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REVISITING CREDIT SCORING MODELS IN A BASEL 2 ENVIRONMENT

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Revisiting Credit Scoring Models in a Basel 2 Environment

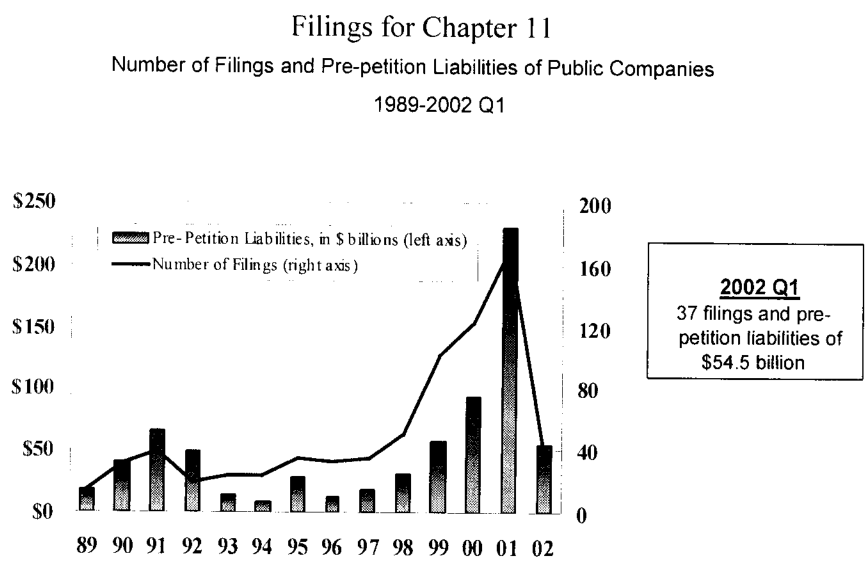
Edward I. Altman

1. Introduction

In recent years, around the turn of the new millennium, credit scoring models have been remotivated and given unprecedented significance by the stunning pronouncements of the new possible Basel Accord on credit risk capital adequacy - - the so-called Basel 2 (Basel [1999] and [2001). Banks, in particular, and most financial institutions worldwide, have either recently developed or modified existing credit risk systems or are currently developing methods to conform with best practice systems and processes for assessing the probability of default (PD) and, possibly, loss-given-default (LGD) on credit assets of all types. Coincidentally, defaults and bankruptcies reached unprecedented levels in 2001 and have continued in 2002. Indeed, companies with sizeable liabilities reached at least $240 billion in 2001 (even with Enron's understatement at the time of filing) and there were 39 firms that filed for protection under the U.S. bankruptcy code with liabilities greater than $1 billion (see Panel A). The pace of these large bankruptcies has continued in 2002 with another 10 firms of such great size filing in the first four months. In the public bond arena, over $63 billion of U.S. domestic public debt defaulted in 2001 and the default rate on U.S. high yield bonds was almost a record 9.8% (see Altman and Arman [2002]).

This chapter primarily discusses a model I developed over 30 years ago, the so-called Z-Score model, and its relevance to these recent developments. In doing so, we will also provide some updated material on the Z-Score model's tests and applications over time as well as some modifications for greater applicability. We also discuss an

Panel A


Filings for Chapter 11
Number of Filings and Pre-petition Liabilities of Public Companies
1989-2002 Q1
Pre-Petition Liabilities, in $ billions (left axis)
Number of Filings (right axis)
$250
$200
$150
$100
$50
$0
200
160
120
80
40
0
89 90 91 92 93 94 95 96 97 98 99 00 01 02
2002 Q1
37 filings and pre-petition liabilities of $54.5 billion

alternative widely used credit risk model, known as the KMV approach, and compare both KMV and Z-Score in the now infamous Enron bankruptcy debacle. Finally, we summarize a recent report (Altman, Resti and Sironi, 2002) on the association between PD and recovery rates on defaulted credit assets.

The major theme of this paper is that the assignment of appropriate default probabilities on corporate credit assets is a three-step process evolving from the development of (1) credit scoring models, (2) capital market risk equivalents - - usually bond ratings, and (3) assignment of PD[1] and possibly LGDs on the credit portfolio. Our emphasis will be on step 1 and how the Z-Score model, (Altman, 1968), has become the prototype model for one of the three primary structures for determining PDs. The other two structures involve either the bond rating process itself or option pricing capital market valuation techniques, typified by the KMV approach, McQuown (1993) and KMV (1995). These techniques are also the backbone of most credit asset value-at-risk (VaR) models. In essence, we feel strongly that if the initial credit scoring model is sound and based on comprehensive and representative data, then the credit VaR model has a chance to be accurate and helpful for both regulatory and economic capital assignment. If it is not, no amount of quantitative sophistication or portfolio analytic structures can achieve valid credit risk results.

2. Credit Scoring Models

Almost all of the statistical credit scoring models in use today are variations on a similar theme. They involve the combination of a set of quantifiable financial indicators

[1] Some might argue that a statistical methodology can combine steps (1) and (2) where the output from (1) automatically provides estimates of PD. This is one of the reasons that many "modelers" of late and major consulting firms prefer the logit-regression approach, rather than the discriminant model that I prefer.

of firm performance with, perhaps, a small number of additional variables that attempt to capture some qualitative elements of the credit process. While this paper will concentrate on the quantitative measures, mainly financial ratios and capital market values, one should not underestimate the importance of qualitative measures in the process.[2] Until recently, sophisticated practitioners, and certainly academicians, had been moving toward the possible elimination of ratio analysis as an analytical technique in assessing firm performance. Theorists downgrade arbitrary rules of thumb (such as company ratio comparisons) widely used by practitioners. Since attacks on the relevance on ratio analysis emanate from many esteemed members of the scholarly world,[3] does this mean that ratio analysis is limited to the world of "nuts and bolts?" Or, has the significance of such an approach been unattractively garbed and therefore unfairly handicapped? Can we bridge the gap, rather than sever the link, between traditional ratio analysis and the more rigorous statistical techniques which have become popular among academicians? Along with our primary interest, credit risk assessment and financial distress prediction, I am also concerned with an assessment of ratio analysis as an analytical technique.

3. Traditional Ratio Analysis

The detection of company operating and financial difficulties is a subject which has been particularly amenable to analysis with financial ratios. Prior to the development of quantitative measures of company performance, agencies were established to supply a qualitative type of information assessing the credit-worthiness of particular merchants.

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[2] Practitioners have reported that these so-called qualitative elements, that involve judgment on the part of the risk officer, can provide as much as 30-50% of the explanatory power of the scoring model.

[3] This is evidenced by the diminished emphasis, indeed almost the entire extinction, of chapters in introductory Corporate Finance textbooks on financial statement analysis and the information one can glean from these statements.

(For instance, the forerunner of the well-known Dun & Bradstreet, Inc. was organized in 1849 in order to provide independent credit investigations). Formal aggregate studies concerned with portents of business failure were evident in the 1930's.

One of the classic works in the area of ratio analysis and bankruptcy classification was performed by Beaver (1967, 1968). His univariate analysis of a number of bankruptcy predictors set the stage for the multivariate attempts, by this author and others, which followed. Beaver found that a number of indicators could discriminate between matched samples of failed and nonfailed firms for as long as five years prior to failure. He questioned the use of multivariate analysis, however, although a discussant recommended attempting this procedure. The Z-Score model, developed by this author at the same time (1968) that Beaver was working on his own thesis, did just that. A subsequent study by Deakin (1972) utilized the same 14 variables that Beaver analyzed, but he applied them within a series of multivariate discriminant models.

The aforementioned studies imply a definite potential of ratios as predictors of bankruptcy. In general, ratios measuring profitability, liquidity, and solvency prevailed as the most significant indicators. The order of their importance is not clear since almost every study cited a different ratio as being the most effective indication of impending problems.[4]

Although these works established certain important generalizations regarding the performance and trends of particular measurements, the adaptation of the results for assessing bankruptcy potential of firms, both theoretically and practically, is questionable. In almost every case, the methodology was essentially univariate in nature

[4] For a more in-depth discussion of other ratio based models, see Altman (1993).

and emphasis was placed on individual signals of impending problems. Ratio analysis presented in this fashion is susceptible to faulty interpretation and is potentially confusing. For instance, a firm with a poor profitability and/or solvency record may be regarded as a potential bankrupt. However, because of its above average liquidity, the situation may not be considered serious. The potential ambiguity as to the relative performance of several firms is clearly evident. The crux of the shortcomings inherent in any univariate analysis lies therein. An appropriate extension of the previously cited studies, therefore, is to build upon their findings and to combine several measures into a meaningful predictive model. In so doing, the highlights of ratio analysis as an analytical technique will be emphasized rather than downgraded. The questions are (1) which ratios are most important in detecting credit risk problems, (2) what weights should be attached to those selected ratios, and (3) how should the weights be objectively established.

4. Discriminant Analysis

After careful consideration of the nature of the problem and of the purpose of this analysis, I chose multiple discriminant analysis (MDA) as the appropriate statistical technique. Although not as popular as regression analysis, MDA has been utilized in a variety of disciplines since its first application in 1930's. During those earlier years, MDA was used mainly in the biological and behavioral sciences. In recent years, this technique has become increasingly popular in the practical business world as well as in academia.

MDA is a statistical technique used to classify an observation into one of several a *priori* groupings dependent upon the observation's individual characteristics. It is used primarily to classify/or make predictions in problems where the dependent variable appears in qualitative from, for example, male or female, bankrupt or nonbankrupt. Therefore, the first step is to establish explicit group classifications. The number of original groups can be two or more. Some analysts refer to discriminant analysis as "multiple" only when the number of groups exceeds two. We prefer that the multiple concepts refer to the multivariate nature of the analysis.

After the groups are established, data are collected for the objects in the groups; MDA in its most simple form attempt to derive a linear combination of these characteristics which "best" discriminates between the groups. If a particular object, for instance, a corporation, has characteristics (financial ratios) which can be quantified for all of the companies in the analysis, the MDA determines a set of discriminant coefficients. When these coefficients are applied to the actual ratios, a basis for classification into one of the mutually exclusive groupings exists. The MDA technique has the advantage of considering an entire profile of characteristics common to the relevant firms, as well as the interaction of these properties. A univariate study, on the other hand, can only consider the measurements used for group assignments one at a time.

Another advantage of MDA is the reduction of the analyst's space dimensionally, that is, from the number of different independent variables to G-1 dimension(s), where G equals the number of original a *priori* groups. This analysis is concerned with two groups, consisting of bankrupt and nonbankrupt firms. Therefore, the analysis is transformed into its simplest form: one dimension. The discriminant function, of the form $Z = V_1X_1 + V_2X_2 + V_nX_n$ transforms the individual variable values to a single discriminant score, or Z value, which is then used to classify the object where:

$V_1, V_2, \ldots . V_n$ = discriminant coefficients, and
$X_1, X_2, \ldots . X_n$ = independent variables

When utilizing a comprehensive list of financial ratios in assessing a firm's bankruptcy potential, there is reason to believe that some of the measurements will have a high degree of correlation or collinearity with each other. While this aspect is not serious in discriminant analysis, it usually motivates careful selection of the predictive variables (ratios). It also has the advantage of potentially yielding a model with a relatively small number of selected measurements which convey a great deal of information. This information might very well indicate differences among groups, but whether or not these differences are significant and meaningful is a more important aspect of the analysis.

Perhaps the primary advantage of MDA in dealing with classification problems is the potential of analyzing the entire variable profile of the object simultaneously rather than sequentially examining its individual characteristics. Just as linear and integer programming have improved upon traditional techniques in capital budgeting, the MDA approach to traditional ratio analysis has the potential to reformulate the problem correctly. Specifically, combinations of ratios can be analyzed together in order to remove possible ambiguities and misclassifications observed in earlier traditional ratio studies. Critics of discriminant analysis point out that most, if not all, financial models using this technique violate several statistical requirements including multivariate normality and independence of the explanatory variables. While valid concerns, my experience has shown that careful bounding of certain extreme value ratios and tests for the models' robustness over time, will mitigate these statistical property criticisms.

5. Development of the Z-Score Model

Sample Selection

The initial sample is composed of 66 corporations with 33 firms in each of the two groups. The bankrupt (distressed) group (Group 1) were all manufacturers that filed a bankruptcy petition under Chapter X of the National Bankruptcy Act from 1946 through 1965. A 20-year period is not the best choice since average ratios do shift over time. Ideally, we would prefer to examine a list of ratios in time period t in order to make predictions about other firms in the following period (t+1). Unfortunately, it was not possible to do this because of data limitations at that time. Recent "heavy" activity of bankruptcies now presents a more fertile environment. Recognizing that this group is not completely homogeneous (due to industry and size differences), I attempted to make a careful selection of nonbankrupt (nondistressed) firms. Group 2 consists of a paired sample of manufacturing firms chosen on a stratified random basis. The firms are stratified by industry and by size, with the asset size range restricted to between $1 and $25 million. Firms in group 2 were still in existence at the time of the analysis. Also, the data collected were from the same years as those compiled for the bankrupt firms. For the initial sample test, the data are derived from financial statements dated one annual reporting period prior to bankruptcy.

An important issue is to determine the asset-size group to be sampled. The decision to eliminate both the small firms (under $1 million in total assets) and the very large companies from the initial sample essentially is due to the asset range of the firms in Group 1. In addition, the incidence of bankruptcy in the large-asset-size firm was quite rare prior to 1966. This changed starting 1970 with the appearance of very large bankruptcies, e.g., Penn-Central R.R. Large industrial bankruptcies also increased in appearance since 1978 (the year of the existing Bankruptcy Code's enactment), and in 2001 alone, 39 companies with liabilities greater than $1 billion filed for protection under the U.S. Chapter 11 Bankruptcy Code (see Panel A).

A frequent argument is that financial ratios, by their very nature, have the effect of deflating statistics by size, and that therefore a good deal of the size effect is eliminated. The Z-Score model, discussed below, has proven to be sufficiently robust to accommodate large firms.

Variable Selection

After the initial groups were defined and firms selected, balance sheet and income statement data were collected. Because of the large number of variables that are potentially significant indicators of corporate problems, a list of 22 potentially helpful variables (ratios) was compiled for evaluation. The variables are classified into five standard ratio categories, including liquidity, profitability, leverage, solvency, and activity. The ratios were chosen on the basis of their popularity in the literature and their potential relevancy to the study, and there were a few "new" ratios in this analysis.

From the original list of 22 variables, five were selected as doing the best overall job together in the prediction of corporate bankruptcy. The contribution of the entire profile is evaluated and, since this process is essentially iterative. There is no claim regarding the optimality of the resulting discriminant function.

In order to arrive at a final profile of variables, the following procedures were utilized: (1) observation of the statistical significance of various alternative functions, including determination of the relative contributions of each independent variable; (2)

evaluation of intercorrelations among the relevant variables; (3) observation of the predictive accuracy of the various profiles; and (4) judgment of the analyst.[5]

The final discriminant function is given in Panel B.

Panel B

The Z-Score Model

$$Z = 1.2\,X_1 + 1.4\,X_2 + 3.3\,X_3 + 0.6\,X_4 + 1.0\,(X_5)$$

X_1 = working capital/total assets,
X_2 = retained earnings/total assets,
X_3 = earnings before interest and taxes/total assets,
X_4 = market value equity/book value of total liabilities,
X_5 = sales/total assets, and
Z = overall Index or Score

Note that the model does not contain a constant (Y-intercept) term. This is due to the particular software utilized and, as a result, the relevant cutoff score between the two groups is not zero. Other software program, like SAS and SPSS, have a constant term, which standardizes the cutoff score at zero if the sample sizes of the two groups are equal.

X_1, Working Capital/Total Asset (RE/TA)

The working capital/total assets ratio, frequently found in studies of corporate problems, is a measure of the net liquid assets of the firm relative to the total capitalization. Working capital is defined as the difference between current assets and current liabilities. Liquidity and size characteristics are explicitly considered. Ordinarily,

[5] Subsequent versions of discriminant model software include step-wise methods which self-select the variables that either enter (forward stepwise) or are excluded (backward) from the final variable profile.

a firm experiencing consistent operating losses will have shrinking current assets in relation to total assets.[6] Two other liquidity ratios tested were the current ratio and the quick ratio. There were found to be less helpful and subject to perverse trends for some failing firms.

In all cases, tangible assets, not including intangibles, are used.

X_2, Retained Earnings/Total Assets (RE/TA)

Retained earnings are the account which reports the total amount of reinvested earnings and/or losses of a firm over its entire life. The account is also referred to as earned surplus. It is conceivable that a bias would be created by a substantial reorganization or stock dividend and appropriate readjustments should be made to the accounts.

This measure of cumulative profitability over time is what I referred to earlier as a "new" ratio. The age of a firm and its use of leverage are implicitly considered in this ratio. For example, a relatively young firm will probably show a low RE/TA ratio because it has not had time to build up its cumulative profits. Therefore, it may be argued that the young firm is somewhat discriminated against in this analysis, and its chance of being classified as bankrupt is relatively higher than that of another older firm, *ceteris paribus*. But, this is precisely the situation in the real world. The incidence of failure is much higher in a firm's earlier years [40 – 50% of all firms that fail do so in the first five years of their existence (Dun & Bradstreet, annual statistics)].

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[6] It is true, however, that this ratio, indeed all liquidity measures using short term assets, can be very misleading in that the ratio can be growing just when a firm is about to fail. This fact highlights the problems of univariate measures of performance.

In addition, the RE/TA ratio measures the leverage of a firm. Those firms with high RE, relative to TA, have financed their assets through retention of profits and have not utilized as much debt. This ratio highlights either the use of internally generated funds for growth (low risk capital) or OPM (other people's money) - - higher risk capital.

X_3, Earnings Before Interest and Taxes/Total Assets (EBIT/TA)

This ratio is a measure of the true productivity of the firm's assets, independent of any tax or leverage factors. Since a firm's ultimate existence is based on the earning power of its assets, this ratio appears to be particularly appropriate for studies dealing with credit risk. Furthermore, insolvency in a bankrupt sense occurs when the total liabilities exceed a fair valuation of the firm's assets with value determined by the earning power of the assets.

X_4, Market Value of Equity/Book Value of Total Liabilities (MVE/TL)

Equity is measured by the combined market value of all shares of stock, preferred and common, while liabilities include both current and long term. The measure shows how much the firm's assets can decline in value (measured by market value of equity plus debt) before the liabilities exceed the assets and the firm becomes insolvent. We discussed this "comparison" long before the advent of the KMV approach (discussed below) and before Merton (1974) put these relationships into an option-theoretic approach to value corporate risky debt. KMV used Merton's work to springboard into their now commonly used credit risk measure - - the EDF (Expected Default Frequency).

This ratio adds a market value dimension which most other failure studies did not consider. At a later point, we will substitute the book value of net worth for the market value in order to derive a discriminant function for privately held firms (Z') and for non-manufacturers (Z").

X_5, Sales/Total Assets (S/TA)

The capital-turnover ratio is a standard financial ratio illustrating the sales generating ability of the firm's assets. It is one measure of management's capacity in dealing with competitive conditions. This final ratio is unique because it is the least significant ratio on an individual basis and on a univariate statistical significance test, it would not have appeared at all. However, because of its relationship to other variables in the model, the sales/total assets ratio ranks high in its contribution to the overall discriminating ability of the model. Still, there is a wide variation among industries and across countries in asset turnover, and we will specify an alternative model (Z"), without X_5 at a later point.

Variable means measured at one financial statement prior to bankruptcy and the resulting F-statistics were observed; variables X_1 through X_4 are all significant at the 0.001 level, indicating extremely significant differences in these variables among groups. Variable X_5 does not show a significant difference between groups and the reason for its inclusion in the variable profile is not apparent as yet. On a strictly univariate level, all of the ratios indicate higher values for the nonbankrupt firms. Also, all of the discriminant coefficients display positive signs, which is what one would expect. Therefore, the greater a firm's distress potential, the lower its discriminant score. While it was clear that four of the five variables displayed significant differences between groups, the importance of MDA is its ability to separate groups using multivariate measures.

Once the values of the discriminant coefficients are estimated, it is possible to calculate discriminant scores for each observation in the samples, or any firm, and to assign the observations to one of the groups based on this score. The essence of the procedure is to compare the profile of an individual firm with that of the alternative groupings. The comparisons are measured by either a chi-square value, or similar test, and group assignments are made based upon the relative proximity of the firms' score to the various group centroids (means).

Testing the Model on Subsequent Distressed Firm's Samples

In three subsequent tests, I examined 86 distressed companies from 1969-1975, 110 bankrupts from 1976-1995 and 120 from 1997-1999. I found that the Z-Score model, using a cutoff score of 2.675, was between 82% and 94% accurate (Panel C). In repeated tests, the accuracy of the Z-Score model on samples of distressed firms has been in the vicinity of 80-90%, based on data from one financial reporting period prior to bankruptcy. The Type II error (classifying the firm as distressed when it does not go bankrupt), however, has increased substantially with as much as 15-20% of all firms and 10% of the largest firms having Z-Scores below 1.81. I advocate using the lower bond of the zone-of-ignorance (1.81) as a more realistic cutoff Z-Score than the score 2.675. The latter resulted in the lowest overall error in the original tests. In 1999, the proportion of U.S. industrial firms, comprised in the Compustat data tapes that had Z-Scores below 1.81 was over 20%.

Panel C Classification & Prediction Accuracy Z-Score (1968) Credit Scoring Model*					
Year Prior To Failure	**Original Sample (33)**	**Holdout Sample (25)**	**1969-1975 Predictive Sample (86)**	**1976-1995 Predictive Sample (110)**	**1997-1999 Predictive Sample (120)**
1	94% (88%)	96% (92%)	82% (75%)	85% (78%)	94% (84%)
2	72%	80%	68%	75%	74%
*Using 2.67 as cutoff score (1.81 cutoff accuracy in parenthesis)					

6. Adaptation for Private Firms' Application

Perhaps the most frequent inquiry that I have received from those interested in using the Z-Score model is, "What should we do to apply the model to firms in the private sector?" Credit analysts, private placement dealers, accounting auditors, and firms themselves are concerned that the original model is only applicable to publicly traded entities (since X_4 requires stock price data). And, to be perfectly correct, the Z-Score model is a publicly traded firm model and *ad hoc* adjustments are not scientifically valid. For example, the most obvious modification is to substitute the book value of equity for the market value and then recalculate V_4X_4.

A Revised Z-Score Model

Rather than simply insert a proxy variable into an existing model to calculate Z-scores, I advocate a complete reestimation of the model, substituting the book values of equity for the Market Value in X_4. One expects that all of the coefficients will change (not only the new variable's parameter) and that the classification criterion and related cutoff scores would also change. That is exactly what happens.

The result of our revised Z-Score model with a new X_4 variable is:

$$Z' = 0.717(X_1) + 0.847(X_2) + 3.107(X_3) + 0.420(X_4) + 0.998(X_5)$$

The equation now looks different than the earlier model; note, for instance, the coefficient for X_1 went from 1.2 to 0.7. But, the model looks quite similar to the one using Market Values. The actual variable that was modified, X_4, showed a coefficient change to 0.42 from 0.60; that is, it now has less of an impact on the Z-Score. X_3 and X_5 are virtually unchanged.

7. Bond Rating Equivalents

As noted in the Introduction, one of the main reasons for building a credit scoring model is to estimate the probability of default and loss given a certain level of risk estimation. Although we all are aware that the rating agencies (e.g., Moody's. S&P, Fitch) are certainly not perfect in their credit risk assessments, in general it is felt that they do provide important and consistent estimates of default - - mainly via their ratings. And, since there has been a long history and fairly large number of defaults which had ratings, especially in the United States, we can "profit" from this history by linking our credit scores with these ratings and thereby deriving expected and unexpected PDs and perhaps LGDs. These estimates can be made for a fixed period of time from the rating date, e.g., one year, or on a cumulative basis over some investment horizon, e.g., five years. And, they can be derived from the rating agencies themselves on an updated basis based on their so-called "static-pool" (S&P) or "dynamic-cohort" (Moody's) approaches.

An alternative is to use Altman's "mortality rate" approach (Altman, 1989) which is based on the expected default from the original issuance date.

With respect to non-rated entities, one can calculate a score, based on some available model, and link it to a bond rating equivalent. The latter then can lead to the estimate of PD. For example, in Pane D we list the bond rating equivalents for various Z-Score intervals based on average Z-Scores from 1995-1999 for bonds rated in their respective categories. For example, one observes that triple-A bonds have an average Z-Score of about 5.0, while singe-B bonds have an average score of 1.70. The latter, incidentally, is in the distress zone and accounts for the largest of the "junk bond" categories.

The analyst can then observe the average one year PD from Moody's/S&P for B rated bonds and find that it is in the 5% - 6% range (Moody's 2002, S&P 2002) or that the average <u>one year after issuance</u> PD is 2.45% (Altman & Arman, 2002). Note that the first year's PD is considerably lower that the PD derived from a "basket" of B-rated bonds which contain securities of many different ages and maturities. We caution the analyst to apply the correct PD estimate based on the qualities of the relevant portfolio of credit assets.

Panel D Average Z-Scores by S&P Bond Rating 1995 – 1999			
	Number of Firms	**Average**	**Standard Deviation**
AAA	11	5.02	1.50
AA	46	4.30	1.81
A	131	3.60	2.26
BBB	107	2.78	1.50
BB	50	2.45	1.62
B	80	1.67	1.22
CCC	10	0.95	1.10

8. A Further Revision – Adapting the Model for Non-Manufacturers and Emerging Markets

The next modification of the Z-Score model analyzed the characteristics and accuracy of a model without X_5 - - sales/total assets. We do this in order to minimize the potential industry effect which is more likely to take place when such an industry-sensitive variable as asset turnover is included. In addition, I have used this model to assess the financial health of non-U.S. corporates. In particular, Altman, Hartzell and Peck (1995, 1997) have applied this enhanced Z"Score model to emerging markets corporates, specifically Mexican firms that had issued Eurobonds denominated in U.S. dollars. The book value of equity was used for X_4 in this case.

The classification results are identical to the revised five-variable model (Z'Score). The new Z"Score model is:

$$Z'' = 6.56 (X_1) + 3.26 (X_2) + 6.72 (X_3) + 1.05 (X_4)$$

All of the coefficients for variables X_1 to X_4 are changed as are the group means and cutoff scores. This particular model is also useful within an industry where the type of financing of assets differs greatly among firms and important adjustments, like lease capitalization, are not made. In the emerging market model, we added a constant term of +3.25 so as to standardize the scores with a score of zero (0) equated to a D (default) rated bond. See Panel E for the bond rating equivalents of this newer, EM-Score model.

Panel E

U.S. Bond Rating Equivalent Based on EM Score

U.S. Equivalent Rating	Average EM Score
AAA	8.15
AA+	7.60
AA	7.30
AA-	7.00
A+	6.85
A	6.65
A-	6.40
BBB+	6.25
BBB	5.85
BBB-	5.65
BB+	5.25
BB	4.95
BB-	4.75
B+	4.50
B	4.15
B-	3.75
CCC+	3.20
CCC	2.50
CCC-	1.75
D	0

Source: In-Depth Data Corp.; average based on over 750 U.S. Corporates with rated debt outstanding: 1995 data.

9. The ZETA® Credit Risk Model

In 1977, Altman, Haldeman and Narayanan (1977) constructed a second-generation model with several enhancements to the original Z-Score approach. The purpose of this study was to construct, analyze and test a new bankruptcy classification model which considers explicitly recent developments with respect to business failures. The new study also incorporated refinements in the utilization of discriminant statistical techniques. The new model, which was called ZETA®, was effective in classifying bankrupt companies up to five years prior to failure on a sample of corporations consisting of manufacturers and retailers. Since the ZETA® model is a proprietary effort, I cannot fully disclose the parameters of the market.

In addition to updating for newer bankruptcies across many industries and adjustments of the financial data for relevant accounting changes (e.g., lease capitalization), the ZETA model tests included non-linear (e.g., quadratic) as well as linear discriminant models. The non-linear model was more impressive in the original, test sample results but less accurate and reliable in holdout or out-of-sample testing.

10. Macro Economic Impact and Loss Estimation

All of the aforementioned models are, in a sense, static in nature in that they can be applied at any point in time regardless of the current or expected performance of the economy and the economy's impact on the key measures - - (1) PDs and (2) LGDs. Aggregate PDs vary over time so that a firm with a certain set of variables will fail more frequently in poor economic times and vice-versa in good periods. This systematic factor is not incorporated directly in the establishment of the scoring model in most cases. Some recent attempts have experimented with including variables, which can capture these exogenous factors - - like GDP growth. Since GDP growth will be the same for the good firms as well as the distressed ones in the model development phase, it is necessary to be creative in including macro-impact variables. One idea is to add an aggregate default measure for each year, for example, to capture a high or low risk environment and observe its explanatory power contribution in the failure classification model. Such attempts have only achieved modest success to date.

Group Prior Probabilities, Error Costs and Model Efficiency

An alternative approach is to adjust the various scores for different risk categories by including explicit estimates for the prior probability of default and the possible costs of the model's errors. That is, assuming multi-normal populations and a common covariance matrix, the optimal cutoff score could be calculated as:

$$Z_c = \ln \frac{q_1 c_1}{q_2 c_{11}}$$

where q_1, q_2 = prior probability of bankrupt (q_1) or nonbankrupt (q_2), and C_1, C_{11} = costs of type I and type II errors, respectively.

Further, if one wanted to compare the efficiency of the ZETA bankruptcy classification model with alternative strategies, the following cost function is appropriate for the expected cost of ZETA (EC_{ZETA}).

$$EC_{ZETA} = q_1 (M_{12} / N_1) C_1 + q_2 (M_{21} / N_2) C_{11},$$

where M_{12}, M_{21} = observed type I and type II errors (misses) respectively, and N_1, N_2 = number of observations in the bankrupt (N_1) and nonbankrupt (N_2) groups.

In our old tests, we implicitly assumed equal prior probabilities and equal costs of errors. We are aware, however, of the potential bias involved in doing so. Instead of attempting earlier to integrate probability priors and error costs, we have assumed equal estimates for each parameter, because to a great extent the two parameters neutralize each other, and it was much easier than attempting to state them precisely. The following is our reasoning.

The 'correct' one-year estimate of q_1 for all firms is probably in the -0.02 –0.06 range. Although the Z-Score model's parameters are based on data from one year prior to bankruptcy, it is not specifically a one-year prediction model. The procedure in this sense is at temporal. In the final analysis, we simply do not know the precise general estimate of bankruptcy priors, since it will depend on the asset's rating, it's age, and the forecasting environment. When we specify these variables, a more precise estimate is attained. Yet, Basel 2 will require one-year estimates.

11. Cost of Classification Errors

Another input that is imperative to the specification of an alternative to the zero cutoff, accept-reject decision is the cost of errors classification. In order to attempt to precise the cost component into an analysis of model efficiency, it is necessary to specify the decision-maker's role. An appropriate reference point is the commercial bank loan function. The type I error bankruptcy classification is analogous to that of an accepted loan that defaults and the Type II error to a rejected loan that would have paid-off successfully. The latter is best assessed as a type of opportunity cost on the foregone investment.

The cost of a Type I error is analogous to the LGD estimate in modern credit risk models. The first attempt to measure LGD was from Altman, et. al. (1977) based on a survey of banks' loan loss files. Since then, there have been a number of studies measuring the recovery rate on bonds (e.g., Moody's [Annual], Fitch [1997]), Altman & Kishore (1996) and a few on bank loans (e.g., Moody's (1997), Van de Castle and Keisman (2000). For bonds, recovery can be measured at the time of default based on the price in the public market or upon emergence from the reorganization process (usually Chapter 11). Altman and Eberhart (1994) report on the annualized return for investors who purchased just after default (the so-called "vultures") and stratify the results by seniority.

Most modern credit risk models and all of the VaR models (e.g., CreditMetrics®), assume independence between PD and the recovery rate. Altman, Resti and Sironi (2002) show, however, that this is an incorrect assumption and simulate the impact on capital requirements when you factor in a significant negative correlation between PD and recovery rates over time. In particular, they find that in periods of high default rates on bonds, the recovery rate is very low relative to the average and losses can be expected to be greater (e.g., in 2000 and 2001). Hu and Perraudin (2002) find similar results and Frye (2000) specified a systematic macro-economic influence on recovery rates. This has caused serious concern of the potential procyclicality of a rating based approach, as is being recommended by Basel 2.

The bottom-line is that Basel 2 has motivated an enormous amount of effort on the part of banks, regulators and others (e.g., academics) to build credit risk models that involve scoring techniques, default and loss estimates, and portfolio approaches to the credit risk problem. We now turn to an alternative approach to the Z-Score type models that we have been discussing.

12. The Expected Default Frequency (EDF) Model

KMV Corporation (1995), purchased by Moody's in 2001, has created a procedure for estimating the default probability of a firm that is based conceptually on Merton's (1974) option-theoretic approach. In three steps, it determines an EDF for a company. In the first step, the market value and volatility of the firm are estimated from the market value of its stock, the volatility of its stock, and the book value of its liabilities. In the second step, the firm's default point is calculated from the firm's liabilities coming due over time. Also, an expected firm value is determined from the current firm value. Using these two values plus the firm's volatility, a measure is constructed that represents the number of standard deviations from the expected firm value to the default point (the distance to default). Finally, a mapping is determined between the distance to default and the default rate based on the historical default experience of companies with different distance-to-default values.

In the case of private companies, for which stock price and default data are generally unavailable, KMV uses essentially the same approach by estimating the value and volatility of the private firm directly from its observed characteristics and values based on market comparables.

The starting point of the KMV model is the proposition that when the market value of a firm drops below a certain level, the firm will default on its obligations. The value of the firm, projected to a given future date, has a probability distribution characterized by its expected value and standard deviation (volatility). The area under the distribution below the book liabilities of the firm is the probability of default.

For a firm with publicly traded shares, the market value of equity may be observed. The market value of equity may be expressed as the value of a call option, as follows:

Market value of equity = f (book value of liabilities,
market value of assets,
volatility of assets, time horizon)

KMV uses a special form of the options pricing approach that they do not disclose. To make their approach more concrete, the Black-Scholes options formula can be substituted for the function f. This results in the following expression:

$$E = VN(d_1) - De^{-T} N(d_2)$$

Where E = is the market value of equity (option value)
D = is the book value of liabilities (strike price)
V = is the market value of assets
τ = is the time horizon
r = is the risk-free borrowing and lending rate
σ_a = is the percentage standard deviation (volatility) of asset value
N = is the cumulative normal distribution function whose value is calculated at d_1 and d_2, where

$$d_1 = \frac{1n\left(\frac{V}{D}\right) + \left(r + \frac{1}{2}\sigma_a^{\;2}\right)t}{\sigma_a\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma_a\sqrt{\tau}$$

The known variables are the market value of equity (E), volatility of equity (σ_e, estimated from historic data), book value of liabilities (D), and the time horizon (τ). The two unknowns are the market value of the assets (V) and the volatility of the assets (σ). Because there are two equations with two unknowns, a solution can be found. This completes the first step.

Next, the expected asset value at the horizon and the default point are determined. An investor holding the asset would expect to get a payout plus a capital gain equal to the expected return. The expected return is related to the systematic risk of the asset. Using a measure of the asset's systematic risk, KMV determines an expected return based upon historic asset market returns. This is reduced by the payout rate determined from the firm's interest and dividend payments. The result is the expected appreciation rate, which, applied to the current asset value, gives the expected future value of the asset.

In the previous analysis it was assumed that the firm would default when its total market value falls below the book value of its liabilities. Based upon empirical analysis of defaults, KMV has found that the most frequent default point is at a firm value approximately equal to current liabilities plus 50% of long-term liabilities (they first tried 25% but it not work well).

Given the firm's expected value at the horizon, and its default point at the horizon, KMV determines the percentage drop in the firm value that would bring it to the default point. By dividing the percentage drop by the volatility, KMV controls for the effect of different volatilities.

The number of standard deviations that the asset value must drop in order to reach the default point is called the distance to default. Mathematically, this can be expressed as

$$Distance\ from\ default = \frac{(expected\ market\ value\ of\ assets - default\ point)}{(expected\ market\ value\ of\ assets)(volatility\ of\ assets)}$$

The distance-from-default metric is a normalized measure and thus may be used for comparing one company with another. A key assumption of the KMV approach is that all the relevant information for determining relative default risk is contained the expected market value of assets, the default point, and the asset volatility. Differences due to industry, national location, size, and so forth are assumed to be subsumed in these measures, notably the asset volatility.

Distance from default is also an ordinal measure akin to a bond rating, but it still does not tell you what the default probability is. In order to extend this risk measure to a cardinal or a probability measure, KMV uses historical default experience to determine an expected default frequency as a function of distance from default. It does this by comparing the calculated distances from default and the observed actual default rate for a large number of firms from their proprietary database. A smooth curve fitted to those data yields the EDF as a function of the distance from default.

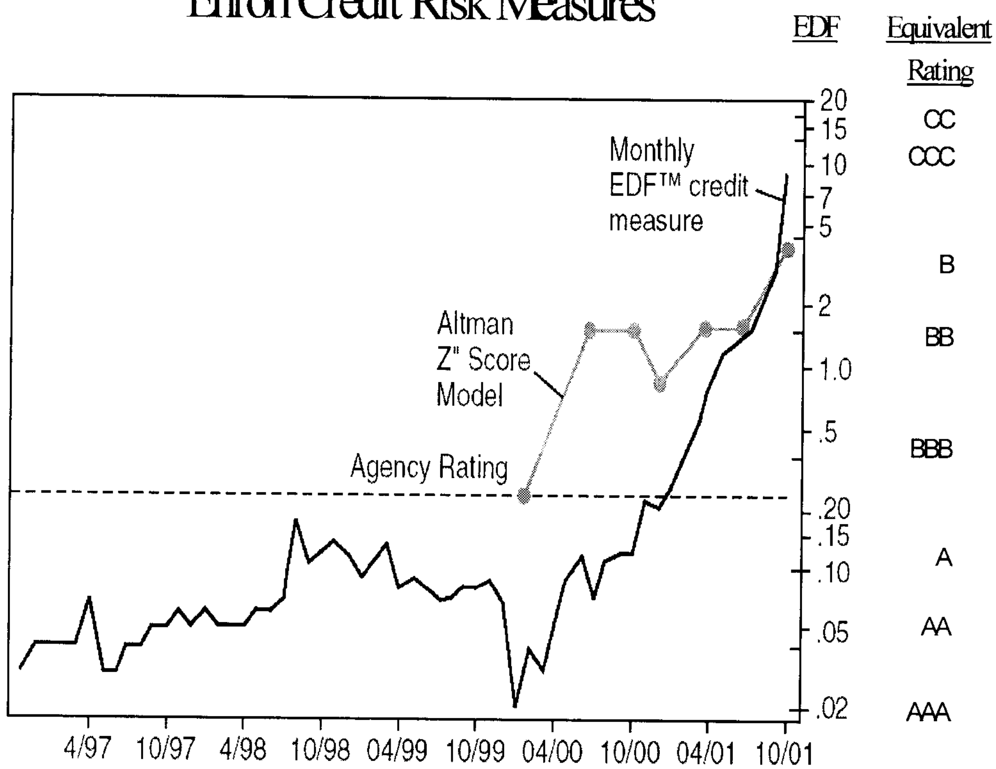
12. The Enron Example: Models Versus Ratings

We have examined two of the more popularly found credit scoring models - - the Z-Score model and KMV's EDF - - and in both cases a bond rating equivalent can be assigned to a firm. Many commentators have noted that quantitative credit risk measurement tools can and have saved banks and other "investors" from losing substantial amounts or at least reducing their risk exposures. A prime example is the recent Enron debacle, whereby billions of dollars of equity and debt capital have been lost. The following illustrates the potential savings involved from a disciplined credit risk procedure.

As Saunders and Allen (2002) report in their new book, on December 2, 2001, Enron Corporation filed for protection under Chapter 11 and became the largest corporate bankruptcy in our history - - with reported liabilities at the filing of over $31 billion and revised liabilities of over $60 billion! Using data that was available to investors over the period 1997-2001, Panel F shows the following: KMV's EDF, with its heavy emphasis on Enron's stock price, rated Enron AAA as of year-end 1999 but then indicated a fairly consistent rating equivalent deterioration resulting in a BBB rating one year later and then a B- to CCC+ rating just prior to the filing. Altman's Z"Score model (the four variable model for non-manufacturing) had Enron as BBB as of year-end 1999 - - the same as the rating agencies - - but then showed a steady deterioration to B as of June, 2001. So, both quantitative tools were issuing a warning long before the bad news hit the market. Although neither actually predicted the bankruptcy, these tools certainly could have provided an unambiguous early warning that the rating agencies were not providing (their rating remained at BBB until just before the bankruptcy). And, both models were using a vastly under-estimate of the true liabilities of the firm. To be fair, the rating agencies were constrained in that a downrating from BBB for Enron could have been the death-signal for a firm like Enron which relied on its all important investment grade rating in its vast counterparty transactions. An unemotional model, based solely on publicly available accounting and market information, is not constrained in that the analyst is free to follow the signal or to be motivated to dig-deeper into what on the surface may appear to be a benign situation.

Panel F

Enron Credit Risk Measures



In the Enron case, and many others that we are aware of, although tools like Z-Score and EDF were available, losses still were incurred by even the most sophisticated investors and financial institutions. Having the models is simply not enough! What is needed is a "credit-culture" within these financial institutions, whereby credit risk tools are "listened-to" and evaluated in good times as well as in difficult situations.

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