investments?," Unpublished working paper, University of Chicago.

Korteweg, A., and M. Sorensen, 2008, "Estimating Risk and Return of Infrequently Traded Assets: A Bayesian Selection Model of Venture Capital," Unpublished working paper, Columbia University.

Lerner, J., and A. Schoar, 2004, "The Illiquidity Puzzle: Evidence from Private Equity Partnerships," *Journal of Financial Economics* 72, 3-40.

Litvak, K., 2009, "Venture Capital Limited Partnership Agreements: Understanding Compensation Arrangements," forthcoming at *University of Chicago Law Review*.

Ljungqvist, A. and M. P. Richardson, 2003, "The Cash Flow, Return and Risk Characteristics of Private Equity," NYU, Finance Working Paper No. 03-001.

Lopez-de-Silanes, F. and L. Phalippou and O. Gottschalg, 2009, "Giants at the Gate: Diseconomies of Scale in Private Equity," working paper.

Meggison, W. L., and K. A. Weiss, 1991, "Venture Capitalist Certification in Initial Public Offerings," *The Journal of Finance*, XLVI, NO. 3, 879-903.

Metrick, A. and Yasuda, A., 2010, "The Economics of Private Equity Funds," *Review of Financial Studies* 23, 2303-2341.

Phalippou, L. and O. Gottschalg, 2008, "The Performance of Private Equity Funds," forthcoming at *Review of Financial Studies*.

Phalippou, Ludovic, 2009, "Venture Capital Funds: Performance Persistence and Flow-

Performance Relation,” forthcoming *Journal of Banking and Finance*.

Phalippou, Ludovic, 2007, “Beware of Venturing into Private Equity,” forthcoming *Journal of Economic Perspectives*.

Quigley, John M. and Susan E. Woodward, 2003, “An Index for Venture Capital,” Department of Economics, UCB working paper.

Sorensen, Morten, 2007, “How Smart is Smart Money: A Two-Sided Matching Model of Venture Capital,” *Journal of Finance*, 62, 2725-2762

Stein, J., 1989, “Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior,” *Quarterly Journal of Economics*, 104, 655-669.

Titman, Sheridan and Brett Trueman, 1986, “Information quality and the valuation of new issues,” *Journal of Accounting and Economics*, vol. 8(2), 159-172.