Structural Modeling in Practice

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Agenda

1. Paper Summary
2. Structural Models in Practice
3. Validation
4. Final Thoughts
5. Appendix

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Paper Summary

Test Two Hypotheses

Hypothesis 1: Black-Scholes-Merton (BSM)-based probability of default is a sufficient statistic for forecasting bankruptcy (default).

Hypothesis 2: When predicting default, the BSM model can be useful. This usefulness potentially arises from two components:

- Functional form implied by the model
- Solution of system of equations

"…it is entirely possible that the proprietary features of KMV's model make its performance superior to what we document here." p. 8

What's in a model name?

Pseudo-“KMV-Merton” (PK): “We do not intend to imply that we are using exactly the same algorithm that Moody’s KMV uses to calculate distance to default.” footnote 2, p.1.

Naïve alternative (NA): Simple model to calculate, but retains some of the functional form of “KMV-Merton.”

Vasicek-Kealhofer (VK): Extension of BSM to barrier, perpetual option and richer capital structure framework. Note the model is not proprietary for clients.

Moody’s KMV (MKMV): Implementation of VK model to produce Expected Default Frequencies™ or EDF™ values.

Hypotheses Tested in the Following Ways on Default Data from 1980 to 2003

1. Incorporate PK into hazard model and compare PK to NA and other default forecasting variables.
2. Compare short-term, out-of-sample forecasting ability of PK and NA.
3. Compare several alternative approaches to calculating a PK value.
4. Test the ability of PK to explain CDS-implied probabilities of default.
5. Investigate via regression the relationship of corporate bond yield spreads and PK, NA, and other variables.

Stated Findings

PK values are not sufficient statistics for bankruptcy (default) prediction.

NA model performs as well as PK.

Test on 80 firms published in CFO magazine in 2003 shows NA has rank correlation of 79% with MKMV’s published EDF values.

Testing for Sufficient Statistic

Authors assume Cox-proportional hazard model for testing statistical sufficiency imposing linearity assumption on functional form.

If the model is mis-specified and certain relationships exist among the variables, this test may lead to false rejection of statistical sufficiency for the independent variable.

Better to use non-parametric tests (see Miller article.)

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Structural Models in Practice

What are some other practical consideration for PD model assessment?

Diagnosability of model behavior.

Corporate transaction analysis.

Data quality and data cleaning.

Adjusting for missing or hidden defaults.

Segmenting sample by firm size.

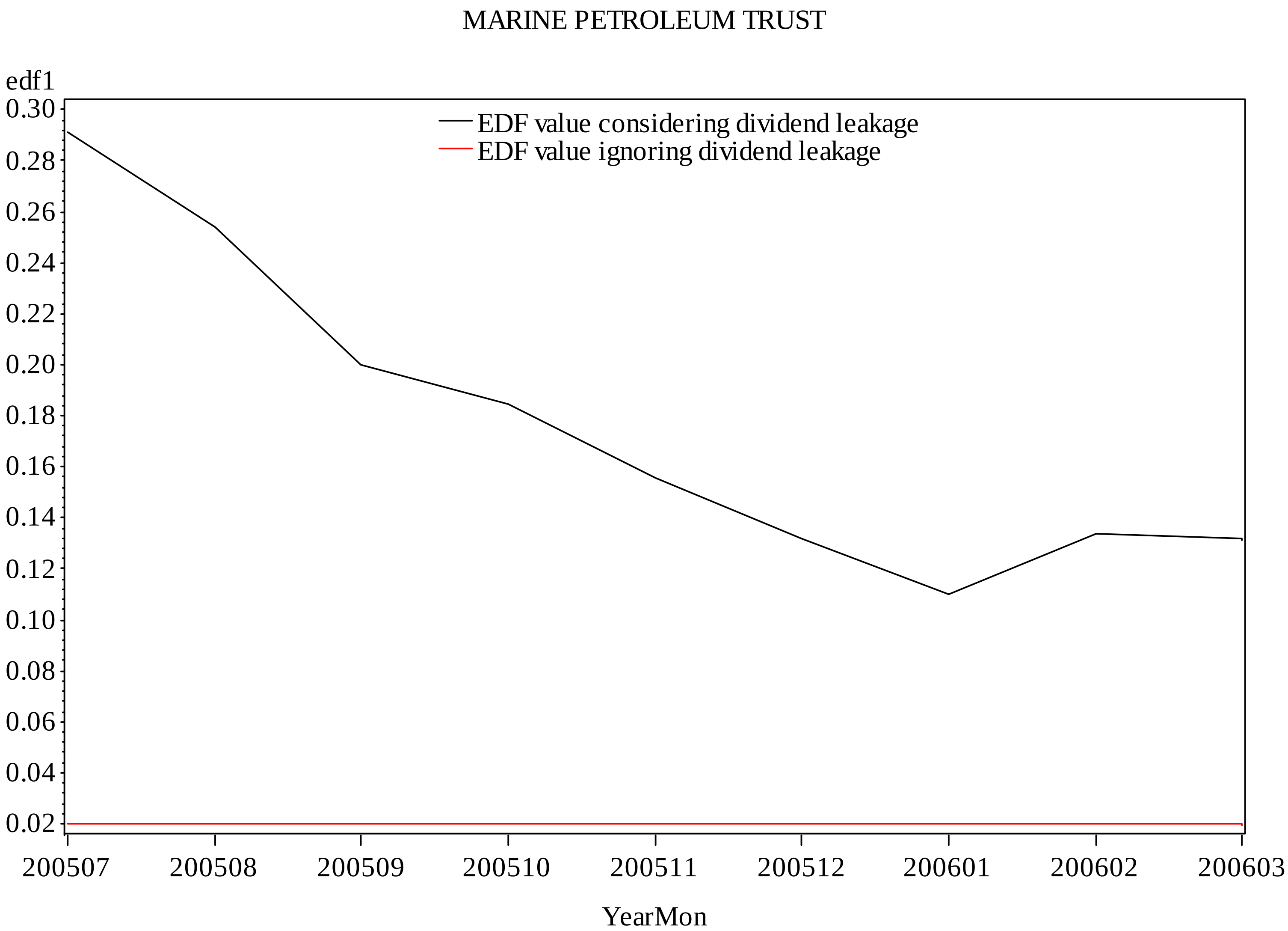
Ability to process thousands (30,000) of firms on a daily basis.

Ability to cover newly established firms.

Vasicek-Kealhofer Model: An Extension of the BSM Framework

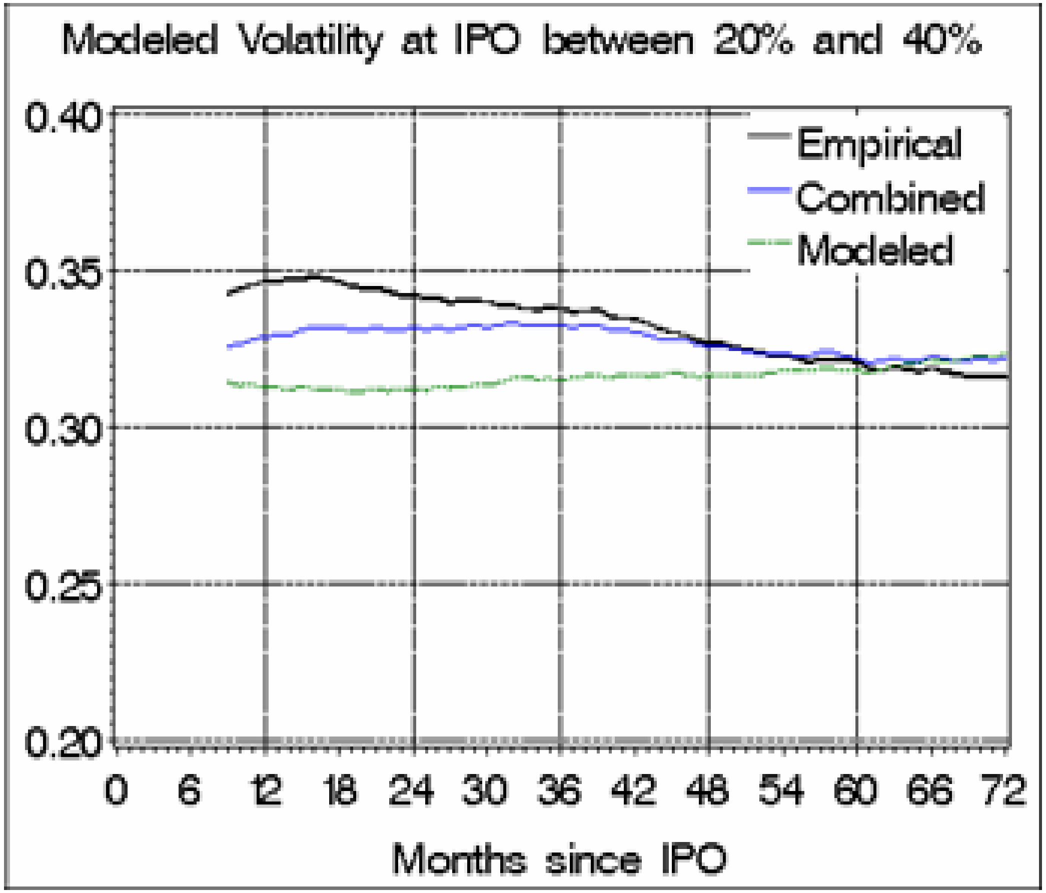
Black-Scholes-Merton	Vasicek-Kealhofer EDF Model
Two classes of Liabilities: Short Term Liabilities and Common Stock	**Five Classes of Liabilities**: Short Term and Long Term Liabilities, Common Stock, Preferred Stock, and Convertible Stock
No Cash Payouts	**Cash Payouts**: Coupons and Dividends (Common and Preferred)
Default occurs only **at Horizon.**	Default can occur **at or before Horizon.**
Default barrier is total **debt**.	Default barrier is **empirically** determined.
Equity is a call option on Assets, **expiring** at the Maturity of the debt.	Equity is a **perpetual** call option on Assets
Gaussian relationship between probability of default (PD) and distance to default (DD).	DD-to-EDF mapping **empirically** determined from calibration to historical data.

Dividend Payout Impact



Modeled Volatility for IPOs: 20% to 40% Asset Volatility

- New firms have unusually volatile equity leading to an over-estimate of asset volatility.
- MKMV's estimate of asset volatility is based on a weighted average of empirical volatility and modeled volatility.
- Modeled volatility is related to the size, industry, and geography of the firm.
- Modeled volatility helps alleviate difficulties with estimating asset volatility for new firms.



Modeled Volatility for IPOs: 40% to 80%

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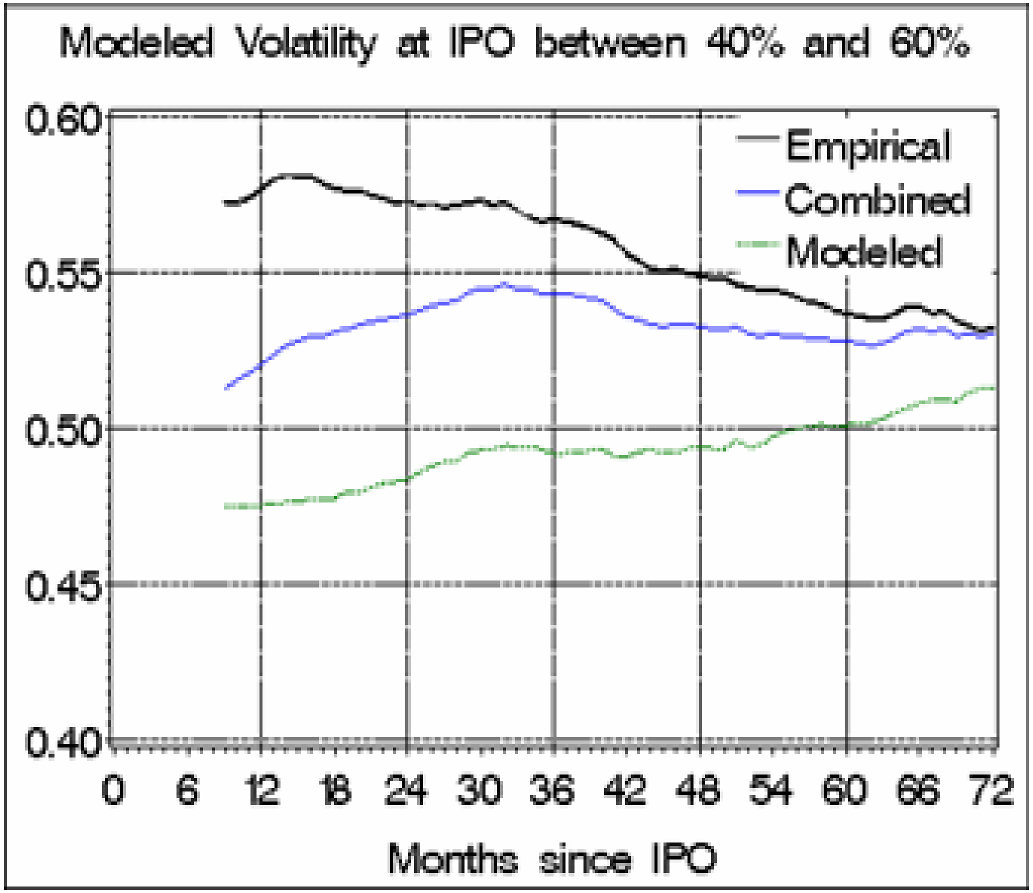
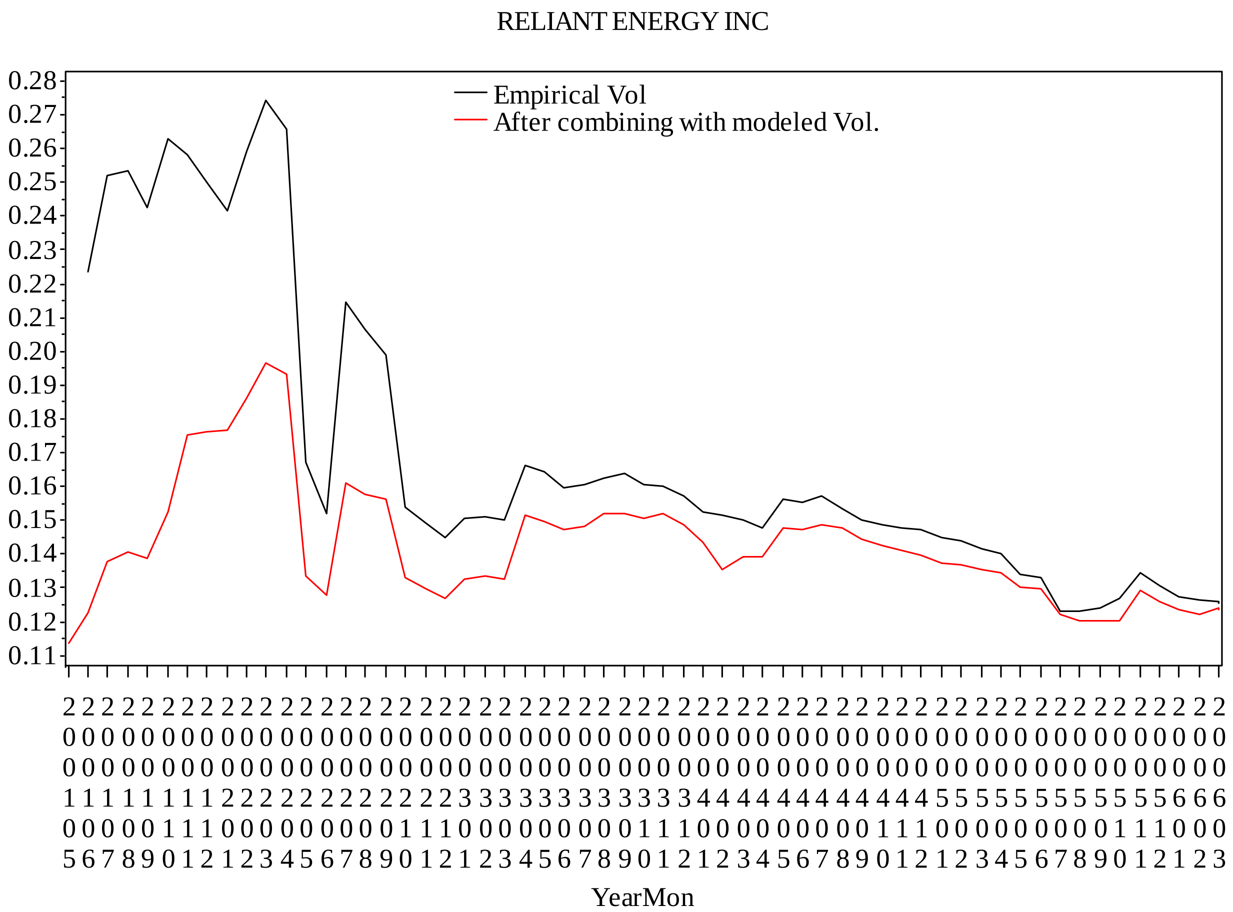
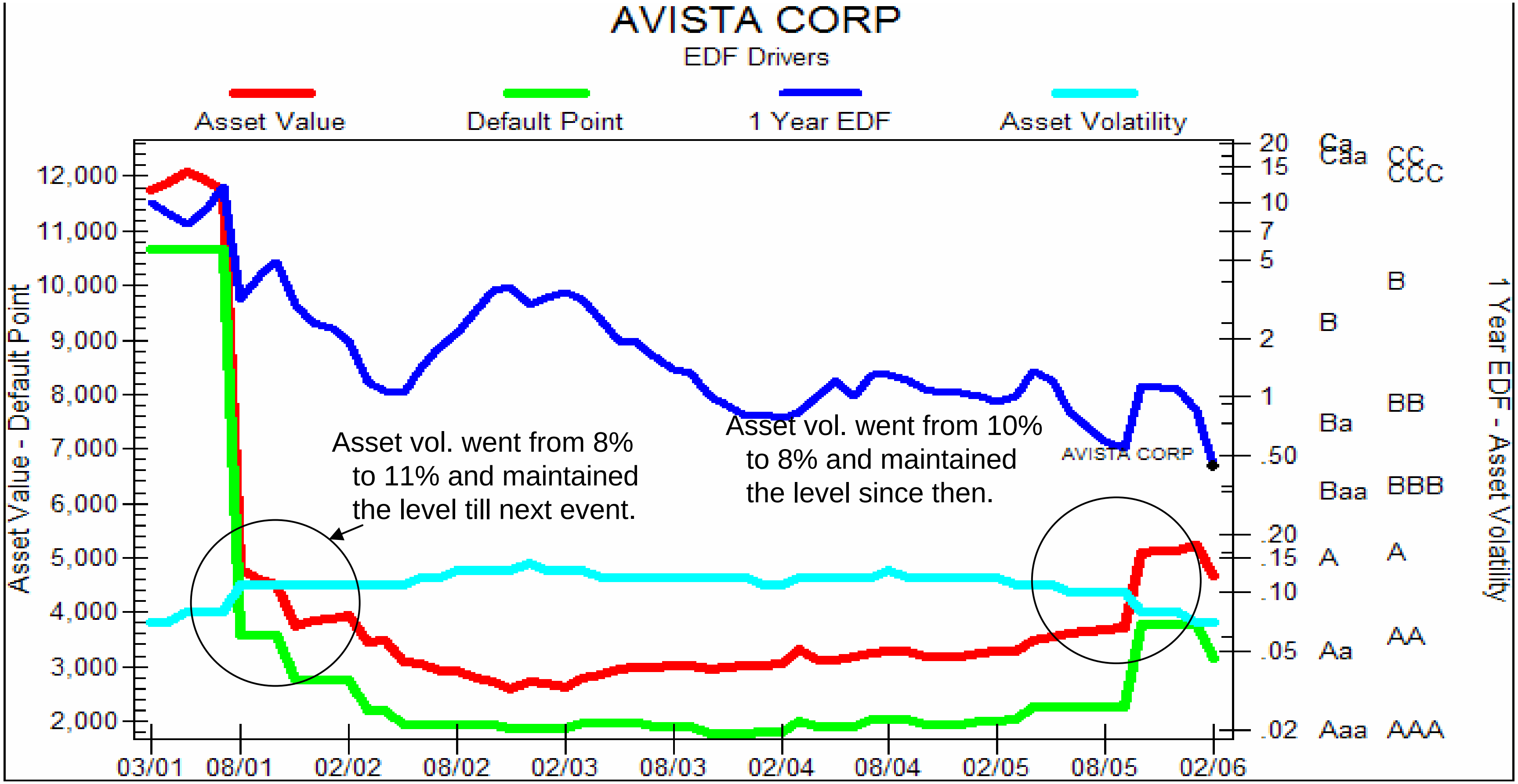


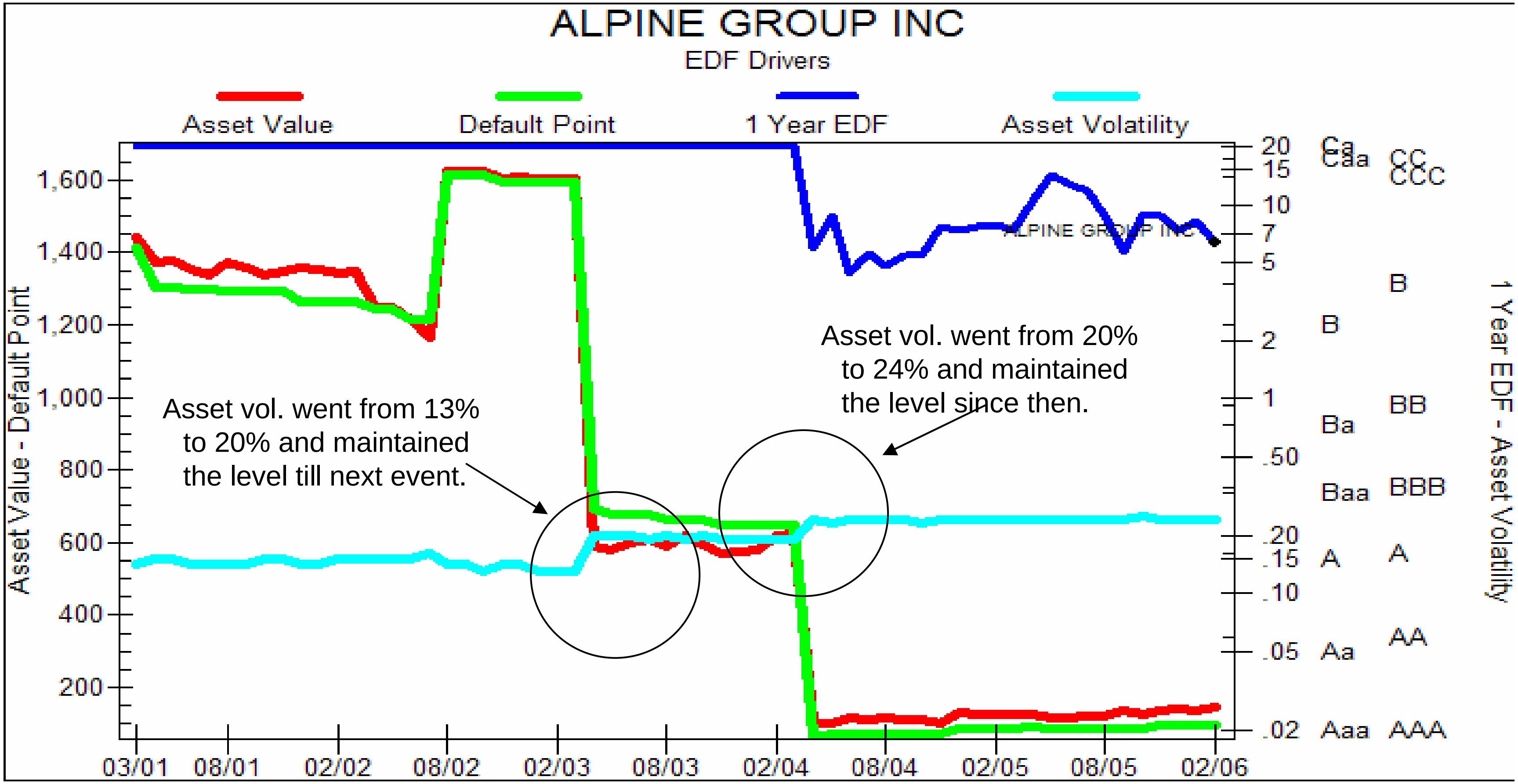
Impact of Modeled Volatility on Companies with Short Histories



Impact of Event Processing: Reducing Debt



Impact of Event Processing: Changing Debt



More Complexity Facilitates More Effective Handling of Actual Companies

Important to account for the total debt breakdown between short-term and long-term debt.

Perpetual down-and-out option assumption consistent with firm's treatment as an "ongoing concern". Provides unique asset value and volatility assessment. Otherwise inferred current asset value and volatility are a function of "time-to-maturity", making them meaningless.

Low volatility firms with decent asset value paying out heavy dividends.

Extensive changes in leverage over time inducing non-stationary equity volatility.

Unusually large amount of convertible securities in capital structure.

Empirical mapping helps capture realistic default experience.

3 Validation

How should PD models be evaluated?

Ranking (Sample dependencies, size of sample)

Timeliness (Aggregates vs. outliers)

Accuracy as a probability (Means vs. medians)

Explaining spreads (Disentangling loss given default, risk premia, other premia)

Corporate transaction analysis (Post transaction asset volatility impact)

Interpretation of inputs and outputs and model transparency (Model risk)

Usefulness in pricing applications (Requires term structure of EDF values)

Functional Form versus Estimation Process

Start with NA as base

Add the following replacement variables representing more sophisticated estimation:

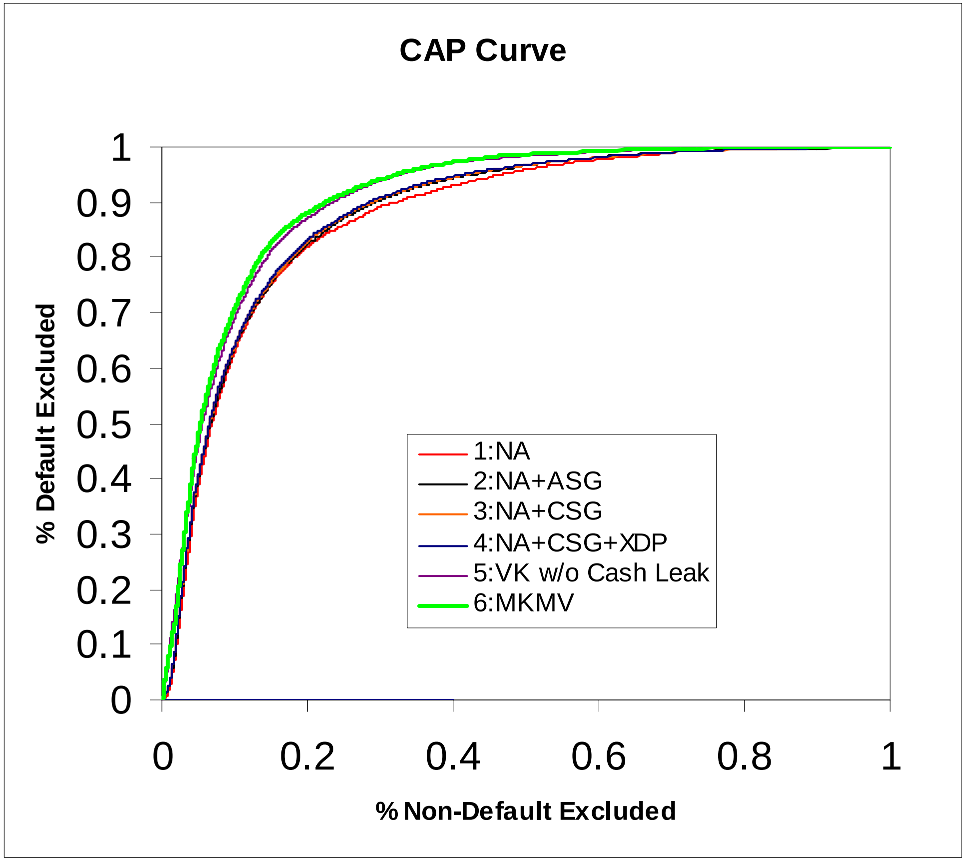
- ASG: “Raw” empirical asset volatility
- CSG: Blended estimation of asset volatility based on ASG and modeled volatility
- XDP: MKMV’s calculated default point reflecting mostly adjustments for financial firms
- VK: Implementation of just the VK model without the benefit of MKMV’s event processing system and data cleaning efforts
- MKMV: Full implementation of VK model with full benefit of the system

Estimation process and solving system significantly improves model performance

Comparing Models: Time Period 1996 to 2006

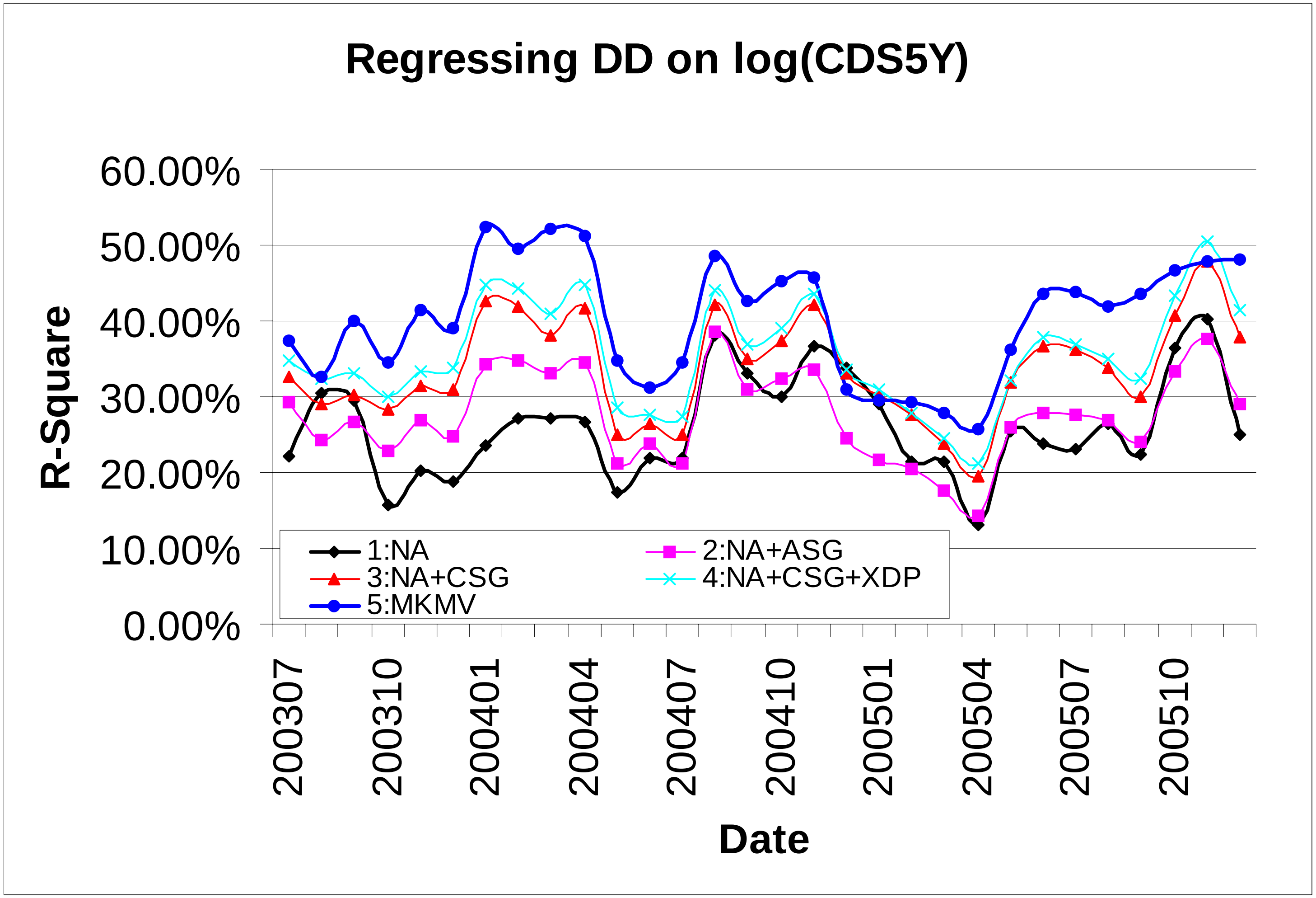
Sample: 3/1996 to 2/2006, U.S. companies, Book Asset > $30 million

(approximately 600,000 firm-month observations)



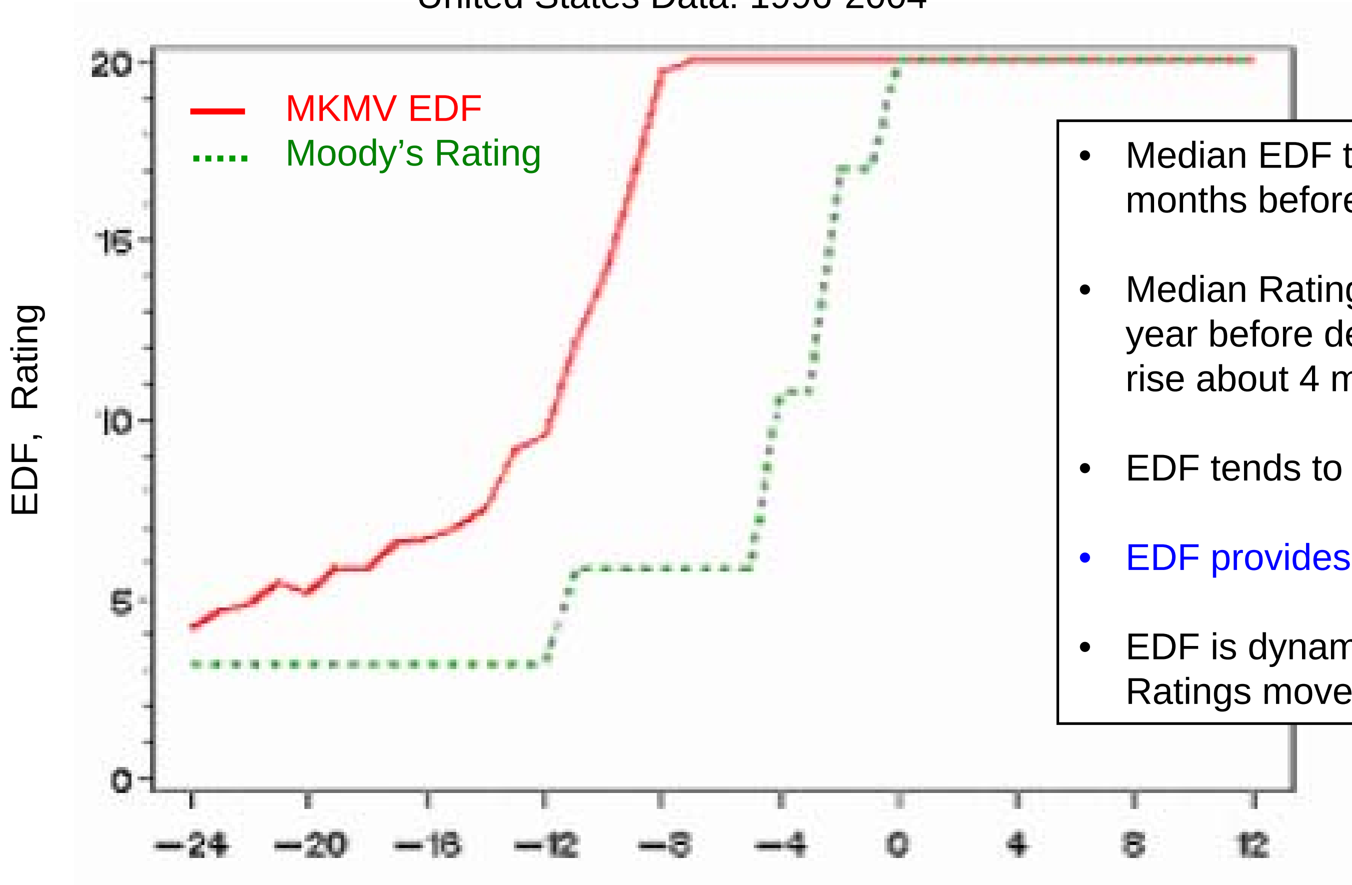
Model	AR
1:NA	0.74739
2:NA+ASG	0.75877
3:NA+CSG	0.7618
4:NA+CSG+XDP	0.76411
5:VK w/o Cash Leak	0.8071
6:MKMV	0.81349

Distance-to-Default (DD) and Credit Default Swap (CDS) Spreads



Early Warning Power of EDF Measures

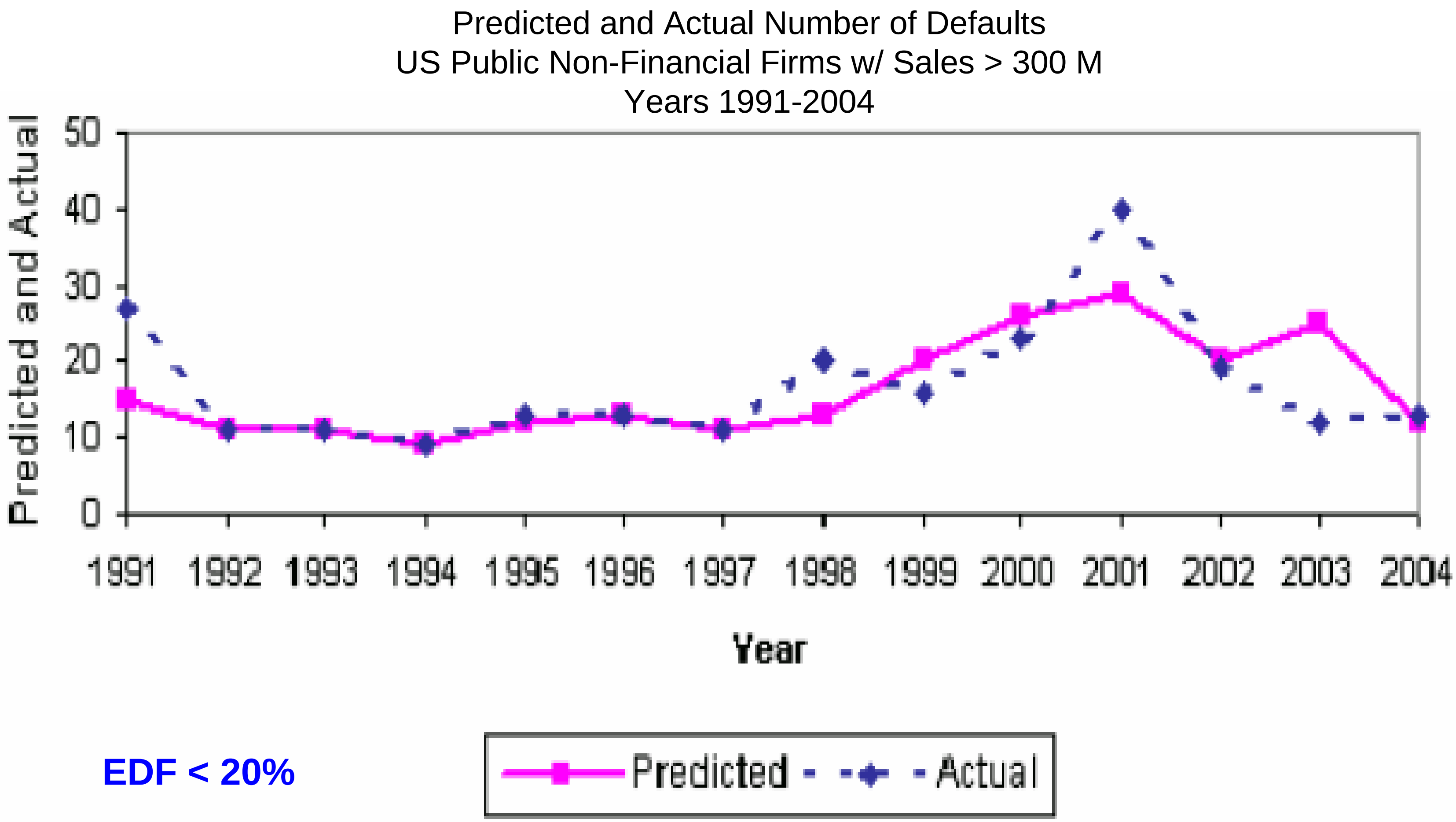
Median EDF and Rating-Implied EDF for Defaulted Firms
United States Data: 1996-2004



Months Before and After Default

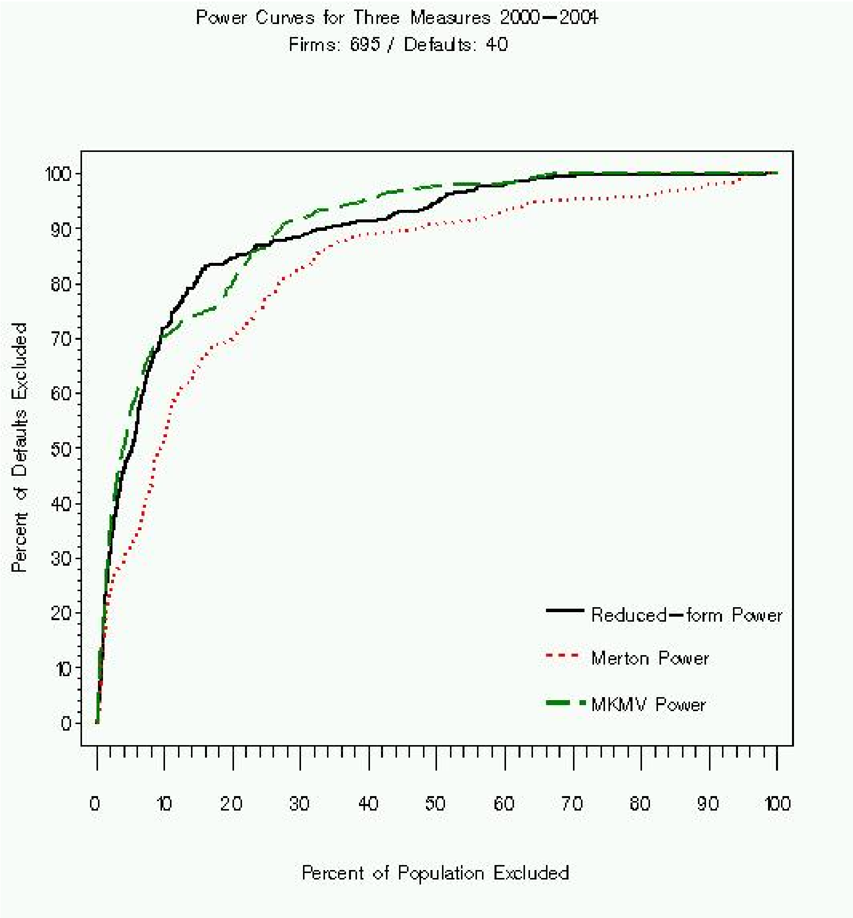
- Median EDF tends to start rising 24 months before default.
- Median Rating tends to stay flat until a year before default, showing a steep rise about 4 months before default.
- EDF tends to lead the Ratings.
- EDF provides early warning power.
- EDF is dynamic and continuous, while Ratings move in discrete steps.

Does the Predicted Default Rate (EDF) match the Actual Default Rate? (Comparison to median prediction generated using a correlation model for firms in sample.)



EDF < 20%

Comparison of Default Predictive Power



Accuracy Ratio	
Model	**Ratio**
Merton	**0.652**
Reduced	**0.785**
MKMV	**0.801**

Explanation of Cross-Sectional Variation

Issuers with 4 or fewer bonds outstanding: 57%

Percentage of cross-sectional variance explained for issuers of 2 to 4 bonds (median):

- Basic Merton: 20%
- Hull-White: 37%
- Vasicek-Kealhofer: 46%

Issuers with 10 or more bonds outstanding: 14%

Percentage of cross-sectional variance explained for issuers of 10 to 15 bonds (median):

- Basic Merton: 46%
- Hull-White: 64%
- Vasicek-Kealhofer: 59%

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Final Thoughts

Improving the Model Further

Enhance modeled volatility estimation

Consider liquidity-based drivers of default at shorter time horizons

Incorporate pro-forma liabilities

Incorporate macro-economic factors

Conclusion

Structural models have provided useful guidance for estimating PD values

Complexity of function and estimation process add value

Validation is both theoretically and practically difficult, but still important

Sample size is important: Small sample produces wide confidence intervals on validation work

Data quality and data cleaning critical to building useful models

5 Appendix

What is added to Black-Scholes-Merton in practice?

Richer characterization of capital structure

Tracking “leakage” of asset value

Barrier option modeling

Expanded asset volatility estimation process

- Iterative solution technique
- Outlier handling
- Corporate event tracking
- Modeled volatility i.e. shrinkage estimation (industry, country, size)

Empirical specification of default point

Empirical calibration of default probability distribution

Implementation of MKMV Model

Use barrier, perpetual option formula to relate asset value and equity value.

Estimate market asset value and market asset volatility using iterative technique on 3 years of weekly data for North American firms and 5 years of monthly data for international firms.

Employ trimming algorithm on outliers.

Track corporate events to adjust for significant changes in capital structure.

Combine “raw” empirical volatility calculated with this process with “modeled” volatility determined from a non-linear regression relating size, country, and industry.

Calculate distance-to-default (DD).

Map DD into EDF credit measure using empirical mapping based on 30 years of default history.

Equity Value as a Function of Asset Value

$E = f(A, \sigma_A, \kappa)$

$\sigma_E = \Delta * (A/E) * \sigma_A$

where E = equity value

A = asset value

σ_E = equity volatility

σ_A = asset volatility

κ = capital structure

Δ = hedge ratio

Distance-to-Default (DD)

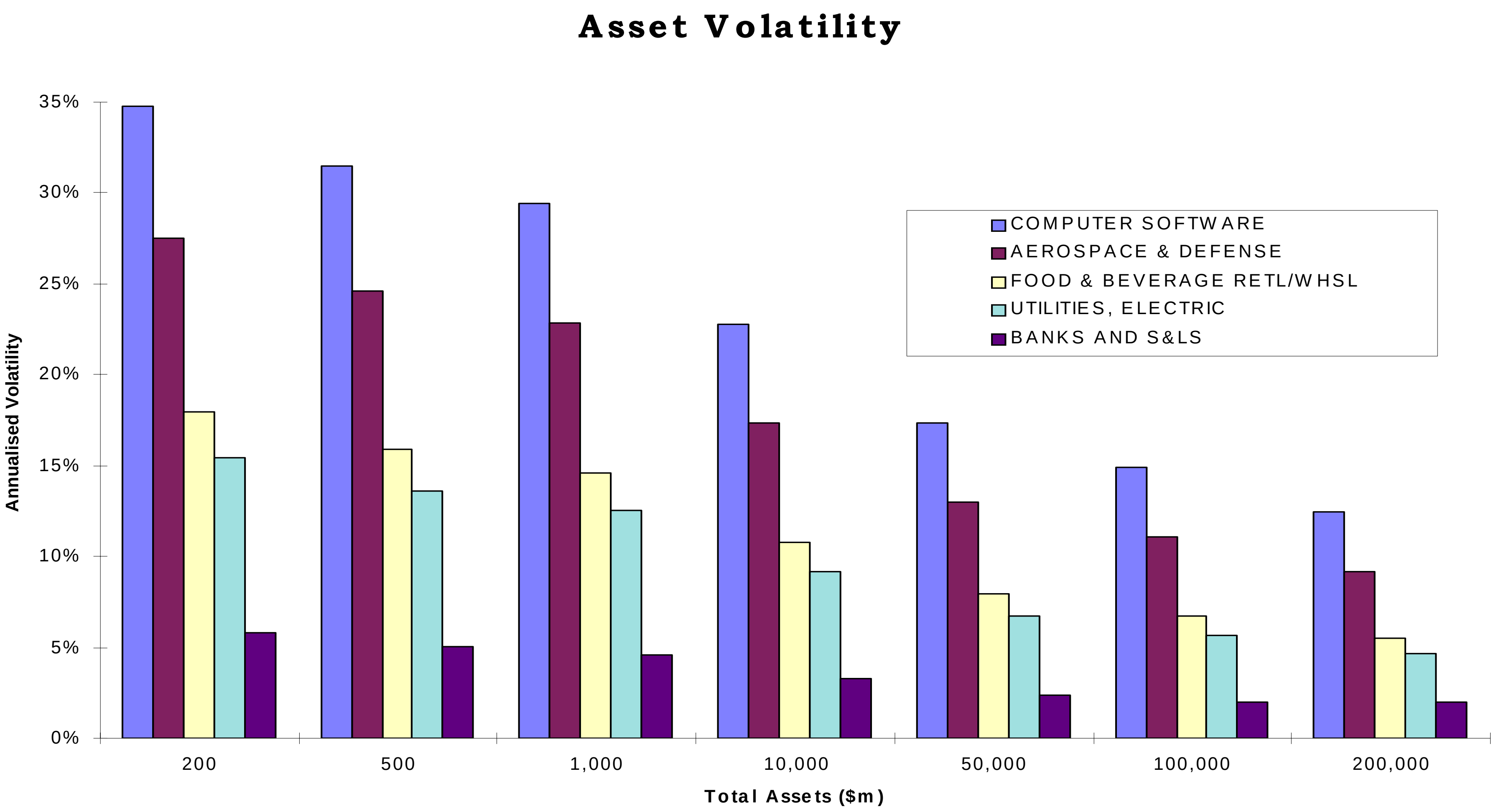
DD is the distance between the market value of assets and default point measured in standard deviations.

$$DD_{0,T} = \frac{\ln A_0 + \left(\mu_A - \frac{\sigma_A^2}{2} \right) T - Payouts - \ln D_T^*}{\sigma_A \sqrt{T}}$$

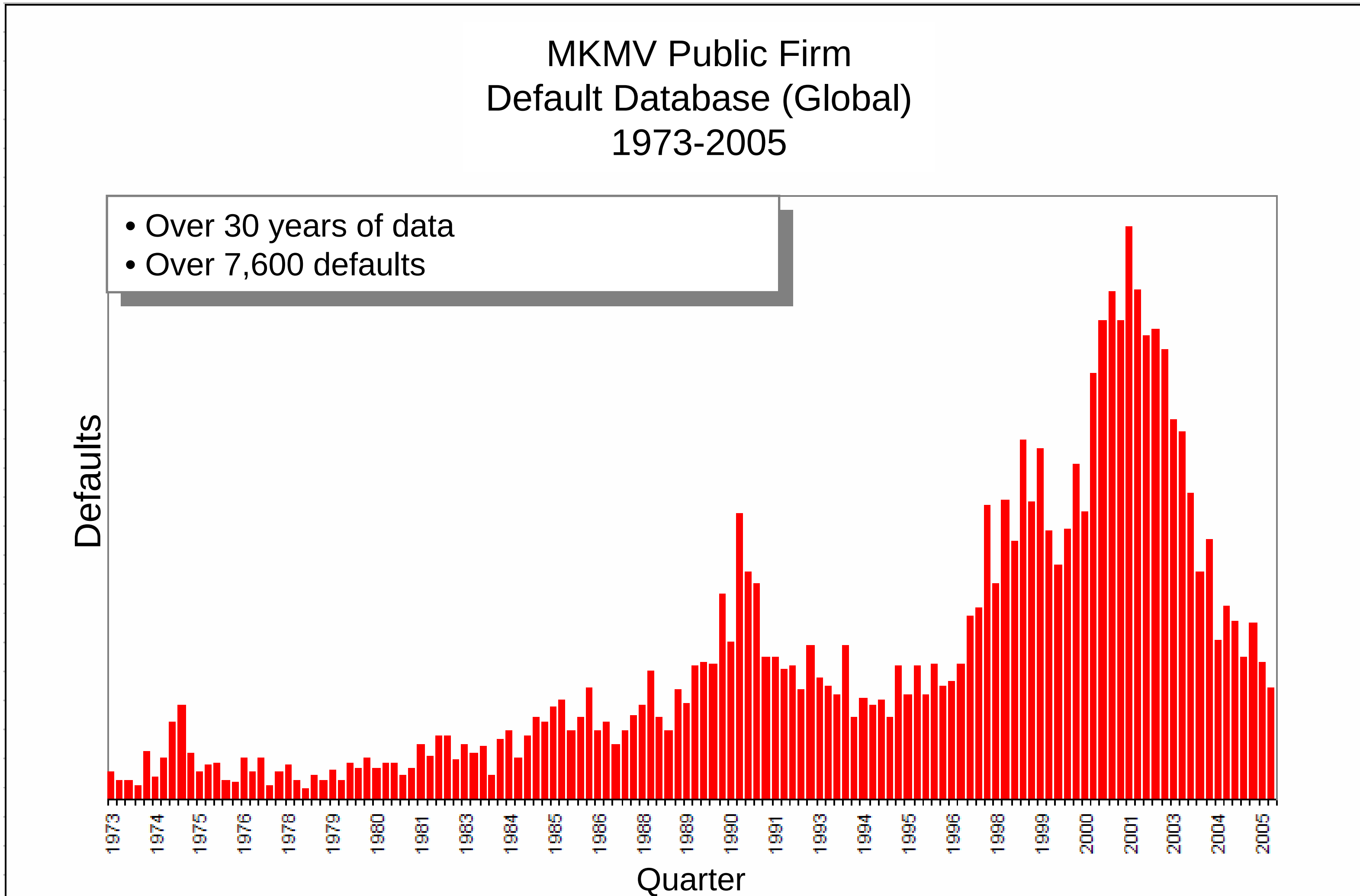
$A \equiv$ Market Value of Assets

$D_T^* \equiv$ Default Point

Asset Volatility: A Measure of Business Risk



MKMV Database



Naïve Alternative (NA) Model Specification

$$NaiveDD = \frac{\ln\left[\left(E+F\right)/F\right] + \left(r_{it-1} - 0.5 Naive\sigma_V^2\right)T}{Naive\sigma_V \sqrt{T}}$$

E = Market value of firm's equity

F = Face value of firm's debt

r_{it-1}= Firm's stock return over the previous year

$$Naive\sigma_V = \frac{E}{E+F}\sigma_E + \frac{F}{E+F}\left(0.05 + 0.25\sigma_E\right)$$

σ_E= Equity volatility

Power Test: Public firm EDF™ power dominates ratings