Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets*

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Abstract

Using a set of structural models, we evaluate the price of default protection for a sample of US corporations. Credit default swaps (CDS) are commonly thought to be less influenced by non-default factors, making them an interesting source of data for evaluating models of default risk. In contrast to previous evidence from corporate bond data, CDS premia are not systematically underestimated. In fact, one of our studied models has little difficulty on average in predicting their level. For robustness, we perform the same exercise for bond spreads by the same issuers on the same trading date. As expected, bond spreads are systematically underestimated, consistent with their being driven by significant non-default components. Considering theoretical and market levels alone is insufficient to evaluate the models' performance, as other factors might be at play in both markets. With this in mind, we relate the models' residuals by means of linear regressions to default and non-default proxies. We find little evidence of any default risk component in either bond or CDS residuals. However, in the residuals for bonds, we find strong evidence for non-default components, in particular an illiquidity premium. CDS residuals reveal no such

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dependence. Taken together with our results on levels of bond spreads and CDS premia, this suggests that structural models are able to capture the credit risk priced in these markets and that they fail to price corporate bonds adequately due to omitted risks.

1 Introduction

A widespread view amongst financial economists is that structural models of credit risk following Black and Scholes (1973) and Merton (1974), although theoretically appealing, underestimate the actual default risk discount on credit risky securities. Several studies from the 1980's onwards document the models producing credit spreads lower than actual corporate bond spreads.[1] However, recent evidence suggests that default risk alone cannot explain the observed yield discrepancies.[2] In turn, this suggests that the apparent underprediction by structural models may not be due to undervaluation of credit risk per se.

To pursue this line of thought, we compare model predicted levels of credit default swap (CDS) premia with their market counterparts. CDS are commonly thought to be less influenced by non-default factors - if at all - and therefore may constitute a better source of data for evaluating structural models' ability to explain default risk. In contrast to what has previously found on corporate bond data, CDS premia are not systematically underestimated. In fact, one of our studied models has little difficulty on average in predicting their level.

For robustness, we also compare the models' theoretical bond spreads to their market counterparts. As expected, the models do systematically underpredict bond spreads, consistent with the hypothesis that these, in contrast to CDS premia, include significant non-default components. However, considering theoretical and market levels alone is insufficient to evaluate the models' performance, as other factors might be at play in both markets. With this in mind, we relate the models' residuals by means of linear regressions to proxies for default and non-default proxies. We find little evidence of any default risk component in either bond or CDS residuals. However, in the residuals for bonds, we find strong evidence for non-default components, in particular an illiquidity premium. CDS residuals reveal no such dependence. Taken together with our results on levels of bond spreads and CDS premia, this is consistent with structural models being able to capture the credit risk priced in these markets and that they fail to price corporate bonds adequately due to omitted risks.

In a recent paper using data on bonds and credit default swaps, Longstaff, Mithal, and Neis (2004) fit a reduced-form model of credit risk jointly to bond spreads and

[1] See Jones, Mason, and Rosenfeld (1984), Jones, Mason, and Rosenfeld (1985), Ogden (1987) and Lyden and Saranati (2000).

[2] See for example Delianedis and Geske (2001), Huang and Huang (2002) and Longstaff, Mithal, and Neis (2004).

default swap premia. By assuming that default swap premia are fully determined by default risk, their model accommodates and estimates a non-default component for corporate bond yield spreads. In a second leg, they show that the estimated non-default spreads are in fact correlated with a set of liquidity proxies. This is consistent with arguments made by Fisher (1959) that bond yield spreads compensate not only for credit risk but also for marketability risk. Further recent work supports this idea: Huang and Huang (2002) and Delianedis and Geske (2001) demonstrate that non-credit risk factors such as illiquidity are the primary drivers of investment grade bond spreads.

Consequently, a structural model should be expected to overprice corporate bonds even if it correctly assesses that part of the yield spread driven by default risk. To determine structural models' ability to capture credit risk, one would ideally use market prices for credit-sensitive financial instruments that reflect only credit risk. Like Longstaff, Mithal, and Neis (2004), we adopt the view that currently, the closest one can get to such data is in the market for credit default swaps.

We use a building block approach to develop a simple pricing formula for credit default swaps (henceforth, CDS). The formula is applied in the framework of three distinct structural models: Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000). By implementing multiple models, we hope to gauge the robustness of our results to specific model assumptions. An important difference between our approach and that of Longstaff, Mithal, and Neis (2004) is that we do not rely on market prices of corporate bonds or CDS as inputs to our estimation. Thus, we need not assume that CDS prices are driven solely by default risk. Instead, we estimate the structural models using firm-specific balance sheet and market data on stock prices, and then compute the term structures of risk-adjusted default probabilities and the corresponding prices for corporate bonds and CDS for the same corporate issuer.[3] By using pairs of contemporaneous transactions and quotes for default swaps and a bond issued by the reference entity, we can compare the estimated bond yield spreads and CDS spreads with two separate sets of data.

As already mentioned, we find that our three models tend to systematically underestimate bond spreads and, more importantly, that this is not the case for CDS

[3]We use a maximum likelihood technique developed by Duan (1994) and evaluated by Ericsson and Reneby (2005). The latter show by means of simulation experiments that the efficiency of this method is superior to the more common approach used in previous empirical studies on the use of structural models for valuing credit risky securities. See for example, Jones, Mason, and Rosenfeld (1984), Ronn and Verma (1986) and Hull (2000).

premia. For our benchmark specifications, the Leland (1994) and Fan and Sundaresan (2000) models underestimate bond spreads by 108 and 91 basis points respectively. The Leland and Toft (1996) model underestimates dramatically less – by ca. 56 basis points. These numbers are not dissimilar to previous findings by, for example, Eom, Helwege, and Huang (2004). When we consider CDS premia, a very different picture emerges. On comparison of model and market CDS premia we find that the Leland (1994) and Fan and Sundaresan (2000) models underestimate CDS premia by 50 and 33 basis points, much less so than for bonds, in particular for the latter. The benchmark implementation of the Leland and Toft (1996) actually overestimates CDS premia by, on average, 8 basis points. This, in our view, constitutes evidence in favour of structural credit risk models, at least when applied to the valuation of default swaps. Two questions remain. First, is there a significant remaining component in CDS premia that the models do not capture? Second, can the improved performance be attributed to differences in market pricing for bonds and CDS?

We therefore proceed to analyze how market and residual spreads are explained by proxies for default and non-default risk. If the structural model indeed manages to capture credit risk, the residual spreads should, of course, be uncorrelated with credit risk factors. Given recent evidence, however, they should be related to proxies for illiquidity and taxes.[4] CDS premia residuals should be related to neither.

We find that (i) residual (market - model) bond spreads are largely unrelated to default proxies but correlated with non-default proxies and (ii) residual CDS premia are uncorrelated with both. Thus the models that still slightly underestimate CDS premia, are nevertheless important in explaining their variation over time. We thus support recent findings that bond spreads– but not premia – are, to a significant extent, driven by non-default factors. More importantly, we provide evidence that the widely documented overpricing of corporate debt by structural credit risk models is unlikely accreditable to an inability to properly capture credit risk, but rather to unmodelled, non-default factors. Thus we find that structural models may well be useful tools for valuing credit risky financial instruments. In particular, without modification, some extant models can be used with some success for the valuation of default swaps.

The remainder of this paper is structured as follows. The following Section introduces the models and Section 3 the empirical methodology. Thenceforth, section 4

[4]For empirical work on the influence of liquidity on corporate bond risk premia, see for example **?**) and **?**). For the influence of taxes on yield spreads see Elton, Gruber, Agrawal, and Mann (2001).

describes both the bond and CDS datasets. Section 5 draws out the implications of the results and finally, section 6 concludes.

2 Setup

Here we initially present the three structural models that we will study. The ensuing description is brief and we refer the reader to the original papers for details. These do not, however, treat the valuation of credit default swaps. Therefore, we utilize a simple building block approach to demonstrate how to value a CDS and its reference bond.

2.1 The Models

Consider first the common characteristics of the three structural models: Leland (1994), Leland and Toft (1996) and Fan and Sundaresan (2000). The fundamental variable in all models is the value of the firm's assets (the unlevered firm), which is assumed to evolve as a geometric Brownian motion under the risk-adjusted measure:

$$d\omega_t = (r - \beta)\,\omega_t dt + \sigma\omega_t dW_t$$

The constant risk-free interest rate is denoted r, β is the payout ratio, σ is the volatility of the asset value and W_t is a standard Wiener-process under the risk-adjusted measure.

Default is triggered by the shareholders' endogenous decision to stop servicing debt. The exact asset value at which this occurs is determined by several parameters as well as the characteristics of the respective models, but is always a constant which we denote by L.

The value of the firm differs from the value of the assets by the values of the tax shield and the expected bankruptcy costs. Coupon payments are tax deductible at a rate τ and the realized costs of financial distress amount to a fraction α of the value of the assets in default (i.e. L). In this setting, the value of the firm ($\mathcal{F}$) is equal to the value of assets *plus* the tax shield ($\mathcal{TS}$) *less* the costs of financial distress ($\mathcal{BK}$). The value of the firm is, of course, split between equity ($\mathcal{E}$) and debtholders ($\mathcal{D}$) and thus all models share the following basic equalities:

$$
\begin{aligned}
\mathcal{F}(\omega_t) &= \omega_t + \mathcal{TS}(\omega_t) - \mathcal{BK}(\omega_t) \\
&= \mathcal{E}(\omega_t) + \mathcal{D}(\omega_t)
\end{aligned}
\tag{1}
$$

Note that the formulae for the components depend on the model, but that they are all independent of time.

In the interest of brevity, the formulae are relegated to the appendix, and this paper merely discusses differences between the applied models. First, Leland (1994) is a natural benchmark model where debt is perpetual and promises a continuous coupon stream C. Financial distress triggers immediate liquidation and no renegotiation is possible.

In Leland and Toft (1996), the firm continuously issues debt of maturity Υ; therefore, the firm also continuously redeems debt issued many years ago. Hence, at any given time, the firm has many overlapping debt contracts outstanding, each serviced by a continuous coupon. Coupons to individual debt contracts are designed such that the total cash flow to debt holders (the sum of coupons to all debt contracts plus nominal repayment) is constant. Letting $\Upsilon \to \infty$, the model converges to the Leland model. For shorter maturities, the need to redeem debt places a higher burden on the firm's cash flows. Consequently, the default barrier in the Leland & Toft model tends to be much higher than in the Leland model for short and intermediate debt maturities.

In Fan and Sundaresan (2000) debt is, as in Leland, single-layered and perpetual but creditors and shareholders can renegotiate in distress to avoid inefficient liquidations. Consequently, the default barrier in the former model is typically lower than in its Leland counterpart. To the extent that equity holders can service debt strategically, shareholder bargaining power is captured by the parameter η. If the bargaining power is nil, no strategic debt service takes place and the model converges to the Leland model.

2.2 Valuing the bond

Next think about what kind of real world 'debt' the authors had in mind when building the three structural models above. A firm's debt consists of bank loans, bonds, accounts payable, salaries due, accrued taxes etc. Dues to suppliers, employees and the government are substitutes for other forms of debt. Part of the price of a supplied good and part of salary paid can be viewed as corresponding to compensation for the debt that, in substance, it constitutes. The cost of debt consequently includes not only regular interest payments to lenders and coupons to bondholders, but also fractions of most other payments made by a company. Clearly, a comprehensive model of all these payments would not be tractable and we think of the three models as portraying firms' aggregate debt rather than a particular bond issue – such as the reference obligation of the CDS.

However, we do need a pricing formula which also accounts for the reference obligation – for robustness, we will let the models price both the CDS and the corresponding bond in order to investigate whether the overestimation of the spread is indeed smaller in the former case. To this end, we apply a bond pricing model that takes discrete coupons, nominal repayment and default recovery into account.[5] To express the value of the bond we make use of two building blocks, a binary option $H(\omega_t, t; S)$ and a dollar-in-default claim $G(\omega_t, t; S)$. The former pays off \$1 at maturity S if the firm has not defaulted before that, the latter pays off \$1 upon default should this occur before S; the value of both depend upon the firms asset value ω_t and current time t. The formulae for the binary option and the dollar-in-default claim are, for a given default barrier L, identical in all three structural models.

Proposition 1 ***A straight coupon bond.*** *The value of a coupon bond with M coupons c paid out at times $\{t_i : i = 1..M\}$ is*

$$\begin{aligned} \mathcal{B}(\omega_t, t) &= \sum_{i=1}^{M-1} c \cdot H(\omega_t, t; t_i) \\ &\quad + (c + P) \cdot H(\omega_t, t; T) \\ &\quad + \psi P \cdot G(\omega_t, t; T) \end{aligned}$$

The formulae for H and G are given in the appendix.

The value of the bond is equal to the value of the coupons (c), the value of

[5] This bond pricing model was used in Ericsson and Reneby (2004) and was shown to compare well to reduced form bond pricing models.

the nominal repayment (P) plus the value of the recovery in a default (ψP). Each payment is weighted with a claim capturing the value of receiving \$1 at the respective date.

Note that the above formula for the reference bond is not directly related to the debt structure of the firm. Specifically, coupon payments to the bond are unaffected by the strategic debt service in the Fan & Sundaresan model, and by the debt redemption schedule elaborated in the Leland & Toft model. The choice of model affects the bond formula solely via the default barrier L.

2.3 Valuing the CDS

A CDS provides insurance for a specified corporate bond termed the *reference obligation*. The firm issuing this bond is designated the *reference entity*. The seller of insurance, the *protection seller*, promises, should a default event occur, to buy the reference obligation from the *protection buyer* at par.[6] The credit events triggering the CDS are specified in the contract and typically range from failure to pay interest to formal bankruptcy. For the CDS, the protection buyer pays a periodic fee rather than an up-front price to the seller. When and if a credit event occurs (at time $\mathcal{T}$), the buyer is also required to pay the fee accrued since the previous payment. Hence fee payments, although due at discrete intervals, fit nicely into a continuous modelling framework. Note also that there is no requirement that the protection buyer actually own the reference obligation, in which case the CDS is used for speculation rather than protection.

The valuation of a CDS thus involves two parts, the premium paid by the protection buyer and the potential buy-back by the protection seller. Letting T^* denote maturity of the CDS and Q the fee, the value of the premium at time t is

$$E' \left[\int_t^{T^*} e^{-r(s-t)} \cdot Q \cdot I_{\mathcal{T} \not< s} \, ds \right]$$

where we let $I_{\mathcal{T} \not< s}$ be the indicator function for nondefault before s and E' denotes the expectations under the standard pricing measure. The maturity of the credit default swap is typically shorter than the maturity of the reference obligation (T). In fact, the by far most common maturity in practice is $T^* = 5$ years.

[6]In practice, there may be cash settlement or delivery of another (non-defaulted) bond in place of direct purchase but we refrain from that complication here.

Assume that a bond holder in the event of bankruptcy recovers a fraction ψ of par, P. The second part of the value of a CDS therefore is the expected value of receiving, upon default of the firm, the difference between the bond's face value and its market price, $P - \psi P$:

$$E' \left[e^{-r(\mathcal{T}-t)} \cdot (P - \psi P) \cdot I_{\mathcal{T}<T^*} \right]$$

The expectation is conditional on default occurring before maturity of the CDS. Using the previously outlined building blocks, we can formalize the value of a CDS in the following proposition.

Proposition 2 *Assume a CDS involves receiving an amount* $P - \psi P$ *if* $\mathcal{T} < T^*$*, and paying a continuous premium* q *until* $\min(T^*, \mathcal{T})$*. The value of the CDS is*

$$\mathcal{CDS}(\omega_t, t) = (P - \psi P) \cdot G(\omega_t, t; T^*) - \frac{Q}{r} (1 - H(\omega_t, t; T^*) - G(\omega_t, t; T^*))$$

The first leg of the CDS-formula captures the value of receiving the bond's face value in case of default. The second leg captures the cost of paying the premium as a risk-free, infinite stream ($\frac{Q}{r}$) less two terms: the first (H) reflecting the discount attributable to the finite maturity of the swap, and the second (G) reproducing the discount due to disrupted payments when and if default occurs.

Typically, the fee is chosen so that the credit default swap upon initiation ($t = 0$) has zero value:

$$Q = \frac{r \cdot (P - \psi P) \cdot G(\omega_0, 0; T^*)}{(1 - H(\omega_0, 0; T^*) - G(\omega_0, 0; T^*))} \tag{2}$$

Often the fee is expressed as a fraction of the reference obligation's face value, and we will refer to the ratio $q = \frac{Q}{P}$ as the credit default swap premium.

Intuitively, holding a CDS together with the reference obligation is close to holding the corresponding risk-free bond only. The positions are not identical, however, since the CDS typically has a different maturity and assures its holder the nominal amount (P), rather than value of the risk free bond (B), upon default. Yet, it is often convenient to think of the default swap premium of a just initiated swap as akin to the spread on the underlying corporate bond.

3 Empirical Method

In the previous section we laid down the pricing formulae for stocks, credits default swaps and bonds. In this section we discuss issues related to the practical implementation of our framework. Table 1 lists the notation used and the assigned parameter values; these are discussed below.

The following inputs are needed to price bonds and CDS using Propositions 1 and 2:

- the bond's principal amount, P, the coupons c , maturity T and the coupon dates
- the recovery rate of the bond, ψ
- the risk-free interest rate, r
- the total nominal amount of debt, N, coupon C and maturity Υ (Leland & Toft only)
- the bargaining power of debtholders η (Fan & Sundaresan only)
- the costs of financial distress, α
- the tax rate, τ
- the rate, β, at which earnings are generated by the assets, and finally
- the current value, ω, and volatility of assets, σ

Details of the bond contract are readily observable. However, the recovery rate of the bond in financial distress is not. We set it equal to 40%, roughly consistent with average defaulted debt recovery rate estimates for US entities between 1985-2001. For the risk-free rate we use constant maturity Treasury yields interpolated to match the maturity of the corporate bond or the CDS.

The nominal amount of debt equals the total liabilities taken from the firms' balance sheets. For simplicity, we assume that the average coupon paid out to all the firm's debt holders equates the risk-free rate: $C = r \cdot N$. To apply the LT model, we also need to specify the maturity of newly issued debt, Υ. This choice turns out to be important and at the same time difficult to pin down; therefore, we will display results for three choices of maturity: 3.38, 5 and 10 years. In Fan & Sundaresan, maturity is, by design, infinite but in contrast, we need the bargaining power of debtholders – we use 0.5. Finally, we assume that 15% of the firm's assets are lost in financial distress before being paid out to debtholders and fix the tax rate at 20%.[7]

Determining the cash flow parameter β is of crucial importance. We, therefore, opt for a dual approach. The first assigns its value exogenously (using 0% and 6%), and the other predicts it as a weighted average of the historical dividend yield and relative interest expense.

We then require estimates of asset value and volatility. The methodology utilized, first proposed by Duan (1994) in the context of deposit insurance, uses price data from one or several derivatives written on the assets to infer the characteristics of the underlying, unobserved, process. In principle, the "derivative" can be any of the firm's securities but in practice, only equity is likely to offer a precise and undisrupted price series.

The maximum likelihood estimation relies on a time series of stock prices, $E^{obs} = \left\{\mathcal{E}_i^{obs} : i = 1...n\right\}$. A general formulation of the likelihood function using a change of variables is documented in Duan (1994). If we let $w\left(\mathcal{E}_i^{obs}, t_i; \sigma\right) \equiv E^{-1}\left(\mathcal{E}_i^{obs}, t_i; \sigma\right)$ be the inverse of the equity function, the likelihood function for equity can be expressed as

$$\begin{aligned} L_{\mathcal{E}}\left(\mathcal{E}^{obs}; \sigma\right) &= L_{\ln \omega}\left(\ln w\left(\mathcal{E}_i^{obs}, t_i; \sigma\right) : i = 2...n; \sigma\right) \\ &\quad - \sum_{i=2}^{n} \ln \omega_i \left. \frac{\partial\, \mathcal{E}\left(\omega_i, t_i; \sigma\right)}{\partial\, \omega_i} \right|_{\omega_i = w\left(\mathcal{E}_i^{obs}, t_i; \sigma\right)} \end{aligned} \tag{3}$$

$L_{\ln \omega}$ is the standard likelihood function for a normally distributed variable, the log of the asset value, and $\frac{\partial\, \mathcal{E}_i}{\partial \omega_i}$ is the "delta" of the equity formula.

An estimate of the asset values is computed using the inverse equity function: $\hat{\omega}_t = w\left(\mathcal{E}_n^{obs}, t_n; \widehat{\sigma}\right)$. Once we have obtained the pair $(\widehat{\omega}_t, \widehat{\sigma})$ it is straightforward to compute the estimated CDS fee using (2). The bond spread is calculated by solving the bond price formula in Proposition 1, computing the risky yield, and subtracting the yield for the corresponding risk free bond.[8]

[7]The choice of 15% distress costs lies within the range estimated by Andrade and Kaplan (1998). The choice of 20% for the effective tax rate is consistent with the previous literature (see e.g. Leland (1998)) and is intentionally lower than the corporate tax rate to reflect personal tax benefits to equity returns, thus reducing the tax advantage of debt.

[8]This bond is valued as a hypothetical bond with the same promised payments as the risky bonds but with yield equal to a linear interpolation of bracketing constant maturity Treasury (CMT) yield indices.

4 Data

To perform our estimation, we require price data on credit default swaps and corporate bonds as well as balance sheet and term structure information.

CreditTrade (CT) market prices are credit default swap (CDS) bids and offers that have either been placed directly into their electronic trading platform by traders, or entered into their database by their voice brokers who receive orders by telephone. Our database includes quotes and trades from June 1997 to April 2003. However, the early years' volumes are minimal and only as of 1999 does the volume become significant. The database distinguishes between bid and offers, and between quotes and trades. The evolution of credit ratings of the underlying debt by Moody's and S&P is recorded from COMPUSTAT. In the early part of our sample, restructuring is considered a credit event. From the very end of 2002 onwards, restructuring is no longer considered a credit event of the CDSs. The underlying debt is almost exclusively senior. All contracts are USD denominated. For consistency, we retain only CDS on senior unsecured debt with restructuring as default event. Very little information is lost with the use of each of these filters.

Our bond transaction data is sourced from the National Association of Insurance Commissioners (NAIC). Bond issue - and issuer - related descriptive data are obtained from the Fixed Investment Securities Database (FISD). The majority of transactions in the NAIC database take place between 1994 and 2001.

Cleaning-up of the raw NAIC database was carried out in three steps. In the first, bond transactions with counterparty names other than insurance companies and Health Maintenance Organizations (HMOs) are removed. Transactions without a clearly defined counterparty are deemed unreliable.

In the second step, we restricted our sample to fixed coupon rate USD denominated bonds with issuers in the industrial sector. Furthermore, we eliminated bond issues with option features, such as callables, putables, and convertibles. Asset-backed issues, bonds with sinking funds or credit enhancements were also removed to ensure bond prices in the sample truly reflect the underlying credit quality of issuers.

The third step involves selecting those bonds for which we have their issuers' complete and reliable market capitalizations as well as accounting information about liabilities. Daily equity values are obtained from DATASTREAM. Quarterly firm balance sheet data is taken from COMPUSTAT.

In total, 98 firms qualify in both the CT and NAIC databases. When we take an intersection of the CT and NAIC data, while requiring a transaction and / or quote for the bonds and default swaps on the same name during the same day, we are left with 731 pairs from 71 distinct entities.[9]

Table 2 identifies descriptive statistics for firm, bond and CDS features. In 2a we see that firm sizes vary from 2.8 billion to 302 billion with an average of 84 billion dollars. Bond issuers' / CDS reference entities' S&P credit ratings range between AA and CCC while the majority lies between BBB+ and BBB. The average bond issue size is 486 million dollars, with 7 million and 3,250 million as extremes. The average transaction size is approximately 4.3 million dollars.[10] On average, bonds were 4.1 years of age and had 9.4 years remaining to maturity. The average coupon rate is 7 percent across all issues. The longest maturity of CDS in our sample is 8 years versus 0.08 years as the shortest. The average maturity is 4.35 years, although the vast majority of the swaps have a 5 year life span.

Table 2b presents the distribution of market CDS and bond spreads over different ratings. As expected, the CDS premia, like bond yield spreads, are largely determined by the reference entity's / issuer's credit quality. Moreover, bond spreads lead over CDS premia across all rating categories save one (BB-, where there are only two observations). CDS premia only represent about a quarter of bond spreads for the AA range, but this fraction increases steadily to ca. 80% for a BB rating. For lower ratings, for which we have less data, the pattern is no longer as clear, but the ratio remains much higher than for high grade issuers. This is suggestive of a proportionally larger non-default component in bond yield spreads for issuers with little default risk.

5 Results

We first briefly characterize the outcome of implementing each model in terms of mean asset value and volatility estimates, before turning to the main results – the corresponding predicted CDS premia and bond spreads. Table 3 shows that the Leland (1994) (L)and Fan and Sundaresan (2000) (FS) models yield approximately the same asset value and volatility estimates on average, whereas the Leland and Toft (1996) (LT) model produces higher asset value but lower asset volatility. The reason is that the higher barrier in the latter model, *ceteris paribus,* increases the theoretical

[9]Note that since we are not using the bond prices and CDS premia in our estimation, we do not require any matching or bracketing between bond and CDS maturities.

[10]Chakravarty and Sarkar (1999) document an average transaction size of 4.4 millions.

equity volatility, and hence the need to predict a higher asset value and/or a lower volatility to match theoretical with observed equity volatility. As a measure of the combined effect of asset value and volatility we also report the KMV-style distance-to-default, i.e. $\frac{\omega - L}{\sigma\omega}$. This shows that the higher asset value and lower volatility estimates in the LT model are not sufficient to compensate the effect of the higher barrier; the model reports the shortest distance-to-default while the Leland model has the highest. This is also evident in the average values of dollar-in-defaults of 2 and 10 years maturity. Therefore, the LT model is expected to produce the highest CDS / bond spreads, followed by FS and finally L.

5.1 CDS premia and bond spreads

Now we turn our attention to the bond spreads and CDS premia estimated by the three models, reported in tables 4a-4c. As expected, the L model estimates the lowest mean bond spread, 60 bps, while the FS and LT models estimate mean bond spreads of 77 bps and 103 bps, respectively. Compared to the mean market bond spread of 168 bps, they substantially underestimate bond yield spreads by 64%, 54% and 39%. This is in line with the findings of previous literature.[11]

The L, FS and LT models produce mean CDS premia of 41, 58 and 89 bps respectively compared to the observed mean CDS premium of 91 bps. Underpredictions are significantly reduced for the L and FS models, whereas the LT model now underestimates CDS premia by 2 basis points on average. Thus, our most plausible model specification produces CDS premia close to market quotes while distinctly underestimating bond yield spreads. Should CDS premia be lower than bond spreads due to the presence of a greater non default component in yield spreads, then the evidence presented in this paper provides some support for structural models. Below, we consider determinants of those components of bond spreads and CDS premia left unexplained by such models.

To assess the sensitivity of our results we also report estimated premia for alternative values of the cash flow rate and, for the LT model, bond maturity. The cash flow rate tends to increase bond spreads and CDS premia because firms with higher payout rates grow less and thus have higher default probabilities. Increasing β can help to reduce the yield spread and CDS premia underestimation for the Leland and

[11]See for example Jones, Mason, and Rosenfeld (1984), Ogden (1987) and Lyden and Saranati (2000). The numbers are also comparable to what Huang and Huang (2002) find for various models in an extensive calibration exercise.

FS models, although not eliminate it. For the LT model, bond yield spread underestimation persists. For CDS premia, the parameter can tilt the balance although at 6%, CDS premia are clearly overstated. The weighted average method of estimating β, provides reasonable results for CDS premia in the LT model.

Note that both 0% and 6% are extreme values. Few if any firms will issue debt and finance coupons entirely out of newly raised equity. A payout rate of 6% is about three times higher than the typical weighted average between firms' dividend yields and interest expenses. Thus numbers reported at these values should illustrate the highest and lowest spreads / premia that can be obtained by varying this parameter.

For debt maturity in the LT model we consider three values: 5, 6.76 and 10 years. Note that the maturity parameter in this model represents the maturity of newly issued debt. It should, therefore, exceed the average maturity of a firm's debt. On the other hand, a firm's debt consists not only of bonds but also of a variety of credits with very short maturity. Consequently, although many firms issue bonds with maturities exceeding 10 years, they are likely to also issue medium-term notes, short-term commercial paper, while also indirectly borrowing from their suppliers, the government and their employees. It seems unlikely that the average maturity of new debt exceeds ten years. We consider the average debt maturity (3.38 years) reported by Stohs and Mauer (1994) which represents a reasonable average maturity of new debt ($6.76 = 3.38 * 2$ years).

An interesting observation is that all structural models residual bond spreads are consistently ca. 60 basis points higher than residual CDS premia. This holds true, regardless of the model. As already discussed, this is in line with the hypothesis that credit default swap premia contain less of a nondefault component. The 60 basis points can be viewed as the difference between the non default components in the bond and CDS markets. This number is lower than the difference between market bond spread and CDS premia which averages 77 basis points. The discrepancy arises because of the difference in maturity between the bonds and the CDS. Bonds are longer than the default swaps by about 5 years. Interestingly, the three models concur that these 5 years should imply yield spreads that are higher by 17 basis points on average, although they predict different absolute levels for the spreads. In addition to gauging the robustness of the different models, this underlines that, merely taking the difference between CDS premia and bond spreads as a proxy for the non-default component in bond spreads without adjusting for differences in maturity, can generate misleading results.

5.2 Default and non-default components of spreads and premia

Table 5 reports the results for a regression analysis of the residual (market minus model) bond spreads and CDS premia. Should the models be able to capture their credit-risk component, the residuals should be uncorrelated to proxies for default risk. Second, if bond spreads are more prone to non default factors than CDS premia, as suggested by the analysis in Longstaff, Mithal, and Neis (2004), then bond residual spreads should be correlated with proxies for these, whereas residual CDS premia should be less so, if at all.

As proxies for default risk, we use aggregate variables such as the difference between Moody's Aaa and Baa bond yield indices, the S&P rating, the VIX, and the return on the S&P 500 index.

To proxy the non-default component, we utilize coupon rates, the size of a bond transaction, size of an issue, a dummy (OTR) for the age of the bond (1 if younger than 6 months). While the coupon could proxy for a tax effect (see Longstaff, Mithal, and Neis (2004)), the remainder represent bond marketability variables.

It has also been documented that bond yield spreads and CDS premia are negatively related to the level of the risk-free rate (see Duffee (1998) and Ericsson, Jacobs, and Oviedo (2004)). Existing literature offers at least three explanations for this. The first relies on the following argument: when risk-free rates increase, both corporate and government bond prices drop. But if the more liquid government bond prices react faster than corporate issues, the former would observe a short term compression of the yield spread. The second explanation is, likewise, related. Since corporate bonds issue higher coupons to compensate for the higher default risk, their lifecycle will be shorter than their Treasuries counterparts. Hence, increasing risk-free rates depresses prices for Treasuries more than for corporates, therewith lowering the yield spread. Such explanations are discussed in Duffee (1998) and Duffie and Singleton (2003). Finally, a structural model similar to Merton (1974), predicts the reduced probability of neutral default as a result of the increase in risk-free rates, thereby also causing yield spread compression.

The first explanation is consistent with the negative impact of the risk-free rate on bond yield spread residuals in Table 5. However, it should not generate any relationship with CDS residuals since CDS premia are not metrics of the difference in yields on different securities. The second explanation implies no relationship between residuals for either bonds or the CDS.[12] The third explanation suggests likewise that there is no relationship, assuming the accuracy of the models' estimation on risk-free rate and risk-neutral asset value dynamics.

We find that the S&P rating is significant only in one case out of six, for CDS residuals in the L model. Moody's Aaa/Baa spread is marginally significant for the FS model on bond residuals, otherwise insignificant. The VIX is insignificant except for the LT model on bond spread residuals. The S&P 500 return shows up significantly for the L and FS models on both bond and CDS residuals, but not at all for the LT model. There are thus indications that the structural models may leave some default-related variation in spreads and premia unexplained. However, this result varies across models and instruments. There appears to be somewhat less unexplained default risk for CDS than for bonds and the LT model is the least afflicted with significance for the VIX on bonds and nothing for CDS. It should also be noted that those variables which predominate significantly may also proxy for other risks not included in the models. For example, in Collin-Dufresne, Goldstein, and Martin (2001), the S&P 500 return is taken to proxy for the state of the overall economy, once they control for firm leverage and return (as our models do).

Bond coupons are important for bond spread residuals. A higher coupon tends to increase the underestimation of bond yield spreads. This is consistent with a tax effect as suggested by Elton, Gruber, Agrawal, and Mann (2001). Higher coupons exacerbate the differential taxation of corporate and Treasury bonds in our sample and a structural model which does not account for this will predict relatively lower yields for those corporate bonds. This finding is consistent with those obtained by Longstaff, Mithal, and Neis (2004) who consider residuals from a reduced-form model. Note, however, that they only consider bond residuals since CDS are, by assumption, devoid of any non-default component in their paper.

The maturity of the bond helps to explain the bond residuals for the L and FS models and, surprisingly, the CDS residuals for the FS model. However, its explanatory power is negligible in the case of the LT model which provides a more realistic

[12]Unless the evaluated structural models do not fully capture duration differences between corporate and Treasury bonds. Again, since CDS premia are not computed as the difference in yields for two securities, this second explanation does not suggest a relation between the risk free rate and CDS premia.

debt maturity structure.

The risk-free rate enters with a negative sign and strong significance for all residuals, bond and CDS alike. This finding holds with previous bond literature on bonds (Duffee (1998)) and CDS (Ericsson, Jacobs, and Oviedo (2004)). Interestingly, it is inconsistent with those stale price and duration explanations discussed above. In the context of the third explanation, the results suggest that either our implemented structural models are missing an aspect of interest rate risk or that the explanation lies elsewhere.

A number of the variables used in the regression can be thought of as proxies for bond or market liquidity. The time series of transactions sizes recorded for the corporate bond data, the size of the bond issue and the issuing firm can all be thought to proxy for the marketability of a given bond. Two proxies that have been used with some success in the literature are the old bond new bond spread and the age of the bond (in our case the OTR dummy). We do not find any significance for the bond transaction sizes. Nor are the issue sizes important in explaining the bond residuals. Perhaps surprisingly, for the L model the CDS residual is marginally significantly related to the bond issue size. The firm size has little explanatory power and is only found to be marginally related to the bond residuals for the L model. The OTR dummy on the other hand is strongly negatively related to bond residuals for all three models– for young bonds the yield spread underestimation is about 20 basis points lower than for older bonds, consistent with the stylized fact that they tend to be less liquid. The 5 year on-off the run spread is also significant for all three models. The larger this spread, the more pronounced the underestimation of bond yield spreads. Again this is consistent with the presence of a liquidity component in bond yield spreads. This variable has no explanatory power for CDS residuals, confirming our prior that these are less influenced by illiquidity and that bond spreads should be higher than CDS premia as we have found in this study.

In summary, we find weak evidence of remaining default risk in the residuals from the three structural models. In contrast, we find strong evidence of the presence of illiquidity in bond residuals but not in CDS residuals. This suggests that rather than an inability to capture default risk, the main shortcoming of structural models to date is that they ignore illiquidity risk

6 Conclusion

We have estimated three structural credit risk models using corporate balance sheet and equity data and compared estimated CDS premia and bond spreads with market data. Like previous studies, we document that the models tend to underestimate bond spreads. This is not the case for CDS premia. For example, the Leland & Toft model, reasonably specified, comes very closed to observed CDS premia. We also provide evidence that (i) residual bond spreads and CDS premia are uncorrelated with default risk proxies and (ii) total and residual bond spreads (in contrast to CDS premia) are correlated with non default factor proxies. Overall, our results question the interpretation of the well established underestimation of bond spreads by models as an inability to properly capture credit risk. Instead, it seems that this underprediction is mainly due to factors not included in the models. Addressing this point should be a priority in the development of new structural credit risk models. However, to value default swaps, current models may well be adequate.[13]

[13]See for example Ericsson and Renault (2000).

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A Appendix

A.1 Building blocks for CDS and bonds

First, define default as the time ($\mathcal{T}$) the asset value hits the default boundary from above, $\ln \frac{\omega_{\mathcal{T}}}{L_{\mathcal{T}}} \equiv 0$. Then define $G(\omega_t, t)$ as the value of a claim paying off \$1 in default:

$$G(\omega_t, t) \equiv E^B \left[e^{-r(\mathcal{T}-t)} \cdot 1 \right]$$

We let E^B denote expectations under the standard pricing measure. The value of G is given by

$$G(\omega_t, t) = \left(\frac{\omega_t}{L_t} \right)^{-\theta}$$

with the constant given by

$$\theta = \frac{\sqrt{\left(h^B\right)^2 + 2r} + h^B}{\sigma}$$

and

$$h^B = \frac{r - \beta - 0.5\sigma^2}{\sigma}$$

Define the dollar-in-default with maturity $G(\omega_t, t; T)$ as the value of a claim paying off \$1 in default *if* it occurs before T

$$G(\omega_t, t; T) \equiv E^B \left[e^{-r(\mathcal{T}-t)} \cdot 1 \cdot \left(1 - I_{\mathcal{T} \nleq T}\right) \right]$$

and define the binary option $H(\omega_t, t; T)$ as the value of a claim paying off \$1 at T if default has *not* occurred before that date

$$H(\omega_t, t; T) \equiv E^B \left[e^{-r(T-t)} \cdot 1 \cdot I_{\mathcal{T} \nleq T} \right]$$

$I_{\mathcal{T} \nleq T}$ is the indicator function for the *survival event,* i.e. the event that the asset value (ω_T) has not hit the barrier prior to maturity ($\mathcal{T} \nleq T$). The price formulae for the last two building blocks are given below. They contain a term that expresses the probabilities (under different measures) of the survival event – or, the *survival* *probability.* To clarify this common structure, we first state those probabilities in the following lemma.[14]

Lemma 1 *The probabilities of the event* $(\mathcal{T} \nleq T)$ *(the "survival event") at* t *under the probability measures* $Q^m : m = \{B, G\}$ *are*

$$Q^m\left(\mathcal{T} \nleq T\right) = \phi\left(k^m\left(\frac{\omega_t}{L_t}\right)\right) - \left(\frac{\omega_t}{L_t}\right)^{-\frac{2}{\sigma}h^m} \phi\left(k^m\left(\frac{L_t}{\omega_t}\right)\right)$$

where

$$\begin{aligned} k^m(x) &= \frac{\ln x}{\sigma\sqrt{T-t}} + h^m\sqrt{T-t} \\ h^G &= h^B - \theta \cdot \sigma = -\sqrt{\left(h^B\right)^2 + 2r} \end{aligned}$$

$\phi(k)$ *denotes the cumulative standard normal distribution function with integration limit* k.

The probability measure Q^G is the measure having $G(\omega_t, t)$ as numeraire (the Girsanov kernel for going to this measure from the standard pricing measure is $\theta \cdot \sigma$). Using this lemma we obtain the pricing formulae for the building blocks in a convenient form. The price of a down-and-out binary option is

$$H(\omega_t, t; T) = e^{-r(T-t)} \cdot Q^B\left(\mathcal{T} \nleq T\right)$$

The price of a dollar-in-default claim with maturity T is

$$G(\omega_t, t; T) = G(\omega_t, t) \cdot \left(1 - Q^G\left(\mathcal{T} \nleq T\right)\right)$$

To understand this second formula, note that the value of receiving a dollar if default occurs prior to T must be equal to receiving a dollar-in-default claim with infinite maturity, less a claim where you receive a dollar in default conditional on it *not* occurring prior to T:

$$G(\omega_t, t; T) = G(\omega_t, t) - e^{-r(T-t)} E^B\left[G(\omega_T, T) \cdot I_{\mathcal{T} \nleq T}\right]$$

[14] The probabilities are previously known, as is the formula the down-and-out binary option in Lemma A.1 (see for example Björk (1998)).

Using a change of probability measure, we can separate the variables within the expectation brackets (see e.g. Geman, El-Karoui, and Rochet (1995)).

$$\begin{aligned} G(\omega_t, t; T) &= G(\omega_t, t) - e^{-r(T-t)} E^B [G(\omega_T, T)] \cdot E^G \left[I_{\mathcal{T} \nless T} \right] \\ &= G(\omega_t, t) \cdot \left(1 - Q^G (\mathcal{T} \nleq T)\right) \end{aligned}$$

A.2 The Leland Model

The value of the firm

$$\mathcal{F}(\omega_t) = \omega_t + \tau \frac{C}{r} \left[1 - \left(\frac{\omega_t}{L} \right)^{-x} \right] - \alpha L \cdot \left(\frac{\omega_t}{L} \right)^{-x}$$

with

$$x = \frac{(r - \beta - 0.5\sigma^2) + \sqrt{(r - \beta - 0.5\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}$$

Value of debt

$$\mathcal{D}(\omega_t) = \frac{C}{r} + \left((1-\alpha) L - \frac{C}{r} \right) \left(\frac{\omega_t}{L} \right)^{-x}$$

The bankruptcy barrier

$$L = \frac{(1-\tau)C}{r} \frac{x}{1+x}$$

A.3 The Leland & Toft Model

The value fo the firm is the same as in Leland (1994). The value of debt is given by

$$\mathcal{D}(\omega_t) = \frac{C}{r} + \left(N - \frac{C}{r} \right) \left(\frac{1 - e^{-r\Upsilon}}{r\Upsilon} - I(\omega_t) \right) + \left((1-\alpha) L - \frac{C}{r} \right) J(\omega_t)$$

The bankruptcy barrier

$$L = \frac{\frac{C}{r} \left(\frac{A}{r\Upsilon} - B \right) - \frac{AP}{r\Upsilon} - \frac{\tau C x}{r}}{1 + \alpha x - (1-\alpha) B}$$

where

$$A = 2ye^{-r\Upsilon}\phi\left[y\sigma\sqrt{\Upsilon}\right] - 2z\phi\left[z\sigma\sqrt{\Upsilon}\right]$$
$$- \tfrac{2}{\sigma\sqrt{\Upsilon}}n\left[z\sigma\sqrt{\Upsilon}\right] + \tfrac{2e^{-r\Upsilon}}{\sigma\sqrt{\Upsilon}}n\left[y\sigma\sqrt{\Upsilon}\right] + (z-y)$$

$$B = -\left(2z + \frac{2}{z\sigma^2\Upsilon}\right)\phi\left[z\sigma\sqrt{\Upsilon}\right] - \frac{2}{\sigma\sqrt{\Upsilon}}n\left[z\sigma\sqrt{\Upsilon}\right] + (z-y) + \frac{1}{z\sigma^2\Upsilon}$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(\omega) = \frac{1}{r\Upsilon}\left(i(\omega) - e^{-r\Upsilon}j(\omega)\right)$$

$$i(\omega) = \phi[h_1] + \left(\frac{\omega}{L}\right)^{-2a}\phi[h_2]$$

$$j(\omega) = \left(\frac{\omega}{L}\right)^{-y+z}\phi[q_1] + \left(\frac{\omega}{L}\right)^{-y-z}\phi[q_2]$$

and

$$J(\omega) = \frac{1}{z\sigma\sqrt{\Upsilon}}\begin{pmatrix} -\left(\frac{\omega}{L}\right)^{-a+z}\phi[q_1]\,q_1 \\ +\left(\frac{\omega}{L}\right)^{-a-z}\phi[q_2]\,q_2 \end{pmatrix}$$

Finally

$$q_1 = \frac{-b - z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$q_2 = \frac{-b + z\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_1 = \frac{-b - y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

$$h_2 = \frac{-b + y\sigma^2\Upsilon}{\sigma\sqrt{\Upsilon}}$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$

$$x = y + z$$

$$b = \ln\left(\frac{\omega}{L}\right)$$

A.4 The Fan & Sundaresan Model

Given the trigger point for strategic debt service S, the value of the firm is

$$\mathcal{F}(\omega_t) = \begin{cases} \omega_t + \frac{\tau C}{r} - \frac{\lambda_+}{\lambda_+ - \lambda_-}\frac{\tau C}{r}\left(\frac{\omega_t}{S}\right)^{\lambda_-}, \text{ when } \omega_t > S \\ \\ \omega_t + \frac{-\lambda_-}{\lambda_+ - \lambda_-}\frac{\tau C}{r}\left(\frac{\omega_t}{S}\right)^{\lambda_+}, \text{ when } \omega_t \leq S \end{cases}$$

The equity value is

$$\mathcal{E}(\omega_t) = \begin{cases} \omega_t - \frac{C(1-\tau)}{r} + \left[\frac{C(1-\tau)}{(1-\lambda_-)r} - \frac{\lambda_-(1-\lambda_+)\eta}{(\lambda_+ - \lambda_-)(1-\lambda_-)}\frac{\tau C}{r}\right]\left(\frac{\omega_t}{S}\right)^{\lambda_-}, \text{ when } \omega_t > S \\ \\ \eta\mathcal{F}(\omega_t) - \eta\,(1-\alpha)\,\omega_t, \text{ when } \omega_t \leq S \end{cases} \tag{4}$$

where η denotes bargaining power of shareholders. The trigger point for strategic debt service is

$$S = \frac{(1 - \tau + \eta\tau)C}{r}\frac{-\lambda_-}{1-\lambda_-}\frac{1}{1-\eta\alpha}$$

and

$$\lambda_{\pm} = 0.5 - \frac{(r-\beta)}{\sigma^2} \pm \sqrt{\left[\frac{(r-\beta)}{\sigma^2} - 0.5\right]^2 + \frac{2r}{\sigma^2}}$$

Table 1: List of notation / Parameter Values

Parameter	Notation	Assigned Value		
		Leland	Leland & Toft	Fan & Sundaresan
Bond principal / coupon / maturity	P / c / T	According to actual contract		
CDS and bond recovery rate	ψ	40%	40%	40%
Riskfree rate	r	Treasury yields interpolated to match bond maturity		
Debt Nominal	N	Total liabilities from firm's balance sheet		
Debt Coupon	C	$r \cdot N$	$r \cdot N$	$r \cdot N$
Debt Maturity	Υ	–	5 /6.76/ 10	–
Bargaining power of debtholders	η	–	–	0.5
Costs of Financial Distress	α	15%	15%	15%
Tax rate	τ	20%	20%	20%
Revenue rate	β	0% / Weighted average[1] / 6%		

[1] The weighted average of the revenue rate is calculated from the balance sheets as

$$\left(\frac{Interest\ Expenses}{Total\ Liabilities}\right) \times Leverage + (Dividend\ Yield) \times (1 - Leverage)$$

where

$$Leverage = \frac{Total\ Liabilities}{(Total\ Liabilities + Market\ Capital)}$$

Table 2a: Descriptive Statistics - Firm Characteristics

Variable	Mean	Std. Dev.	Minimum	Maximum
Firm size[2] (billions of $)	84	76	2.8	303
Leverage[3]	45%	19%	7.7%	86%
Historic stock volality	46%	14%	23%	100%
S & P rating[4]	8.7	2.7	3	18
Bond transaction size (dollars)	4.3”	10.3”	10,000	191”
Bond issue size (dollars)	486”	413”	7”	3,250”
Bond age	4.1	3.0	0.01	14.7
Bond maturity	9.4	8.5	1.8	98.0
Bond coupon	7.1	1.1	4.5	10.4
CDS maturity	4.4	1.2	0.1	8.0

[2] Market value of equity + total liabilities

[3] Leverage is defined as

$$Leverage = \frac{Total\ Liabilities}{(Total\ Liabilities + Market\ Capital)}$$

[4] AAA is represented by a numeric rating of 1, AA+ by 2 and so on until our highest value of 18 which corresponds to a CCC rating. The mean rating of 8.7 thus roughly corresponds to BBB.

Table 2b: Descriptive Statistics - Bond and CDS characteristics

Rating	Number of transactions	Bond yield spreads				CDS premia			
		Mean	Stdev	Max	Min	Mean	Std	Max	Min
AA	23	77	25	149	32	19	6	40	13
AA-	6	85	27	120	49	18	1	20	17
A+	37	96	37	191	34	35	15	63	15
A	73	117	39	205	19	33	14	90	11
A-	81	148	39	295	85	67	38	260	27
BBB+	126	147	62	360	40	76	53	375	11
BBB	202	175	57	371	35	93	59	295	22
BBB-	84	202	99	487	62	122	84	342	17
BB+	30	349	86	641	247	281	143	750	110
BB	1	456	na	na	na	365	na	na	na
BB-	2	326	11	334	318	477	250	654	300
B+	16	183	132	610	54	122	116	475	22
B	17	322	171	873	128	198	197	777	55
B-	32	170	120	547	43	100	163	900	15
CCC	1	369	na	na	na	178	na	na	na

Bond spreads and CDS premia are presented in basis points.

Table 3: Asset value and volatility estimates

Variable	Model	Mean	Std. Dev.	Min	Max
Asset volatility	Leland	31%	15%	10%	96%
	Fan and Sundaresan	31%	15%	11%	96%
	Leland and Toft ($T = 6.76$)	27%	13%	9%	83%
Asset value (billions of $)	Leland	75	71	2.4	290
	Fan and Sundaresan	75	71	2.4	290
	Leland and Toft ($T = 6.76$)	80	74	2.6	294
Distance to default	Leland	2.7	0.7	1.0	4.6
	Fan and Sundaresan	2.6	0.6	1.0	4.5
	Leland and Toft ($T = 6.76$)	2.5	0.6	1.1	4.3
Dollar in default year 2 (%)	Leland	0.4	1.3	0.0	11
	Fan and Sundaresan	0.7	2.0	0.0	17
	Leland and Toft ($T = 6.76$)	1.3	2.8	0.0	19
Dollar in default year 10 (%)	Leland	9.3	10.8	0.0	58
	Fan and Sundaresan	11.2	11.6	0.0	60
	Leland and Toft ($T = 6.76$)	14.0	12.4	0.0	65

Table 4a: Bond spread and CDS premia estimates – The Leland Model

	Market Bond Spreads $y-r$	Model Bond Spreads $\widehat{y}-r$	Residual Bond Spreads $y-\widehat{y}$	Market CDS Premia q	Model CDS Premia $\widehat{q}$	Residual CDS Premia $q-\widehat{q}$
$\beta = 0\%$						
Mean	168	54	114	91	39	52
Std	91	88	102	94	77	95
Max	873	601	743	900	559	645
Min	19	0	-318	11	0	-415
$\beta = w.ave.$						
Mean	168	60	108	91	41	50
Std	91	93	104	94	98	101
Max	873	614	744	900	624	568
Min	19	0	-322	11	0	-490
$\beta = 6\%$						
Mean	168	89	79	91	54	37
Std	91	113	111	94	96	99
Max	873	672	630	900	624	568
Min	19	0	-413	11	0	-490

Table 4b: Bond spread and CDS premia estimates – The Leland & Toft Model

$T = 6.76$	Market Bond Spreads $y - r$	Model Bond Spreads $\widehat{y} - r$	Residual Bond Spreads $y - \widehat{y}$	Market CDS Premia q	Model CDS Premia $\widehat{q}$	Residual CDS Premia $q - \widehat{q}$
$\beta = 0\%$						
Mean	168	92	76	91	81	10
Std	91	128	130	94	125	121
Max	873	756	568	900	864	606
Min	19	0	-579	11	0	-655
$\beta = w.\ ave.$						
Mean	168	103	65	91	89	2
Std	91	132	131	94	129	123
Max	873	769	540	900	880	576
Min	19	0	-582	11	0	-659
$\beta = 6\%$						
Mean	168	161	7	91	129	-38
Std	91	161	145	94	159	138
Max	873	847	352	900	976	375
Min	19	0	-665	11	0	-751

$\beta = w.\ ave.$	Market Bond Spreads $y - r$	Model Bond Spreads $\widehat{y} - r$	Residual Bond Spreads $y - \widehat{y}$	Market CDS Premia q	Model CDS Premia $\widehat{q}$	Residual CDS Premia $q - \widehat{q}$
$T = 5$						
Mean	168	112	56	91	99	-8
Std	91	139	137	94	139	130
Max	873	791	531	900	931	562
Min	19	0	-624	11	0	-696
$T = 6.76$						
Mean	168	103	65	91	89	2
Std	91	132	131	94	129	123
Max	873	769	540	900	880	576
Min	19	0	-582	11	0	-659
$T = 10$						
Mean	168	93	75	91	78	13
Std	91	124	125	94	119	116
Max	873	743	549	900	820	590
Min	19	0	-530	11	0	-615

Table 4c: Bond spread and CDS premia estimates – The Fan & Sundaresan Model

	Market Bond Spreads $y-r$	Model Bond Spreads $\widehat{y}-r$	Residual Bond Spreads $y-\widehat{y}$	Market CDS Premia q	Model CDS Premia $\widehat{q}$	Residual CDS Premia $q-\widehat{q}$
$\beta = 0\%$						
Mean	168	69	99	91	55	36
Std	91	103	106	94	97	99
Max	873	628	535	900	633	564
Min	19	0	-399	11	0	-496
$\beta = w.ave.$						
Mean	168	77	91	91	58	33
Std	91	108	108	94	101	101
Max	873	641	508	900	649	539
Min	19	0	-444	11	0	-545
$\beta = 6\%$						
Mean	168	105	63	91	69	22
Std	91	127	118	94	113	107
Max	873	698	526	900	740	471
Min	19	0	-514	11	0	-617

Table 5: Default and non-default components of spreads and premia

Dependent variables are given in column headings, explanatory variables in first column; the variables are explained in table 2a. Percentages below coefficient estimates are p-values. Regressions are random effects GLS with an AR(1) error structure and use a method developed by Baltagi and Wu (1999) to adjust for the unbalanced nature of the data panel. L denotes the Leland (1994) model, FS the Fan & Sundaresan (2000) model and finally LT the Leland & Toft (1996) model implemented with $T = 10$. Note that a superscript $*$ or $**$ denote statistical significance at the 10% and 5% levels respectively.

	Residual bond spreads			Residual CDS premia		
	Leland	Fan & Sundaresan	Leland & Toft	Leland	Fan & Sundaresan	Leland & Toft
Coefficient estimates						
S&P rating	6.085	2.900	3.824	8.347**	4.252	3.128
Transaction size ($ Million)	0.128	0.101	-0.012	0.103	0.135	0.218
Bond issue size ($ Million)	0.001	0.000	-0.002	-0.006	-0.006	-0.004
Firm size ($ Billions)	0.000*	0.000	0.000	0.000	0.000	0.000
Bond maturity (years)	-1.150**	-1.035**	-0.278	-0.276*	-0.333**	0.016
Bond on-the-run dummy	-22.549**	-21.981**	-19.677**	-6.355	-6.097	-4.151
Bond coupon	0.090**	0.091**	0.087**	-0.022	-0.027	-0.030
US 5 year generic Treasury rate	-0.293**	-0.319**	-0.285**	-0.273**	-0.309**	-0.309**
Moody's Aaa-Baa index	0.418	0.495*	0.509	0.245	0.266	0.199
5 year on-off the run Treasury spread	0.855**	0.894**	0.911**	-0.080	-0.170	-0.350
VIX	-0.786	-1.317	-2.004**	0.017	-0.373	-1.522
S&P 500 return	2.821**	2.556**	2.266*	1.845**	1.655*	0.912
P-values						
S&P rating	11%	45%	38%	4%	32%	53%
Transaction size ($ Million)	44%	57%	95%	41%	33%	27%
Bond issue size ($ Million)	80%	95%	67%	9%	15%	44%
Firm size ($ Billions)	10%	22%	26%	28%	54%	91%
Bond maturity (years)	0%	0%	25%	7%	5%	95%
Bond on-the-run dummy	0%	1%	2%	25%	32%	64%
Bond coupon	0%	0%	0%	15%	12%	23%
US 5 year generic Treasury rate	0%	0%	0%	0%	0%	1%
Moody's Aaa-Baa index	14%	10%	12%	27%	29%	57%
5 year on-off the run Treasury spread	1%	1%	2%	74%	53%	37%
VIX	35%	14%	4%	98%	61%	15%
S&P 500 return	1%	3%	8%	3%	8%	50%
R^2	12.0%	10.1%	8.8%	13.9%	10.9%	5.0%