Liquidity Risk in the Corporate Bond Markets[1]

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Abstract

A great deal of work has focused on market microstructure, but relatively little work has been devoted to the study of risk associated with liquidity. The work that has been done has almost exclusively focused on US equities - primarily because that market is fairly liquid and therefore data is plentiful. However, because that market is liquid, the empirical results been mixed - the effects of illiquidity are likely small in a market with a large amount of trading (and therefore transactions data). For our work, we turn to the US corporate bond market. Because the corporate bond market is several orders of magnitude more illiquid than the equity market, this seems a much more appropriate setting to study the effects of illiquidity. To get around the problem of a lack of trading and transactions data, we construct a new measure of liquidity which does not require trading. Using this measure, we show that not only is liquidity risk priced, but that the effects of liquidity risk are quite pervasive and need to be controlled for carefully when doing virtually any analysis of security returns.

1 Introduction

Any investor holding a security or a portfolio of securities or considering purchasing a security is exposed to liquidity risk. As in Chacko & Stafford (2004), we use the following reduced-form definition of liquidity: liquidity is simply the gap between the fundamental value of a security and the price at which the security is actually transacted at; high liquidity means this gap is small and vice versa. Thus, liquidity *risk* is the uncertainty of how wide or narrow this gap will be at any point in time. For all investors and potential investors, liquidity risk is a real risk that they bear. Every transaction is essentially a negative NPV project for the buy-side investor. If the investor knew how negative the NPV would be, then this would not be a risk - the investor could simply perform his asset allocation optimization by factoring in the transaction costs. The risk comes in not knowing how far off the investor will transact from the fundamental value of the asset he is buying or selling. Furthermore, this is a risk that is not fully diversifiable.[1] The natural next question then is whether the systematic portion of liquidity risk is priced, i.e., do investors command a risk premium for bearing liquidity risk? In this paper, we address this question.

We will call the gap between the transactable price of an asset and its fundamental value as the half-spread. The bid-ask spread is simply the sum of the buy half-spread (the gap between the price for which a buy-side investor pays to purchase an asset and the asset's fundamental value) and the sell half-spread (fundamental value minus the sell price). There are two factors that give rise to half-spreads. The first is that for a transaction to occur a match must be made between a buyer and a seller. It is very likely however that when one wants to sell a particular quantity of a specific asset, there will not be a buyer who wants precisely that same asset in the same quantity at the same point in time. This gives rise to a financial intermediary, namely a market maker. The market maker bridges the asset-type, quantity, and time gap between buyers and sellers by using his balance sheet to store assets. For this inventory service, the market maker requires a fee, which he collects through the half spreads. This notion of a market maker was initially applied by Demsetz (1968) in the context of market specialists and expanded on by a number of other papers [Garman (1976), Stoll (1978), Ho & Stoll (1981),

[1] With their options-based transaction cost model, Chacko & Stafford (2004) produce a clear definition of diversifiable vs. systematic risk in the context of liquidity.

O'Hara & Oldfield (1986), Amihud & Mendelson (1986, 1988), Grossman & Miller (1988), Biais (1993), and Madhavan & Smidt (1993)]. The second factor that gives rise to half-spreads is asymmetric information. In a transaction, one can never be sure whether the counterparty in a transaction is informed. To compensate for the possibility of transacting with an informed trader, market makers charge an additional fee to all traders, which is once again collected through the half-spreads. This notion of information-based transaction costs has been examined in many papers as well - see, for example, Glosten & Milgrom (1985), Easley & O'Hara (1987, 2001), and Easley, Hvidkjaer, & O'Hara (2002).

While the question of whether a bid-ask spread exists has an easy answer, of much more contention is the question of whether the bid-ask spread is time-varying and whether this time variation is systematic, i.e., whether it is priced. Intuitively, one might expect liquidity risk to be priced. Major "liquidity" shocks such as during the LTCM crisis or just after the bursting of the technology bubble, resulted in both low liquidity (very high half-spreads) and poor (stock) market performance. A security that experiences low returns precisely when an investor's wealth drops must offer a premium to investors to induce them to hold the security. While these are only two datapoints, they are suggestive of liquidity risk being priced. However, the theoretical counterargument to this naive inference has been posed by a number of papers, including Constatinides (1986), Aiyagari & Gertler (1991), Vayanos & Vila (1991), and Vayanos (1998), who argue that liquidity costs can only be a second-order determinant of asset prices because half-spreads are too small relative to the equilibrium risk premium.

Whether liquidity risk is an important source of systematic risk is extremely important to practioners and academics alike. However, there has been relatively little empirical research devoted to this topic and the results appear to be mixed. The main problem is that liquidity is correlated to trading, and the downside risk associated with liqudity is that very little trading exists in a particular security that an investor wishes to buy or sell. With limited trading, any given buy or sell order will have a larger deviation from fundamental price - therefore, a wider effective half-spread. Consequently, it is precisely securities that are thinly traded where we would like to study liquidity and liquidity risk. However, thinly traded securities by definition have very little trade data associated them. Therefore, much of the empirical and theoretical research that has been done on liquidity risk has been done with highly liquid securities; namely, US equities. So it is not surprising that the question of whether liquidity is priced has not been settled. Amihud & Mendelson (1986), Brennan & Subrahmanyam (1996), Brennan, Chordia & Subrahmanyam (1998), Datar, Naik, & Radcliffe (1998), Chordia, Roll, & Subrahmanyam (2000) have all found positive relationships between stock returns and overall liquidity as measured by spreads, depth, and volume. Meanwhile Chordia, Subrahmanyam & Anshuman (2001) find a negative relationship between liquidity and expected returns, while Hasbrouck & Seppi (2001) find no relationship. Finally, Huberman & Halka (2001) and Pastor & Stambaugh (2003) have examined the more relevant question of whether liquidity risk is a systematic factor.

In this paper, we look at the issue of whether liquidity risk is priced using data from the US corporate bond markets. Compared to US equities, corporate bonds are extremely illiquid. While the median stock trades once every few minutes, the median US corporate bond trades approximately once every two months. In this market, liquidity is a problem for most market participants. Therefore, to the extent investors command a premium for liquidity risk it should be more easily discerned in this market.

Unlike equities, corporate bonds are traded in a dealer market. Therefore, the US coporate bond market is essentially an over-the-counter market. Obtaining data on this market is difficult. No single dealer has enough share and therefore sees enough transactions for a meaningful analysis to be conducted. For this reason, our dataset will come from one of the world's largest custody banks. As part of the custody process, custody banks record the transactions conducted by their clients; thus, the largest custody banks essentially see across the transactions databases of multiple dealers. While not being able to see all of the transactions of the corporate bond market, custodians can see a substantial part of it. Thus, as long as their view of the market is not biased, this should provide a satisfactory database for analyzing the question of liquidity.

A substantial problem still remains. Even if we could look across the whole market a measure of liquidity would not adequately capture the difference in liquidity between most bonds. Because trading volume is so low for most bonds, measured differences in liquidity using traditional methods would only measure small differences between most bonds.

To address the question of a liquidity measure we construct a new liquidity measure that assesses the accessibility of a bond, rather than its trading volume. Because corporate bonds trade in a dealer network, dealers rely on being able to access their buy-side clients' inventories either to purchase or sell bonds. If a bond is readily accessible, meaning a dealer can call up one of a number of buy-side clients and obtain the bond easily, the bond can be thought of as liquid even though it may not actually trade very much. Specifically, if a bond issue is held primarily by a set of investors with high portfolio turnover, the bond may be thought of as more accessible - essentially, it is easier for a dealer to call up one of the investors holding this bond and convince them to sell it. On the other hand, if a bond issue is held primarily by investors with extremely low portfolio turnover (long term buy-and-hold investors, such as insurance companies) it is more difficult for dealers to call up the typical holder of one of these bonds and convince them to sell it. Thus, our measure of liquidity is a bond's *accessibility.* To utilize this concept, we construct a statistic known as latent liquidity, which measures the accessibility of a bond to dealers based on the aggregate trading characteristics of investors holding bonds.

With a suitable measure of liquidity available, we then follow a standard process used in the equity literature: we form factor portfolios. In the fixed income world, the commonly used factors are interest rate risk and credit risk, and we will append liquidity risk to these two factors. We will therefore take the US corporate bond universe and split up all of these bonds into one of six portfolios. From these six portfolios, we form three factor portfolios. The interest rate factor portfolio will have a long position in long duration bonds and a short position in short duration bonds. The credit factor portfolio has a long position in low credit quality bonds and a short position in high credit quality bonds. Finally, the liquidity factor portfolio has a long position in bonds with low liquidity and a short position in bonds with high liquidity.

These factor portfolios then mimick each of the three factors thought to be of importance in the fixed income markets. With the time series of the factors, a number of interesting properties of these factors can be analyzed. First, of course, the question of whether liquidity risk is priced can be answered by running factor regressions and analyzing how individual bonds load on the liquidity factor. We can also test whether credit risk is priced.[2] Another interesting question is how does the risk premium for credit risk and interest rate risk compare to the premium for liquidity risk. For example, most practitioners use the yield spread of a bond, the difference between a bond's yield and the corresponding duration Treasury's yield, as a proxy for

[2] Given that credit risk is really nothing more than a put option on the assets of firms economy-wide, it would be surprising if credit risk were not priced.

credit risk - thus the yield spread has traditionally been thought of as entirely being due to credit risk. However, if liquidity risk is priced, then the yield spread is composed of two pieces, a credit spread and a liquidity spread, which add to form the total yield spread. It would be interesting to know how much of a typical corporate bond's yield spread is composed of credit spread and how much is liquidity spread. Finally, time-series properties of the factors such as their persistence and volatility will be analyzed.

The organization of the paper is as follows. The first section will be similar to this section and will serve to introduce the topics of the paper to the reader. The second section will then analyze the trading properties of the US corporate bond market using our database. We will also use this section to determine if there are any significant biases in our database. The third section will describe the construction of the factors and present some intial statistics on the factors. The fourth section will then answer the question of whether liquidity risk is priced. The fifth section will look at how liquidity risk is related to credit and interest rate risk. The sixth section will look further at the time series properties of these factors. Finally, the seventh section will then conclude the paper.

2 Data & Liquidity Measurement

While the corporate bond markets seem like an ideal place to understand the importance of liquidity, two primary reasons exist for the lack of research on liquidity in corporate bond markets. The first is that the corporate bond market is a dealer market (essentially an OTC market) so no central data source exists for all of the transactions occurring in the market. Each dealer sees and keeps track of the transactions that he participates in, but a dealer does not see other dealers' transactions. Therefore, it is difficult for any one entity to accumulate a comprehensive bond transaction database.

The second, and more important, problem is how to measure liquidity. In the presence of lots of trading, we can proxy for a security's liquidity by the trading volume in that security, e.g., a security with a high average daily trading volume may be assumed to be more liquid than a security with a low average daily trading volume. However, the definition of an illiquid market is that very little or no trading exists - therefore, the question arises how to measure the degree of a corporate bond's illiquidity without the availability of transaction data? We answer this question by deriving a new measure of liquidity called *latent liquidity*. While we develop this measure in the context of corporate bonds, the calculation of this measure is equally easy for virtually any other security.

2.1 Bond Database

Because the corporate bond market is essentially an OTC market, no single comprehensive source of trading data exists.[3] Therefore, to conduct research one has to go to an individual dealer and collect and analyze the transactions participated in by that dealer. However, this leaves open the possibility of biases: for example, one dealer might be a market leader in executing high-yield transactions, in which case the database the researcher puts together from that dealer's transactions will be biased towards high-yield bonds. To avoid this problem, we rely on data, both transactions and holding characteristics, collected directly about the individual buy-side firms rather than data from financial transaction intermediaries. To this end, we use data from the National Association of Insurance Commisioners (NAIC), Spectrum, and a securities custodian. Therefore, because we see all types of buy-side firms in our database, from insurance companies to hedge funds to mutual funds, the database is much larger than that comprised by any individual dealer or custodian[4], and is much more representative of the aggregate market.

Using our database, we give an indication in Table 1 about the characteristics of corporate bonds that trade in the marketplace. In general, we see that bond issues are split into one of eight industry categories that we devise. We see that the financial services industry is the biggest issuer of corporate debt - in 2004, more than one-third of all debt issues came from firms within this industry. This should not be a surprising result as most financial services firms such as banks and insurance companies are highly levered entities on the right-hand sides of their balance sheets. In contrast, we see that the technology industry is the smallest issuer of corporate debt. This result should also not be surprising. Technology companies tend to have very low leverage and whatever leverage that they do have tends to be

[3]The NASD has initiated a program known as TRACE where the individual members of the NASD report to NASD all of their corporate bond transactions. However, the TRACE effort is not comprehensive yet; furthermore, because the program is new, a substantial history will not be available for many years.

[4]Chacko, Mahanti, Mallik, & Subrahmanyam (2006) use a similar approach to that used in this paper but rely on data exclusively from a securities custodian.

private, such as bank loans or private placements.

Table 2 shows the how the bond rating of corporate bonds have been changing through time. During the late '90s, a higher percentage of outstanding bonds were rated investment grade by Moody's. For example, in 1999 55% of bond issues were rated as investment grade. As we progress through time, however, we see that the mix shifts the other way. By 2004, only 44% of bond issues were rated as investment grade. This result is not surprising, if we take into account the occurrences in the general marketplace. Equity markets dropped substantially during the early 2000s, indicating that the probability of default of most firms likely increased as well - this is supported by the fact that credit spreads increased significantly during this time period. Therefore, if rating agencies were doing a reasonable job, the result that more bonds in the marketplace were getting rated below investment grade is natural.

Table 3 displays the maturity structure of corporate debt in the marketplace. It is interesting to note that the average maturity of debt has been steadily decreasing through time. For example, bonds in the 50th percentile have gone from an average 8.3-year maturity in 1994 to a 5.0 year maturity in 2004.

Table 4 shows that the time since issuance of outstanding debt has been steadily increasing from '94 until the present. If we think of a timeline for currently outstanding debt, there are two interesting points in time to consider: the first is when it was issued relative to today, and the second is when it matures relative to today. From Table 3, we know that the time-to-maturity of the typical bond has decreased through time. However, from Table 4 we see that the time since the debt was issued has increased considerably through time. The explanation that reconciles these two facts is that the number of new debt issues has decreased through time, and firms are financing themselves with shorter term debt than in the past. The decreasing number of debt issues means that the average debt issue in the marketplace is not getting replaced (refinanced); thus the time since issuance of the average debt issue is increasing. Furthermore, the ones that are getting refinanced are being refinanced with shorter maturity debt causing the average maturity of bonds outstanding to decrease. This is a surprising result given that interest rates during this time period were coming down and so we would expect to see lots of new issuance - and therefore the time since issuance statistic to decrease through time. However, the next table explains part of this result.

Table 5 shows the face value distribution of all debt issues in the market.

The table shows that the average face value amount has increased substantially over the last ten years. In reconciling this result with the results of the previous table, we see that while the number of new bond issues has been decreasing (as Table 4 shows) the face value of the average issuance must have increased substantially. This would account for the average increased size of face value of debt remaining in the markets.[5] Thus, we seem to be observing the following financing trend: firms are doing fewer bond issuances, but each new issuance is larger than in the past and of shorter maturity.

Tables 6 through 8 give us a sense of the amount of trading that occurs in the corporate bond markets. Table 6 shows the average number of days that passes between trades for a bond issue. For the median traded bond, a bond in the 50th percentile, after a trade occurred in 2003 an average of 61 days (three months[6]) lapsed with no trading before another trade in that issue occurred. For the median stock, in comparison, this value is more on the order of minutes. Therefore, what we see is that the corporate bond market is orders of magnitude more illiquid than the stock market. An interesting thing to note is that despite the degree of illiquidity that currently exists, the current state of liquidity appears to be better than existed previously. For example, in 1995 there were an average of 122 days that elapsed between trades for the median bond. Furthermore, this increase in trading is seen throughout the market, except in the most illiquid bonds. Thus, for most of the time period the median corporate bond has traded less than once every three months. Table 7 shows that despite the increase in the number of trades, the average trade size has been getting smaller since 1999. The median bond, for example, used to have an average trade size of \$2.02 million in 1995 and this value increased steadily to \$2.34 million by 1998; however, this value has decreased steadily since then to a value of \$1.52 million in 2003. We see this same pattern across the bond market. One explanation for this is that since 1999 investors have been breaking up their trades into smaller sizes. This would explain both the increase in the number of trades as well as the decrease in the trade size. The question then remains that if the number of trades is increasing but the trade size is decreasing, is the

[5]Another explanation is that the debt maturing through time had unusually small face value, thus causing the average face value of the remaining debt to increase in face value. However, if this were the case, the total amount of debt outstanding would not increase, which it has done. Furthermore, we would not expect to see increased daily trading volume (in dollars) through time, which we also observe.

[6]Roughly twenty trading days equals one month.

dollar amount of trading (the product of the two) increasing or decreasing. Table 8 answers this question. It does seem that trading volume has been increasing during the whole time period covered by the database. For bonds in the 50th percentile, for example, trading volume has gone from $36,177 per day in 1995 to $43,002 per day in 1999 to $69,279 per day in 2003.

2.2 Liquidity Measurement

The previous section showed that the corporate bond market is an extremely illiquid market, though the degree of liquidity seems to be improving. As compared to equity markets, this seems a much better setting to study liquidity risk. However, one important problem remains. Most corporate bonds rarely trade. This is both a good thing and a bad thing for studying issues of illiquidity. It is good in that if illiquidity is an important economic factor, then the corporate bond market is an excellent setting to measure and evaluate the effects of illiquidity. However, it is also a bad thing because the lack of trading means there is little transaction data available, which means that traditional measures of liquidity such as trading volume and bid-ask spread cannot be calculated with any reliability for most of the bonds in the database. A much more statistically reliable liquidity measure is needed here. To this end, we define a statistic called *latent liquidity* to measure the liquidity of any security held by investors.

We approach this problem by analyzing how a securities dealer views liquidity. In such a market what really determines the liquidity of a security is the ease in which a dealer can access a security. For example, if a buy order comes in to a dealer he will supply that order out of his own inventory, or he will try to source the bonds from the inventory of one of his other customers, i.e., the dealer will "work the order" by calling up customers to see if he can convince someone to sell him the bonds to fill the buy order.[7] Consider the case when he is trying to call up customers to fill the buy order. If the bond issue of interest is held primarily by funds with high turnover (hedge funds, for example), it should be easier for the dealer to call up one of these investors and convince the investor to sell him the needed bonds than if the bonds were held primarily by funds with low turnover (insurance companies, for example). This is because the high turnover funds are used to trading in

[7] The dealer will, of course, look to buy the bonds at a lower price from the customer than the price at which he will fill the buy order. Thus he earns a fee for his "search services".

and out of securities with high frequency[8] and can be more easily convinced to trade a particular security they are holding. Therefore, whether a bond issue experiences a great deal of trading volume or not, we can say that a bond issue is more liquid if it is more accessible by dealers, i.e., if the funds holding the bond issue tend to be higher turnover funds.

This measure of accessibility of a security is not a direct measure of liquidity, but rather a more latent measure. In order to measure latent liquidity, we need to be able to determine for each bond issue, which of two types of investor holds the issue and the aggregated weighted average turnover of all the funds holding the issue. If the weighted average turnover of all the funds holding a particular bond issue is high, then we say that the bond issue has high latent liquidity, i.e., it is more accessible, relative to another bond that has lower latent liquidity. In other words, a bond's accessibility can be thought as the degree to which an issue is held by investors who are expected to trade more frequently relative to those expected to trade less frequently, or who are predominately buy and hold investors, based on historical trading patterns.

In our database, we have not only transactions but also portfolio holdings and changes in portfolio holdings through time. This makes it possible to calculate certain characteristics of investors; specifically, we can calculate their propensity to trade. We start by calculating the twelve-month historical turnover number for all portfolios in our database. For any bond issue then, we aggregate all of the funds holding that issue to calculate a weighted average turnover value for that bond issue. This then becomes our latent liquidity measure for that bond.

The most convenient feature of this measure is that it is based entirely on aggregate investors' holdings and does not require transaction details. This allows us to study illiquidity issues precisely in situations where illiquidity issues are likely to be important: where there is little to no trading occurring.

Figures 3 thru 5 present how latent liquidity changes with certain bond characteristics. For these figures, after calculating a latent liquidity number for each bond, we classified bonds into one of 5 quintiles, with quintile 1 representing bonds with the highest latent liquidity and quintile 5 representing bonds with the lowest latent liquidity. Figure 3 plots the latent liquidity of bonds from the time they were first issued. What we observe

[8]At least relative to most fixed income investors, who tend to be "buy and hold till maturity" type of investors.

here is that bonds are at their peak latent liquidity levels when they are just issued. Their latent liquidity levels decrease after issuance. This is consistent with the casual evidence that "on the run" bonds are the most liquid "off the run" bonds. The story that emerges is that many bonds are initially placed into high turnover funds, who then "flip" the bonds to lower turnover (usually, buy-and-hold) funds. Figure 4 plots the latent liquidity of a bond issue versus its issue size. What we observe is that larger issues tend to have higher latent liquidity levels. Specifically, there appears to be a considerable decrease in liquidity for issues that are $500 million or less. One explanation for this is that smaller issuance amounts mean fewer bonds will be available for buying and selling. High turnover funds tend to stay away from these bond issues because these funds rely on bond issues that are plentiful and widely distributed for easier accessibility. The question remains whether a liquidity premium is therefore charged for small bond issues. As will be shown later, there is in fact a higher liquidity premium for smaller bond issues. This magnifies the size vs. latent liquidity pattern because high turnover funds generally pay liquidity premia and therefore seek bonds with low liquidity premia. Figure 5 provides a plot of latent liquidity versus time to maturity for bond issues. What we observe is that the longer the maturity of the bond, the higher the latent liquidity. The jumps in this figure are initially surprising but easily explained - they are due to bond issues of that maturity level. Essentially bonds at the 5-year maturity are composed of two types: bonds that were issued in the past and are now down to 5 years left to maturity, off-the-run bonds, and bonds that have been just issued, on the run bonds. However, bonds at the 5.1 year maturity are composed only of off-the-run bonds (because 5.1 years is seldom chosen as a maturity time for newly issued bonds). Therefore, the significantly higher latent liquidity of the on-the-run bonds at the 5-year maturity level result in a substantially higher latent liquidity measure at the 5-year level vs. the 5.1-year; hence the observed jump in the graph. The same result holds at typical maturity points for new issues, such as at 3 and 7 years.

3 Empirical Results

In this section we now try to answer the question whether liquidity risk is priced or not. Our methodology is fairly straightforward now that we have a liquidity measure. We sort the universe of bonds into categories by duration risk, credit risk, and liquidity risk. From the loadings on these factors, we form long-short portfolios to construct the time series of the duration, credit, and liquidity factors. With these factors we then conduct some simple regressions to determine whether the liquidity factor is priced. We find evidence that it is. We also do some out-of-sample testing by taking the liquidity factor and testing it in another asset class: Treasury bonds. We find that the liquidity factor is extremely important in explaining Treasury bond returns; specifically, we find that the slope factor, which usually considered the most important factor in explaining Treasury returns, is highly correlated with the liquidity factor.

3.1 Liquidity Factor Construction

The first step in determining whether liquidity risk is priced is to calculate the time series of the liquidity risk factor. Once this factor is determined, we simply assess the importance of this factor in explaining corporate bond returns.

To construct this factor, we first sort the universe of corporate bonds over the last 10 years, approximately 25,000 bonds[9], into 27 buckets on a monthly basis. For each month, the sort is done by first placing each bond issue existing at that point in time into high (H), medium (M), and low (L) duration buckets according to the duration of the bond. The sort is done in an equal-weight manner, i.e., the sort is done so that each bucket contains the same number of bond issues. Therefore, the duration cutoff to go from one bucket to another varies through time.

Similarly, we construct three credit buckets and three liquidity buckets. For credit, we use a combination of Moody's and State Street's credit rating, and for liquidity we use the latent liquidity measure described earlier. In each of these sorts, we start with each of the three duration buckets. We then sort each duration bucket into one of three equal-weight credit buckets - H, M, and L. This gives a total of nine equal-weight buckets. Finally, we take each of these nine buckets and sort into one of three equal-weight liquidity buckets. This process yields a total of 27 buckets, each with unique duration, credit, and liquidity risk characteristics.

[9] The universe of bonds covered in the database is actually much larger - closer to 70,000 bonds. However, for most of these other bonds, we see fewer than three trades in the entire 10-year period. Therefore, we throw out these bonds due to lack of sufficient pricing information.

From these 27 buckets, we then form three factor portfolios - a duration factor portfolio, a credit factor portfolio, and a liquidity factor portfolio. To form the duration factor portfolio, we take a long position in the high duration portfolio and a short position in the low duration portfolio - HML (High minus Low) duration. Similarly, to form the credit factor portfolio, we take a long position in the low credit portfolio and a short position in the high credit portfolio - LMH (Low minus High) credit. Finally, to form the liquidity factor portfolio, we take a long position in the low latent liquidity portfolio and a short position in the high latent liquidity portfolio - LMH liquidity.

The time series of each of these portfolio returns represent the returns from the duration, credit, and liqudity risk factors. Figure 6 shows the graph of the liquidity factor in the form of a monthly index, i.e, it shows how \$100 invested in a pure liquidity index would have performed over the last ten years. Specifically, the index rises in value from \$100 to around \$180, a compounded return of approximately 3.4%. Notice that the index shows a fairly smooth and steady rise with exceptions coming during the mid to latter part of 1998, when the Russian default and the subsequent LTCM crises hit the markets, and in 2002, in the midst of the burst of the technology "bubble". These blips denote times when the liquidity premium increased substantially.

3.2 Is Liquidity Risk Important?

We now answer the important questions of whether liquidity risk is important for bond pricing and whether this risk is priced, i.e., whether the market pays a premium for bearing liquidity risk. The latter question does not automatically follow from the first. For example, a risk could be one that does not command a risk premium (e.g., it could be diversifiable, as with idiosyncratic volatility) but still remain important for pricing securities, i.e., assuming the risk to be zero would lead to the wrong price. In this section we look at whether liquidity risk is important, and in the next section we analyze whether this risk commands a premium.

To answer both of these questions, we first conduct factor regressions. We regress each security against the three factors and calculate the duration, credit, and liquidity betas of the security. We run these regressions on a daily basis using a rolling 2-year lookback window. Thus, each day for every bond in the database we have the bond's three betas with respect to the risk factors. We then sort the set of bonds by their respective betas. We form 5 portfolios of equal-weight liquidity portfolios: H, H/M, M, M/L, L. These correspond to portfolios with high (H) betas, high/medium (H/M) betas, medium (M) betas, medium/low (M/L) betas, and low (L) betas with respect to the liquidity risk factor. As in the sort we did earlier, we then take each of these 5 liquidity portfolios and split each one up into three credit portfolios: H, M, L, denoting high credit, medium credit, and low credit. Finally, we then take each of the fifteen buckets we now have and split each one up into three more portfolios (H, M, and L) based on duration. In this process we effectively do a 5 by 3 by 3 sort of the data (5 liquidity portfolios, 3 credit portfolios, and 3 duration portfolios).

The first way we try to answer whether liquidity risk is important is to run a regression of each of the 45 portfolios against the factors we just created. Table 9 presents one set of these regression results. Specifically, this table shows the average coefficient and t-statistic of the liquidity factor in regressions in which all three factors and an intercept term are the explanatory variables. Each coefficient represents the average of 3 individual regressions in which the only difference is the left-hand-side variable - the left-hand-side variable is a portfolio whose credit and liquidity levels are the ones indicated in the table for that coefficient, and the duration level varies across H, M, and L. In this table, the important thing to notice is that as the liquidity level of the portfolio decreases (as we go from H to L), the liquidity factor coefficient increases along with its t-statistic. This result is expected if the liquidity factor in an important component in the pricing of bonds. In addition, notice that the coefficients increase as we go from high to low credit. This indicates that while we have done a good job of sorting on liquidity, we have not done a perfect job: latent liquidity is not a complete measure of liquidity - the credit of a bond drives liquidity as well, though it appears to be substantially less important than latent liquidity.

Table 10 presents the increase in R^2 that occurs in the regressions done in Table 9 when we include the liquidity factor, i.e., it shows the incremental increase in R^2 due to the liquidity factor. Similar to Table 9, what we observe is that as the liquidity level of the portfolio decreases the incremental R^2 produced by the liquidity factor increases, as we would expect if liquidity was an important factor. Also similar to Table 9, we consistently notice that the R^2 increases slightly as we go from H to L in credit. This indicates that there are some residual liquidity effects in the credit factor.

Tables 11 and 12 present a different cross-section of results from the same regressions run in Tables 9 and 10. Instead of looking at how the liquidity coefficients and incremental R^2 vary with liquidity and credit, these tables present how these statistics vary by liquidity and duration - therefore, in the regressions producing a single entry in this table, the left-hand-side variable varies only in credit level. Essentially, we see the same results as in Tables 9 and 10, further reinforcing the point that liquidity appears to be an important factor.

3.3 Is Liquidity Risk Priced?

Tables 9 and 10 indicate that liquidity risk is clearly an important factor in the pricing of corporate bonds. These tables also hint at this risk being priced, i.e., that this risk is not diversifiable. We now conduct a simple and more direct test for whether liquidity risk is priced. We take the beta-sorted portfolios used in the previous section and run them through several asset pricing models. Specifically, we take the five liquidity portfolios created through the sort done above and run regressions of these portfolios against common measures of systematic risk factors. Table 11 presents the alpha from these regressions.

The first row of this table presents the alphas of running each of the five liquidity portfolios against the bond market. As a proxy for the bond market, we use the Lehman corporate bond index. The first row's alphas are ordered from the regression involving the most liquid portfolio to the least liquid portfolio. The final column presents the alpha of the least liquid portfolio minus the alpha of the most liquid portfolio. What we see from this row is that the alphas increase as the portfolios proceed from more to less liquid. The alphas of the last three portfolios are all positive and statistically significant. The pattern of the alphas strongly suggests that liquidity is not only important in explaining returns, but more imporantly that liquidity risk is priced.

In the next two rows of the table, we regress each of the liquidity portfolios against the duration and credit factors that we created. We observe that the alphas still rise as the left-hand-side variable decreases in liquidity and that the alphas are strongly significant. Therefore, even if we expand the risk model to incorporate important factors such as duration and credit, the alpha does not disappear.

The alphas of the duration, credit regression are very similar to the alphas in the CAPM regression. This is to be expected because the Lehman corporate bond index is composed primarily of duration and credit risk. The index is composed of the most liquid bonds, and therefore incorporates little liquidity risk. However, the alphas from the CAPM regression are lower than those from the duration regression and the duration, credit regression - despite the fact that the Lehman index is composed of the most liquid bonds, there is still some liquidity risk in the index. This causes the liquidity portfolios to load a bit more on the index, reducing the alpha slightly.

The alphas from the duration regression are higher than those from the duration, credit regression because, as we saw from other tables, the credit risk factor incorporates some liquidity risk. Therefore, as we move from the duration regression to the duration, credit regression, the liquidity risk in the portfolios load slightly on the credit factor, thereby reducing the alphas.

3.4 Out of Sample Test

The above tests strongly suggest that liquidity risk is an important risk factor and that it is priced. However, if we have done a good job of capturing the liquidity risk factor and this factor is in fact priced, then this risk factor should work in explaining the returns of other asset classes. In this section we conduct a simple test of whether the liquidity risk factor explains the returns of US Treasury bonds.

Much empirical work in US Treasuries have revealed that there appear to be three important factors in explaining bond returns: these are the so-called level, slope, and curvature factors.[10] The level factor is simply the level of the short end of the yield curve, i.e., the short rate. The slope factor is simply the return of the long end of the yield curve, the 10-year bond for example, less the return of the short end of the yield curve, the interbank overnight deposit rate for example. Finally, the curvature factor is simply a measure of the average convexity of the yield curve.

The Treasury market is generally thought of as a very liquid market. Despite this view, there are nevertheless significant differences in prices between on-the-run and off-the-run bonds. This leads us to suspect that liquidity risk might be an important explanatory factor in the returns of these bonds. We run a simple test to check this hypothesis.

Table 12 presents the results from a simple horserace between the level, slope, and curvature factors and the liquidity factor we have constructed in

[10]See ??? for more details on these factors.

this paper. For the test, we construct a four-factor affine term structure model such as those in Chen and Scott (1992), Balduzzi, Das, and Foresi (1998) and Chacko (2004). The model we use is the following:

$$\begin{aligned} dr &= \kappa(m-r)dt + vdZ_r \\ dm &= \alpha(\theta-m)dt + \delta dZ_m \\ dv &= \beta(\sigma-v)dt + \gamma dZ_v \end{aligned} \tag{1}$$

In this model, r represents the short rate, or level factor; m represents the conditional mean of the short rate, and proxies for the slope factor; and v denotes the short rate's volatility, and proxies for the curvature factor. The pricing kernel is assumed to have the following dynamcis:

$$\frac{dM}{M} = -rdt - ldZ_r - \phi_m dZ_m - \phi_v dZ_v \tag{2}$$

$$dl = \pi(\lambda - l)dt + \eta dZ_l \tag{3}$$

where l is a time-varying price of risk. We use this time-varying price of risk to proxy for the liquidity factor. Using yield curve data, we calculate the time series of the short rate, the slope factor (10yr yield minus 3mo yield), and curvature (slope of 5yr to 3mo minus the slope of 10yr to 5yr). We then use the liquidity factor derived in this paper as our liquidity factor. Each day from 1994 to 2004, we estimate the time series models of (1) and (3). We then estimate the pricing kernel parameters, ϕ_m and ϕ_v, by calibrating the yield curve model implied by (1) and (3) to the actual yield each day.[11]

Table 12 presents the results of repeating the above exercise by leaving out one of the factors. Essentially, we assume one of the factors is a constant and redo the yield curve calibration exercise. Leaving out a factor results in error in the model yield curve versus the actual yield curve. Table 12 reports these errors. Generally, the more important a factor is in explaining bond yields the greater the error that should result when excluding that factor. What we initially see from Table 12 is the usual story: the slope factor on average is more important than the curvature factor in explaining bond yields. Furthermore, the curvature factor is more important at explaining bond yields in the middle part of the yield curve while the slope factor is

[11] We use the 3 month, 1 year, 3 year, 5 year, 7 year, and 10 year points for the calibration. Essentially, we find the values of the two pricing kernel factors each day that minimizes the sum of squared errors between the yield curve model and the actual yield curve.

better at explaining yields at the longer end of the yield curve. The new result in this table is that the errors from leaving out the liquidity factor are much higher than leaving out the slope factor. In fact, the liquidity factor behaves very much like the slope factor in that it is most important for explaining yields at the long end of the yield curve.

What Table 12 is hinting at is that what most people traditionally think of as the slope factor is in fact a poorly measured liqudity factor. There is a larger liquidity premium for longer maturity bonds than for shorter maturity bonds not only due to the longer maturity of the bond but also due to the fact that there is less trading volume at the long end of the curve. Therefore, when the traditional slope factor is calculated (by subtracting the long bond yield from the short bond yield), what is left over are two components: a term premium and a liquidity premium. Therefore, the slope factor is really a noisy version of a liquidity factor. What we observe in Table 12 is that when we put a less noisy liquidity factor into the empirical test, the data loads much more strongly on this better-measured liquidity factor. Thus, it appears the main reason that the traditionally-measured slope factor does well in explaining bond yields is that it is partially measuring liquidity risk in the markets.

This test also indicates the importance of the liquidity factor that we have calculated. The test we conducted here is a pure out-of-sample test in that we have taken the liquidity factor constructed using corporate bond holding characteristics and applied it successfully to treasury bonds, an entirely different asset class.

4 Liquidity-Based Strategies

In this section, we apply the analysis conducted in the previous sections to a couple of very popular trading strategies. One is convertible arbitrage and the second is capital structure arbitrage. Convertible arbitrage involves going long a convertible bond and shorting the equity of the firm issuing the bond, while capital structure arbitrage involves going long any corporate bond and shorting the bond's corresponding equity. Both of these trading strategies have done quite well over the last few years, generating almost double digit alphas.

The motivation for these strategies is provided by Figures 5 and 6. Figure 5 shows the risk-neutral probability of default calculated from the equity markets for Worldcom.[12] What we conclude from this is that the probability of default for Worldcom is increasing (as predicted by the equity markets). However, looking from January of 2001 to January of 2002, we see in Figure 6 that the yield spread for Worldcom's bonds did not change. It appears from the debt markets that the probability of default for Worldcom has not changed. Therefore, it appears that the debt and equity markets are not very well integrated, at least for Worldcom's securities. An arbitrageur would take a short position in the bond and a long position in equity and then wait until the discrepancy in pricing in the two markets resolved itself.

One explanation for the discrepancy in the Worldcom securities is that the spread for Worldcom is not entirely due to credit risk but also due to liquidity risk. In other words, it is possible that liquidity for Worldcom increased from January, 2001 to January, 2002, thus causing a decrease in liquidity spread. Meanwhile the credit spread for Worldcom increased, consistent with the information in the equity markets. So, it is possible that the increase in credit spread was offset by the decrease in liquidity spread, leaving the total yield spread for Worldcom bonds unchanged during this time period. Therefore, the explanation for this seeming arbitrage is that credit and liquidity risk are not being correctly attributed. Another way to say this is that simply taking a short position in Worldcom's bonds, which were very illiquid and therefore have a good degree of liquidity risk, and a long position in Worldcom's equity, which was very liquid, is not a pure arbitrage as this leaves liquidity risk in the portfolio. Therefore, one explanation for the seeming outperformance of capital structure arbitrage is simply that the returns from capital structure arbitrage are simply fair compensation for the liquidity risk being held in the arbitrage portfolio.

Similarly, in the case of convertible arbitrage one explanation for the arbitrage from a long position in a convertible, usually an extremely illiquid instrument, and a short position in the issuing firm's equity, usually a very liquid instrument, is that the seemingly abnormal returns are simply just fair compensation for the liquidity risk being borne by these portfolios. If this is the explanation, then if one tested the returns from capital structure arbitrage with an appropriate benchmark, i.e., one containing a liquidity risk factor, the returns would not seem abnormal.

In this section, we put the liquidity risk factor derived in this paper as an

[12] The methodology for calculating is through an application of the Merton (1974) risk model as commonly done in practice.

explanatory variable in regressions of convertible arbitrage returns against standard fixed income risk factors.

4.1 Convertible Arbitrage

Table 13 (from Batta, Chacko, & Dharan (2005)) contains the results of regressing convertible arbitrage returns against various explanatory standard explanatory variables including a default factor, a term structure factor, four equity market factors (the equity market, a book-to-market factor, a size factor, and a momentum factor). Finally, we include the liquidity factor in the regressions. The table is presented such that the top half contains regression results with the liquidity factor while the bottom half contains the exact same regressions but excluding the liquidity factor. This allows us to see the effect of the liquidity factor on covertible arbitrage returns.

What we immediately conclude is that the liquidity factor appears to be very important in explaining the outperformance of convertible arbitrage. Concentrating on the middle set of regression results (the ones with all of the factors included), we see that when the liquidity factor is not included, the alpha of convertible arbitrage is approximately 13.5% annualized (the results use weekly data) and this value has strong statistical significance. However, when we include the liquidity factor in the same regression, the alpha drops to 5.7% annualized and is not statistically significant. Furthermore, including the liquidity factor increases the R^2 of all of the regressions considerably.

Therefore, what we conclude here is that the outperformance of convertible arbitrage may simply be due to leaving out an important risk factor in performance evaluation. Because the strategy involves taking a considerable amount of liquidity risk, the strategy earns returns for this risk. However, this is not outperformance as the returns are simply fair compensation for the risk being taken. In order to evaluate outperformance, one needs to fully risk-adjust the returns by incorporating a liquidity factor in the performance evaluation regression (thereby liquidity-adjusting the returns). Doing so seems to eliminate all of the alpha, or outperformance, from the strategy.

5 Conclusion

In conclusion, this paper analyzes whether liquidity risk is a priced factor. The asset class in which this question is answered is important. Generally, we don't expect a liquid security to contain very much liquidity premium. Therefore, it is important to answer this question in an asset class with a good deal of illiquidity. However, in the presence of illiquidity there are few transactions with which conventional liquidity measures such as trading volume or bid-ask spread can be constructed. Therefore, we created a new measure of liquidity in this paper called latent liquidity which measures the accessibility of a bond. By using a database of corporate bond holdings, we constucted a latent liquidity measure for every US corporate bond over the last ten years. From these measures, we were then able to develop a liquidity factor time series, which we used for various tests.

In general, we find very strong evidence that the liquidity risk factor we constructed is an important determinant of bond returns and that it is priced. We even took this risk factor outside the corporate bond asset class from which it was constructed and assessed its importance in determining the returns of another asset class, US Treasury bonds. This out-of-sample test showed that the liquidity risk factor can be important in explaining returns in a number of asset classes and, therefore, can be thought of as a universal risk factor.

If one believes in the results in this paper, the obvious next question to ask is how important is liquidity risk relative to credit risk. In other words, of the measured yield spread, the total spread between a bond's yield and the corresponding duration Treasury, how much is true credit spread (liquidity-adjusted credit spread) and how much is liquidity spread. If participants in the markets could discern between these two components of a bond's yield spread, they would better able to take the risks that they want and avoid those that they do not. For example insurance companies, who typically provide liquidity into the markets and earn a risk premium, may be more interested in purchasing a bond with a greater liquidity spread than credit spread. On the other hand, hedge funds, who typically require liquidity and pay the liquidity premium, may be interested in earning the same spread as the insurance company but on a bond where a greater proportion of the spread is coming from the credit spread rather than the liquidity spread. In addition, being able to discern between liquidity and credit risk should allow for much more efficient portfolio allocation.

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TABLES

Table 1.

US Corporate Debt: Sector Breakdown.

	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
Unique issue count	**4235**	**5557**	**6892**	**8808**	**10027**	**10357**	**9747**	**9878**	**9447**	**8472**	**5275**
Sector breakdown:											
% Basic Materials	**5%**	**5%**	**5%**	**4%**	**4%**	**4%**	**4%**	**4%**	**5%**	**5%**	**5%**
% Communications	**10%**	**10%**	**10%**	**10%**	**12%**	**12%**	**12%**	**12%**	**12%**	**11%**	**12%**
% Consumer Cyclical	**12%**	**11%**	**11%**	**10%**	**10%**	**10%**	**10%**	**10%**	**11%**	**10%**	**11%**
% Consumer Non-cyclical	**10%**	**9%**	**9%**	**8%**	**9%**	**9%**	**8%**	**9%**	**10%**	**10%**	**11%**
% Energy	**6%**	**6%**	**5%**	**5%**	**5%**	**6%**	**6%**	**6%**	**7%**	**8%**	**8%**
% Financial	**33%**	**38%**	**39%**	**40%**	**39%**	**38%**	**40%**	**37%**	**35%**	**34%**	**33%**
% Industrial	**9%**	**8%**	**8%**	**8%**	**9%**	**9%**	**8%**	**9%**	**9%**	**9%**	**9%**
% Technology	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**	**1%**
% Utilities	**12%**	**11%**	**9%**	**9%**	**7%**	**8%**	**8%**	**9%**	**8%**	**9%**	**8%**

Notes:
Unique issue count represents the number of issues that traded at least once during the year.
Sector breakdown provides the proportion of issues within each sector.
*1994 and 2004 are not full years

Table 2.

US Corporate Debt: Bond Rating Breakdown.

	Bond Rating Breakdown					
	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
% Aaa	2%	3%	3%	4%	4%	4%
% Aa	10%	10%	11%	11%	11%	11%
% A	26%	29%	27%	25%	25%	27%
% Baa	16%	18%	18%	20%	22%	25%
% Below Baa	16%	18%	17%	21%	26%	30%

Notes:
Bond rating breakdown provides the proportion of issues within various Moody’s bond rating categories.
* 2004 is not a full year

Table 3.

US Corporate Debt: Maturity Percentiles.

	Maturity Percentiles (years)										
	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
MIN	0.1	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10%	3.2	2.2	1.6	1.3	1.4	1.1	1.0	1.0	1.0	1.0	1.1
20%	4.6	3.4	2.9	2.6	2.5	2.1	1.7	1.8	2.0	2.1	2.0
30%	5.6	4.7	4.1	3.9	3.8	3.4	2.8	2.9	3.2	3.1	3.1
40%	7.0	5.9	5.2	5.3	5.1	4.9	4.0	4.2	4.5	4.4	4.0
50%	8.4	7.2	6.8	6.7	6.8	6.4	5.3	5.4	5.6	5.3	5.0
60%	9.4	8.4	8.0	8.2	8.3	8.0	7.0	6.9	6.9	6.7	6.5
70%	10.0	9.8	9.7	9.8	9.6	9.3	8.3	8.2	8.7	8.8	8.1
80%	15.0	12.0	11.4	11.5	10.0	10.1	10.0	10.0	10.0	12.1	12.2
90%	27.4	26.1	25.6	26.6	25.1	24.4	23.5	23.6	24.0	23.9	23.4
MAX	99.8	100.0	100.0	100.0	100.0	99.2	98.2	97.3	96.1	95.1	94.0

Notes:
Maturity percentiles provide the percentiles for the bond issue maturity in years.
*1994 and 2004 are not full years

Table 4.

US Corporate Debt: Time Since Issuance Percentiles.

	Time Since Issuance Percentiles (years):										
	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
MIN	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	0.6
10%	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	when-issued	0.2	1.2
20%	0.2	0.1	0.1	0.0	0.0	0.2	0.6	0.5	0.4	0.8	1.7
30%	0.5	0.7	0.5	0.2	0.2	0.7	1.2	1.3	1.2	1.6	2.4
40%	0.9	1.3	1.1	0.8	0.6	1.1	1.7	2.1	2.2	2.6	3.0
50%	1.3	1.7	2.0	1.6	1.2	1.7	2.2	2.8	3.2	3.8	4.3
60%	1.8	2.3	2.7	2.5	2.1	2.4	2.9	3.4	4.0	4.7	5.2
70%	2.4	2.8	3.3	3.6	3.1	3.4	3.9	4.3	4.8	5.5	6.0
80%	3.4	3.7	4.1	4.4	4.6	5.2	5.4	5.9	6.3	6.8	7.0
90%	5.7	5.8	5.8	5.8	6.1	6.8	7.5	8.3	8.9	9.3	8.9
MAX	33.4	49.1	41.8	51.4	43.2	44.5	45.6	46.3	47.5	48.3	49.1

Notes:
Time Since Issuance percentiles provide the percentiles for the bond issue's time since issuance in years.
*1994 and 2004 are not full years

Table 5.

US Corporate Debt: Face Value Percentiles.

	Face Value Percentiles (millions of US dollars):										
	YR1994*	**YR1995**	**YR1996**	**YR1997**	**YR1998**	**YR1999**	**YR2000**	**YR2001**	**YR2002**	**YR2003**	**YR2004***
MIN	1	0	0	0	0	0	0	0	0	0	0
10%	32	20	18	18	25	25	25	35	50	70	100
20%	75	50	43	40	50	60	75	99	100	100	150
30%	100	86	75	75	100	100	100	100	135	150	200
40%	105	100	100	100	100	110	125	150	153	200	227
50%	150	125	125	125	130	150	150	175	200	220	272
60%	160	150	150	150	150	180	200	200	250	260	300
70%	200	200	200	200	200	200	250	250	300	316	400
80%	250	250	248	250	250	250	300	302	400	500	500
90%	300	300	300	300	300	350	450	500	600	750	950
MAX	5026	5501	5501	5501	5026	5501	5501	5501	6500	6500	6500

Notes:
Face value percentiles provide the percentiles for the entire bond issue face value in millions of US dollars.
*1994 and 2004 are not full years

Table 6.

US Corporate Debt: Average Number of Days between Trades Percentiles.

	Average # of Days between Trades Percentiles:										
	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
MIN	5	5	4	4	4	3	3	3	2	2	2
10%	30	29	28	27	26	24	20	15	11	10	8
20%	48	47	46	45	41	40	36	27	20	17	13
30%	66	68	69	67	60	58	54	43	32	27	20
40%	88	92	94	91	84	82	78	63	48	42	27
50%	114	122	128	126	112	112	107	89	70	61	37
60%	140	147	152	150	143	144	138	126	99	90	50
70%	186	220	262	261	218	225	187	157	140	132	62
80%	260	260	262	261	261	261	260	261	261	201	99
90%	260	260	262	261	261	261	260	261	261	261	99
MAX	260	260	262	261	261	261	260	261	261	261	99

Notes:
Average # of days between trades percentiles provide the percentiles for the average number of trading days between trades per issue.
Those issues that trade only once during the year are assigned a value equal to the number of trading days for the particular year (typically around 260).
*1994 and 2004 are not full years

Table 7.

US Corporate Debt: Average Trade Size Percentiles.

	Average Trade Size Percentiles (millions of US dollars):										
	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
MIN	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10%	0.36	0.44	0.43	0.48	0.50	0.43	0.40	0.42	0.37	0.35	0.28
20%	0.75	0.83	0.84	0.94	0.97	0.82	0.72	0.73	0.67	0.66	0.55
30%	1.06	1.11	1.18	1.23	1.32	1.12	1.01	1.03	0.94	0.91	0.78
40%	1.43	1.50	1.63	1.68	1.78	1.54	1.38	1.43	1.22	1.16	1.03
50%	1.84	2.02	2.09	2.16	2.34	2.08	1.93	1.98	1.66	1.52	1.30
60%	2.30	2.63	2.71	2.85	3.10	2.88	2.56	2.65	2.21	1.97	1.65
70%	3.02	3.59	3.61	3.72	4.15	3.89	3.45	3.59	2.99	2.50	2.17
80%	4.10	4.99	4.97	5.06	5.56	5.31	5.02	5.12	4.30	3.46	2.88
90%	6.20	7.22	7.33	8.00	9.16	8.93	8.23	8.42	7.06	5.75	4.55
MAX	100.31	99.92	100.67	111.99	224.98	249.93	152.53	199.98	271.99	199.98	100.28

Notes:
Average trade size percentiles provide the percentiles for the average trade size per issue in millions of US dollars, where size is defined as the market value of the transaction.
*1994 and 2004 are not full years

Table 8.

US Corporate Debt: Average Daily Trading Volume Percentiles.

	Average Daily Trading Volume Percentiles (US dollars)										
	YR1994*	YR1995	YR1996	YR1997	YR1998	YR1999	YR2000	YR2001	YR2002	YR2003	YR2004*
MIN	0	2	0	3	0	1	0	0	0	0	0
10%	2561	3096	2767	3251	3720	2935	3088	3826	3118	3189	3396
20%	6817	7093	6976	7591	9085	8028	7770	9672	9819	10924	9124
30%	12248	13175	13427	14466	18838	16458	15677	19688	22055	22497	19093
40%	20998	22042	22764	24917	30482	27333	26180	34337	40287	41523	35153
50%	32703	36177	37863	41398	49098	43002	41957	55496	66525	69279	56206
60%	49998	55588	59841	66908	79131	66589	64058	87965	110283	110243	87257
70%	78231	89305	95622	111033	122649	101513	103501	140787	182740	171125	130295
80%	128188	154659	177335	202012	203898	172114	174869	250757	326696	284251	206040
90%	240816	337538	401592	432456	401242	372501	392877	629944	713805	542730	366581
MAX ($mil.)	20	12	30	23	31	80	40	50	43	11	9

Notes:
Average daily trading volume percentiles provide the percentiles for the average daily trading volume per issue, where average daily trading volume is defined as the market value of all transactions that occurred during the year divided by the number of trading days that the bond issue was outstanding in the year.
*1994 and 2004 are not full years

Table 9.

Contribution of Liquidity: I

Incremental R^2 of Liquidity Factor

		Liquidity Portfolios				
		H	**H/M**	**M**	**M/L**	**L**
Credit Portfolios	**H**	**5%**	**12%**	**18%**	**23%**	**30%**
	M	**5%**	**13%**	**21%**	**25%**	**32%**
	L	**4%**	**13%**	**22%**	**26%**	**34%**

H: High
H/M: High/Medium
M: Medium
M/L: Medium/Low
L: Low

Table 10.

Contribution of Liquidity: II

Incremental R^2 of Liquidity Factor

		Liquidity Portfolios				
		H	**H/M**	**M**	**M/L**	**L**
Duration Portfolios	**H**	**4%**	**14%**	**21%**	**27%**	**36%**
	M	**3%**	**16%**	**20%**	**28%**	**37%**
	L	**6%**	**17%**	**23%**	**30%**	**39%**

H: High
H/M: High/Medium
M: Medium
M/L: Medium/Low
L: Low

Table 11.

Liquidity Risk Alpha

Alphas of Portfolios Sorted on Historical Liquidity Betas

	1	2	3	4	5	5-1
CAPM	-0.54%	0.71%	1.25%	1.94%	2.36%	2.90%
Duration	-0.36%	0.69%	1.31%	2.13%	2.78%	3.14%
Duration, Credit	-0.56%	0.63%	1.09%	1.68%	2.15%	2.71%

1: Lowest Liquidity Beta Bin
5: Highest Liquidity Beta Bin

Table 12.

Contribution of Liquidity Factor to Treasury Bond Yields

Average Contribution of Individual Factors to Bond Yields (RMSE)

Maturity	Curvature	Slope	Liquidity
0.5	2	3	5
1	3	7	10
2	7	9	16
3	13	16	27
5	29	37	56
7	38	46	73
10	21	64	97

Table 13.

Convertible Arbitrage

Alpha	DEF	TERM	Rm-Rf	SMB	HML	UMD	Liq.	Adj.R^2
0.0029	**-0.66**	**-0.33**					**0.27**	**0.3859**
1.39	***-1.43***	***-1.21***					***3.65***	
0.0011	**-0.02**	**0.09**	**-0.19**	**0.07**	**0.08**	**-0.02**	**0.24**	**0.4897**
0.59	***-0.13***	***1.1***	***-2.45***	***2.45***	***1.28***	***-0.09***	***2.93***	
0.0012			**-0.19**	**0.06**	**0.1**	**0.01**	**0.26**	**0.4565**
0.67			***-2.58***	***1.82***	***1.54***	***0.24***	***3.47***	
0.0004	**-0.66**	**-0.33**						**0.055**
0.58	***-1.43***	***-1.21***						
0.0026	**-0.02**	**0.08**	**-0.15**	**0.07**	**0.08**	**-0.03**		**0.1598**
3.51	***-0.15***	***1.08***	***-2.74***	***2.44***	***1.26***	***-0.09***		
0.0035			**-0.17**	**0.06**	**0.09**	**0.01**		**0.1566**
3.32			***-2.07***	***1.8***	***1.51***	***0.25***		

FIGURES

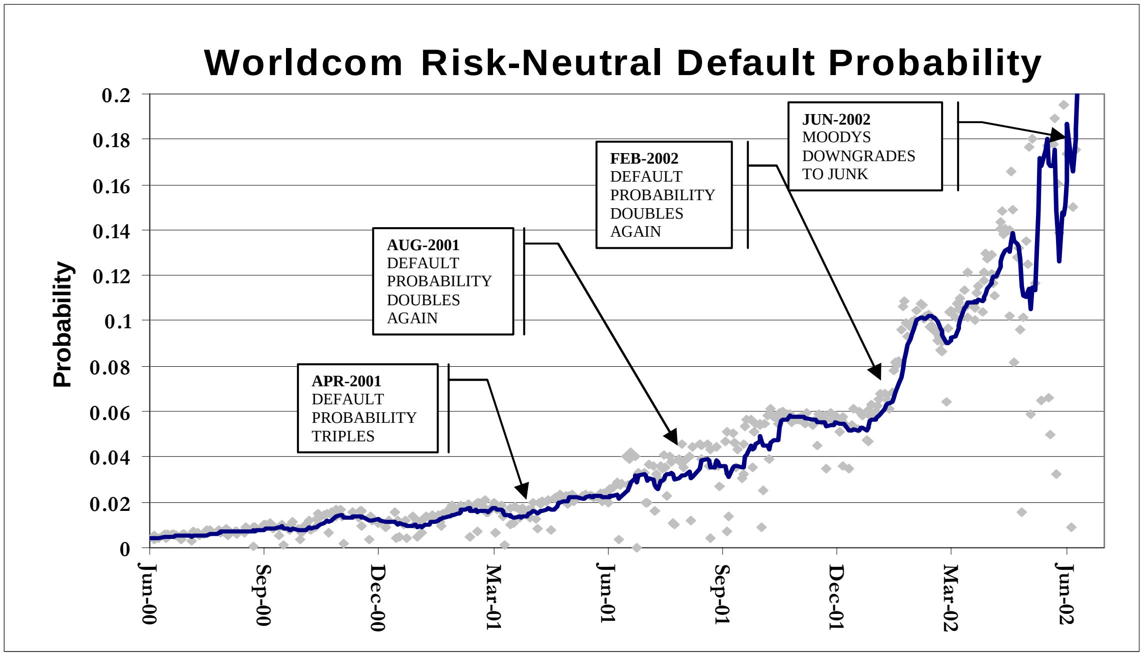

Fig. 1. Time-Series Plot of WorldCom's Risk-Neutral Default Probability.

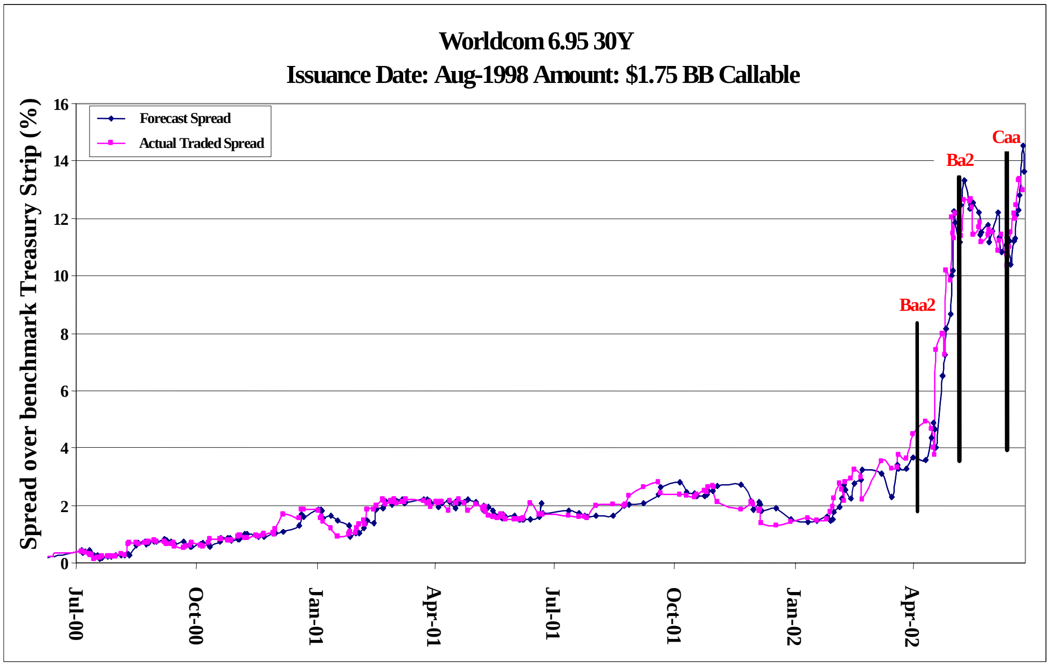

Fig. 2. Time-Series Plot of WorldCom's Yield Spread over Benchmark Treasury. The diamond solid line shows the forecasted yield spreads while the square solid line shows the actual traded yield spreads. Three Moody's rating downgrade events are marked on the plot.

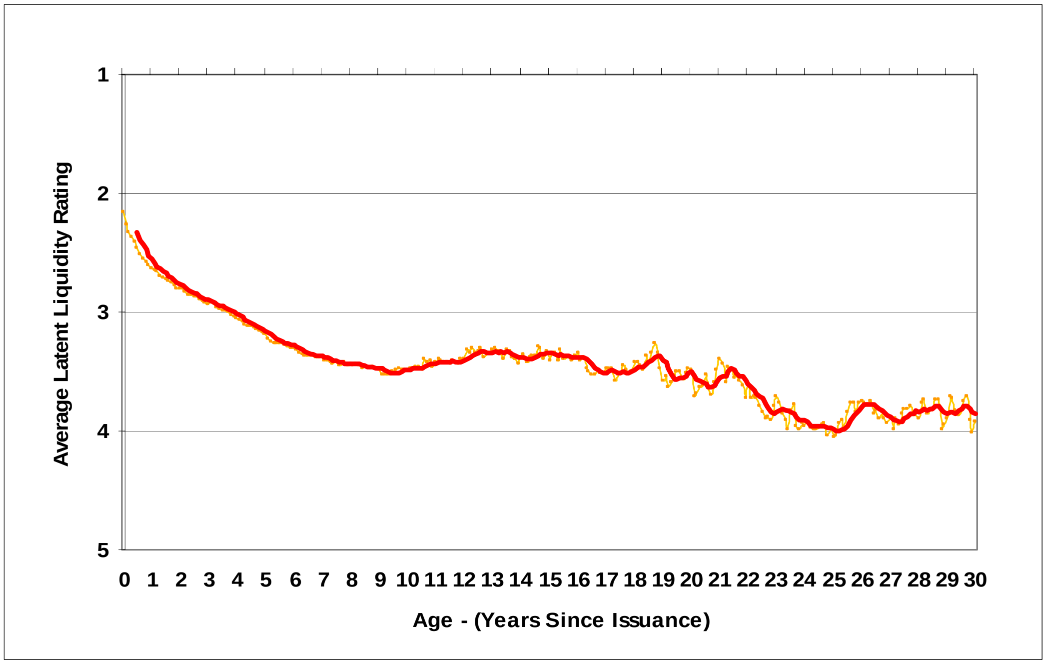

Fig. 3. Latent Liquidity Rating Increases or Liquidity decreases as Bond Ages. Bond age is defined as the number of years since issuance date. Latent liquidity rating ranges from 1 to 5, with 1 being the most liquidity and 5 being the least liquid.

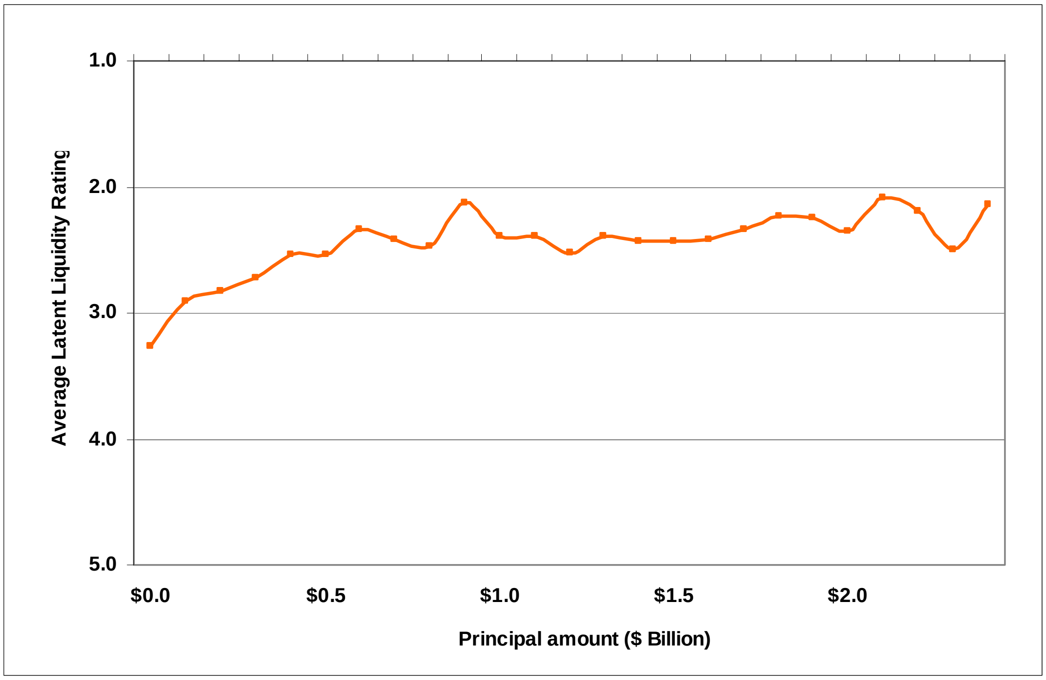

Fig. 4. Lower Latent Liquidity Rating or Higher Liquidity with Larger Issue Size. Issue size is defined as the amount of principal at issuance. Latent liquidity rating ranges from 1 to 5, with 1 being the most liquidity and 5 being the least liquid.

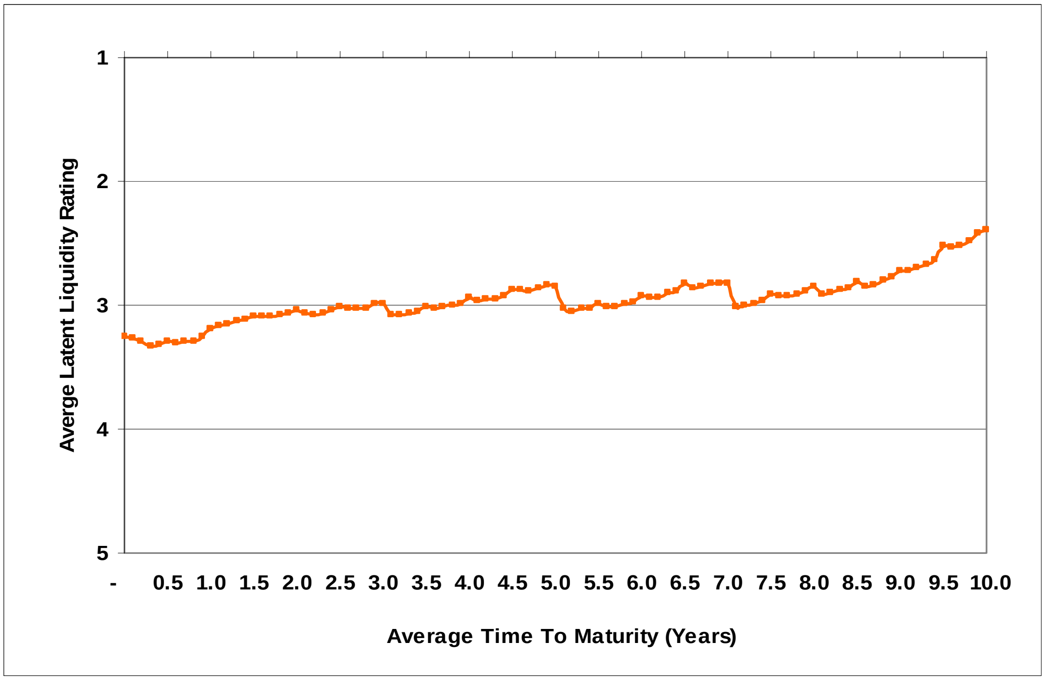

Fig. 5. Lower Latent Liquidity Rating or Higher Liquidity as Bond is Farther Away from Maturity. Latent liquidity rating ranges from 1 to 5, with 1 being the most liquidity and 5 being the least liquid.

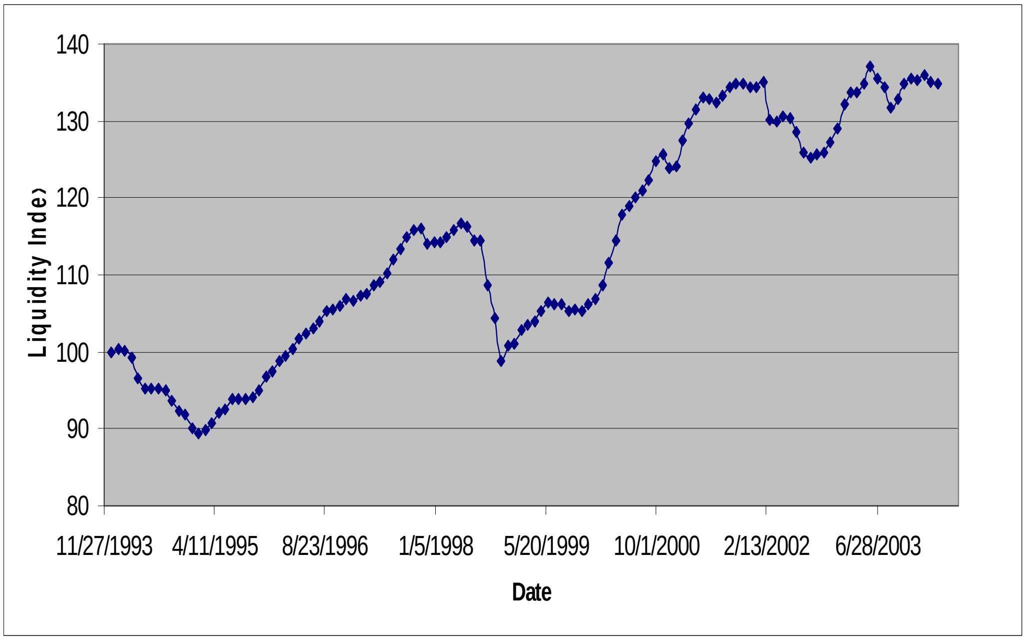

Fig. 6. Time-Series Plot of Accumulative Liquidity Index.

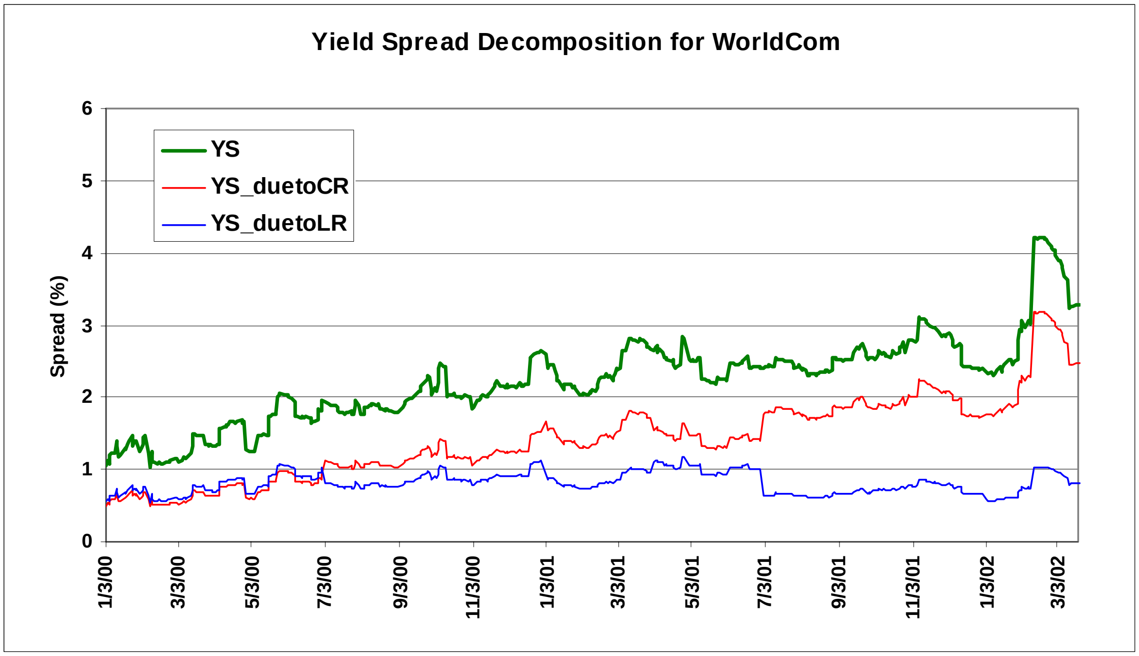

Fig. 7. Decomposition of Yield Spread into Credit and Liquidity Components for WorldCom.