Equity Correlation Trading

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Outline

- Equity Correlation: Definitions, Products and Trade Structures
- Rationale: Evidence and Models
- Opportunities: an Historical Perspective

Correlation Products

Building Blocks: Vol Products

- Realized variance:

$$RV = \frac{1}{n}\sum_{t=1}^{T}(\ln(\frac{S_t}{S_{t-1}}))^2$$

- OTC products to trade realized variance:
  - Delta-hedged options (straddles)
  - Volatility swap
  - Variance swap

- Listed Products
  - Futures on realized variance

Implied Correlation

- From index and single-stock implied vols, one can extract the average pairwise *Implied Correlation* (= *IC*) embedded in option prices by the market.

- Let FVV = Fair Value of Variance, then *IC* is

$$IC = \frac{FVV_{Index} - \sum_{i=1}^{n} w_i^{\,2} FVV_i}{\left(\sum_{i=1}^{n} w_i \sqrt{FVV_i}\right)^2 - \sum_{i=1}^{n} w_i^{\,2} FVV_i}$$

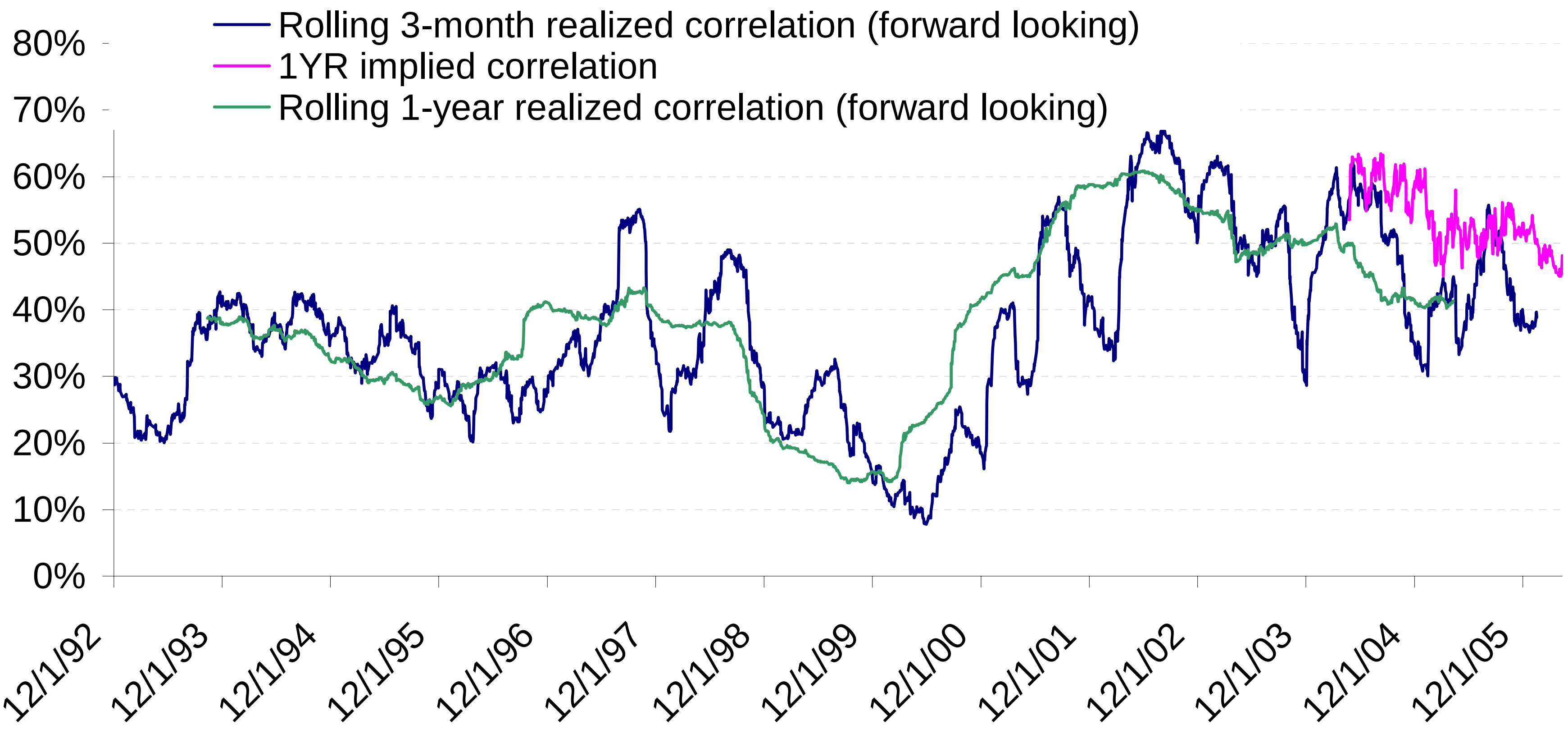
Basic Trade Idea

- Mechanics: a dispersion trade consists of
  - selling vol on the index, while simultaneously
  - buying vols on the component

- Appeal:
  - historically index volatility has traded rich, while
  - individual stock volatility has been fairly priced
  - ➔ implied correlation has historically been above realized

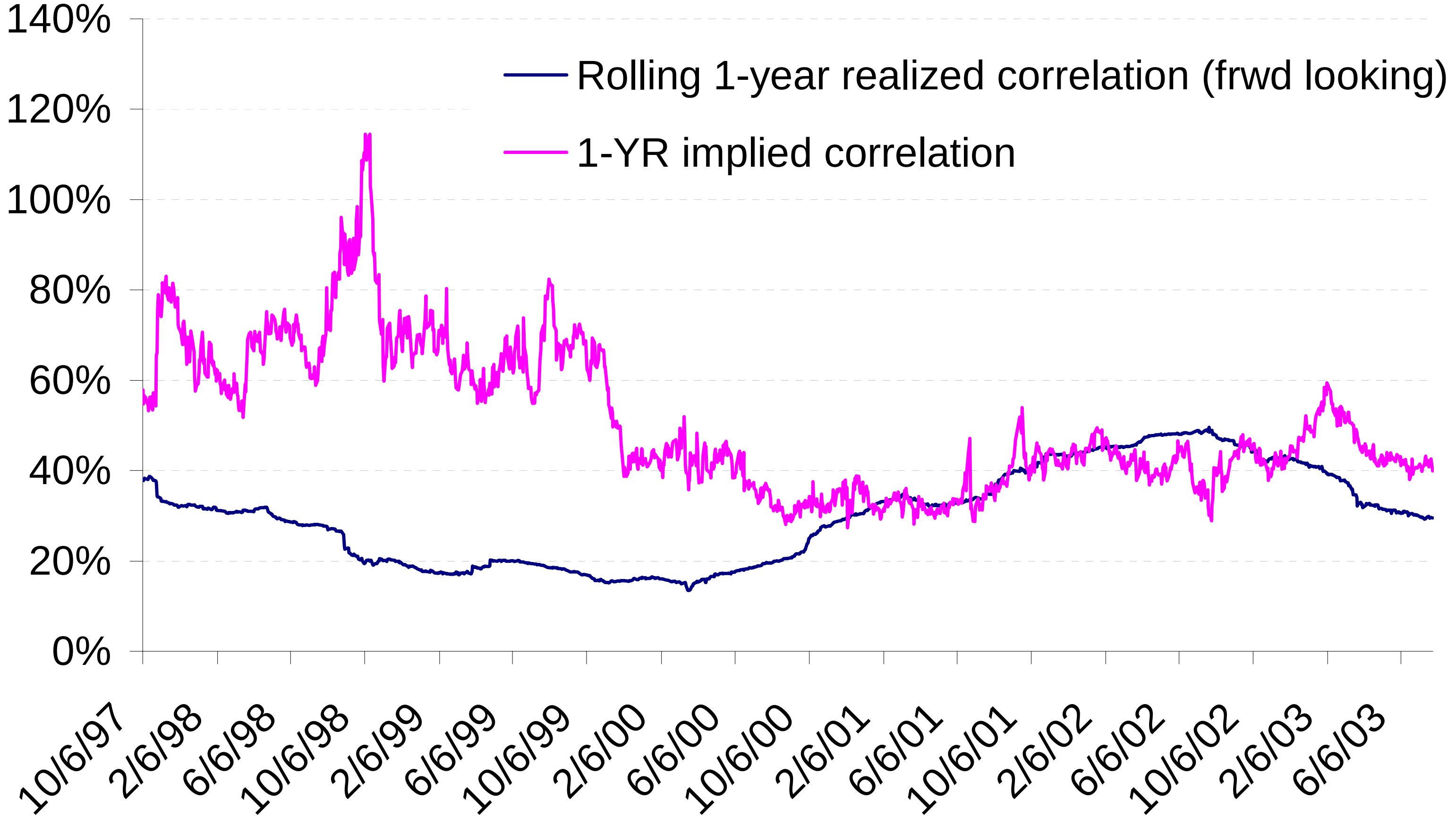
Correlation Market “Anomaly”

Index = Eurostoxx



Correlation Market "Anomaly"

Index = Dow Jones



Correlation Trading: Products

- Correlation swaps: pay the difference between an implied correlation strike and the average pairwise correlation in a basket of stocks. Correl-swaps are not a natural hedge for dealers' or structurers' books, as theses books are mostly exposed to covariance risk.

- Delta-hedged straddles: sell index straddles, buy single-stock straddles. Delta-hedging a book of 50-100 options is expensive and complicated for a hedge fund.

- Index var-swaps against single-stock var-swaps: it is the most popular way to structure the trade over the last 2/3 years has been to trade. This structure fits broker-dealer books relatively well and is manageable from a hedge fund point of view as no delta-hedging is necessary.

Dispersion Trading: Var-swaps

- Sell a var-swap on an index, buy variance swaps on the individual components of the index.

- On the single stock side, vega notionals are typically proportional to index weights.

- By adjusting the ratio of index vega notional to stock vega notional, one can modify the return distribution profile of the portfolio. Most people like the trade "vega neutral" (sum of single stock vega notional = - index vega) or "premium neutral" (sum of variance notional * variance strikes on the index side = index variance notional * index variance strike).

- As the next 2 slides will show, a "premium neutral" trade is a good way to replicate a covariance exposure.

Vega-Neutral trade

Payoff Matrix (in $M, for a $1M index vega dispersion trade on DJIA,43% implied correlation)

Correlation	VOL RATIO																			
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.4	1.6	1.8	2	2.4	2.8	3.2	3.6	4
0	(4.09)	(3.74)	(3.15)	(2.33)	(1.28)	0.01	1.53	3.29	5.28	7.50	12.65	18.74	25.76	33.72	42.61	63.21	67.79	66.05	64.08	61.87
0.05	(4.10)	(3.77)	(3.23)	(2.48)	(1.50)	(0.32)	1.09	2.71	4.54	6.60	11.35	16.97	23.45	30.79	39.00	58.01	60.71	56.80	52.38	47.43
0.1	(4.10)	(3.81)	(3.31)	(2.62)	(1.73)	(0.64)	0.65	2.13	3.81	5.69	10.05	15.20	21.13	27.87	35.39	52.81	53.63	47.56	40.68	32.98
0.15	(4.11)	(3.84)	(3.39)	(2.76)	(1.95)	(0.97)	0.20	1.55	3.08	4.79	8.75	13.43	18.82	24.94	31.78	47.61	46.56	38.32	28.98	22.07
0.2	(4.12)	(3.88)	(3.48)	(2.91)	(2.18)	(1.29)	(0.24)	0.98	2.35	3.89	7.45	11.66	16.51	22.02	28.17	42.41	39.48	29.07	22.07	22.07
0.25	(4.13)	(3.92)	(3.56)	(3.05)	(2.41)	(1.62)	(0.68)	0.40	1.62	2.99	6.15	9.89	14.20	19.09	24.55	37.21	32.40	22.07	22.07	22.07
0.3	(4.14)	(3.95)	(3.64)	(3.20)	(2.63)	(1.94)	(1.12)	(0.18)	0.89	2.08	4.85	8.12	11.89	16.17	20.94	32.01	25.32	22.07	22.07	22.07
0.35	(4.15)	(3.99)	(3.72)	(3.34)	(2.86)	(2.27)	(1.57)	(0.76)	0.16	1.18	3.55	6.35	9.58	13.24	17.33	26.81	22.07	22.07	22.07	22.07
0.4	(4.16)	(4.02)	(3.80)	(3.49)	(3.08)	(2.59)	(2.01)	(1.34)	(0.57)	0.28	2.25	4.58	7.27	10.32	13.72	21.61	22.07	22.07	22.07	22.07
0.45	(4.17)	(4.06)	(3.88)	(3.63)	(3.31)	(2.92)	(2.45)	(1.91)	(1.31)	(0.63)	0.95	2.81	4.96	7.39	10.11	16.41	22.07	22.07	22.07	22.07
0.5	(4.18)	(4.10)	(3.96)	(3.78)	(3.53)	(3.24)	(2.89)	(2.49)	(2.04)	(1.53)	(0.35)	1.04	2.65	4.47	6.50	15.98	22.07	22.07	22.07	22.07
0.55	(4.19)	(4.13)	(4.04)	(3.92)	(3.76)	(3.57)	(3.33)	(3.07)	(2.77)	(2.43)	(1.65)	(0.73)	0.33	1.54	2.89	15.98	22.07	22.07	22.07	22.07
0.6	(4.19)	(4.17)	(4.13)	(4.06)	(3.99)	(3.89)	(3.78)	(3.65)	(3.50)	(3.33)	(2.95)	(2.50)	(1.98)	(1.38)	(0.72)	15.98	22.07	22.07	22.07	22.07
0.65	(4.20)	(4.20)	(4.21)	(4.21)	(4.21)	(4.22)	(4.22)	(4.22)	(4.23)	(4.24)	(4.25)	(4.27)	(4.29)	(4.31)	(4.33)	15.98	22.07	22.07	22.07	22.07
0.7	(4.21)	(4.24)	(4.29)	(4.35)	(4.44)	(4.54)	(4.66)	(4.80)	(4.96)	(5.14)	(5.55)	(6.04)	(6.60)	(7.23)	(5.90)	15.98	22.07	22.07	22.07	22.07
0.75	(4.22)	(4.28)	(4.37)	(4.50)	(4.66)	(4.87)	(5.10)	(5.38)	(5.69)	(6.04)	(6.85)	(7.81)	(8.91)	(10.16)	(5.90)	15.98	22.07	22.07	22.07	22.07
0.8	(4.23)	(4.31)	(4.45)	(4.64)	(4.89)	(5.19)	(5.55)	(5.96)	(6.42)	(6.94)	(8.15)	(9.58)	(11.22)	(13.08)	(5.90)	15.98	22.07	22.07	22.07	22.07
0.85	(4.24)	(4.35)	(4.53)	(4.79)	(5.11)	(5.52)	(5.99)	(6.54)	(7.16)	(7.85)	(9.45)	(11.35)	(13.53)	(15.34)	(5.90)	15.98	22.07	22.07	22.07	22.07
0.9	(4.25)	(4.39)	(4.61)	(4.93)	(5.34)	(5.84)	(6.43)	(7.11)	(7.89)	(8.75)	(10.75)	(13.11)	(15.84)	(15.34)	(5.90)	15.98	22.07	22.07	22.07	22.07
0.95	(4.26)	(4.42)	(4.69)	(5.08)	(5.57)	(6.17)	(6.87)	(7.69)	(8.62)	(9.65)	(12.05)	(14.88)	(18.15)	(15.34)	(5.90)	15.98	22.07	22.07	22.07	22.07
1	(4.27)	(4.46)	(4.78)	(5.22)	(5.79)	(6.49)	(7.32)	(8.27)	(9.35)	(10.56)	(13.35)	(16.65)	(20.46)	(15.34)	(5.90)	15.98	22.07	22.07	22.07	22.07

- A vega neutral trade can loose money even if realized correlation is below implied correlation, in case realized vols are very low

Premium-Neutral trade

Payoff Matrix (in $M, for a $1M index vega dispersion trade on DJIA,43% implied correlation)

	VOL RATIO																			
Correlation	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.4**	**2.8**	**3.2**	**3.6**	**4**
0	0.07	0.30	0.67	1.20	1.87	2.70	3.67	4.80	6.07	7.50	10.80	14.70	19.20	24.30	30.00	43.20	45.72	43.98	42.01	39.80
0.05	0.07	0.26	0.59	1.06	1.65	2.37	3.23	4.22	5.34	6.60	9.50	12.93	16.89	21.37	26.39	38.00	38.64	34.74	30.31	25.36
0.1	0.06	0.23	0.51	0.91	1.42	2.05	2.79	3.64	4.61	5.69	8.20	11.16	14.58	18.45	22.78	32.80	31.57	25.49	18.61	10.91
0.15	0.05	0.19	0.43	0.77	1.20	1.72	2.35	3.07	3.88	4.79	6.90	9.39	12.27	15.52	19.17	27.60	24.49	16.25	6.91	(0.00)
0.2	0.04	0.16	0.35	0.62	0.97	1.40	1.91	2.49	3.15	3.89	5.60	7.62	9.96	12.60	15.55	22.40	17.41	7.00	(0.00)	(0.00)
0.25	0.03	0.12	0.27	0.48	0.75	1.07	1.46	1.91	2.42	2.99	4.30	5.85	7.64	9.67	11.94	17.20	10.33	(0.00)	(0.00)	(0.00)
0.3	0.02	0.08	0.19	0.33	0.52	0.75	1.02	1.33	1.69	2.08	3.00	4.08	5.33	6.75	8.33	12.00	3.25	(0.00)	(0.00)	(0.00)
0.35	0.01	0.05	0.11	0.19	0.30	0.42	0.58	0.76	0.96	1.18	1.70	2.31	3.02	3.82	4.72	6.80	(0.00)	(0.00)	(0.00)	(0.00)
0.4	0.00	0.01	0.02	0.04	0.07	0.10	0.14	0.18	0.22	0.28	0.40	0.54	0.71	0.90	1.11	1.60	(0.00)	(0.00)	(0.00)	(0.00)
0.45	(0.01)	(0.03)	(0.06)	(0.10)	(0.16)	(0.23)	(0.31)	(0.40)	(0.51)	(0.63)	(0.90)	(1.23)	(1.60)	(2.03)	(2.50)	(3.60)	(0.00)	(0.00)	(0.00)	(0.00)
0.5	(0.02)	(0.06)	(0.14)	(0.24)	(0.38)	(0.55)	(0.75)	(0.98)	(1.24)	(1.53)	(2.20)	(2.99)	(3.91)	(4.95)	(6.11)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.55	(0.02)	(0.10)	(0.22)	(0.39)	(0.61)	(0.88)	(1.19)	(1.56)	(1.97)	(2.43)	(3.50)	(4.76)	(6.22)	(7.88)	(9.72)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.6	(0.03)	(0.13)	(0.30)	(0.53)	(0.83)	(1.20)	(1.63)	(2.13)	(2.70)	(3.33)	(4.80)	(6.53)	(8.53)	(10.80)	(13.33)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.65	(0.04)	(0.17)	(0.38)	(0.68)	(1.06)	(1.53)	(2.08)	(2.71)	(3.43)	(4.24)	(6.10)	(8.30)	(10.84)	(13.73)	(16.95)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.7	(0.05)	(0.21)	(0.46)	(0.82)	(1.28)	(1.85)	(2.52)	(3.29)	(4.16)	(5.14)	(7.40)	(10.07)	(13.16)	(16.65)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.75	(0.06)	(0.24)	(0.54)	(0.97)	(1.51)	(2.18)	(2.96)	(3.87)	(4.89)	(6.04)	(8.70)	(11.84)	(15.47)	(19.58)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.8	(0.07)	(0.28)	(0.63)	(1.11)	(1.74)	(2.50)	(3.40)	(4.44)	(5.63)	(6.94)	(10.00)	(13.61)	(17.78)	(22.50)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.85	(0.08)	(0.31)	(0.71)	(1.26)	(1.96)	(2.83)	(3.85)	(5.02)	(6.36)	(7.85)	(11.30)	(15.38)	(20.09)	(24.76)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.9	(0.09)	(0.35)	(0.79)	(1.40)	(2.19)	(3.15)	(4.29)	(5.60)	(7.09)	(8.75)	(12.60)	(17.15)	(22.40)	(24.76)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
0.95	(0.10)	(0.39)	(0.87)	(1.54)	(2.41)	(3.48)	(4.73)	(6.18)	(7.82)	(9.65)	(13.90)	(18.92)	(24.71)	(24.76)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)
1	(0.11)	(0.42)	(0.95)	(1.69)	(2.64)	(3.80)	(5.17)	(6.76)	(8.55)	(10.56)	(15.20)	(20.69)	(27.02)	(24.76)	(18.51)	(4.03)	(0.00)	(0.00)	(0.00)	(0.00)

- The sign of the P&L only depends on whether realized correlation is above or below implied correlation. The magnitude of the P&L increase with realized vol levels. The trade is essentially a covariance play

Mark-to-Market

- For longer-dated trades P&L will come more from mainly from remarking of implied correlation than from differences between implied and actual correlation.
- For a var swap, the P&L between t1 and t2 is

$$PNL = Var_{t_2} - Var_{t_1}$$, where $$Var_t = \frac{t}{T} RV_t + \frac{T-t}{T} FVV_t$$

Therefore

$$PNL = \frac{t_2 - t_1}{T}\left(RV_{t_1}^{t_2} - FVV_{t_1}\right) + \frac{T-t_2}{T}\left(FVV_{t_2} - FVV_{t_1}\right)$$

- Similarly, for a correlation trade, we have

$$PNL \cong \frac{t_2 - t_1}{T}\left(RC_{t_1}^{t_2} - IC_{t_1}\right) + \frac{T-t_2}{T}\left(IC_{t_2} - IC_{t_1}\right)$$

Where *IC* is implied correlation and *RC* is realized correlation

Puzzle(1): Long-Dated Implied Correlation … Too Low?

- Mark-to market risk for long-dated volatility structures, including correlation trades, is possibly not compensated ... enough

- Market segmentation: there is no demand for short-dated correlation (structurers use long-dated)

Puzzle(2): Long-dated Index-Volatility Skews Too High

- Index-volatility skews do not flatten with longer maturities

- True for all markets (world-wide Crash-o-Phopia, see Foresi-Wu, JOD 2005): put options are more expensive than the corresponding call options ➔ index returns have a risk-neutral return distribution that, unlike empirical distribution, is asymmetric

- This is likely to be consistent with systematic risk, in the form of "bad" correlation, or market (world-wide market) crash risk, an eminently un-diversifiable equity-market risk

- Is the size of the premium reasonable, when one considers that the "market" is much more than just the equity-market?

Correlation Trading: Motivation

- Why trade correlation? Is it a bet on correlation being mean-reverting or a premium for "beta", possibly exotic beta?

- A reasonable model of correlation has correlation time varying (Driessen et al, 2005)

$$d\rho_t = \kappa(\rho_\infty - \rho_t)dt + \alpha\sqrt{(1-\rho_t)\rho_t}\,dw_t$$

- The equivalent risk-neutral expression embeds a correlation premium. The data suggests that this premium is large – which is reasonable if market crash-risk is not diversifiable

$$d\rho_t = \kappa^*({\rho^*}_\infty - \rho_t)dt + \alpha^*\sqrt{(1-\rho_t)\rho_t}\,dw_t$$

Correlation Modeling

- There is a relation between market-wide realized vols and realized pairwise correlation

$$dS_t / S_t = \sigma_t du_t$$
$$d\sigma_t = g(\rho_t, \sigma_t)dt + f(\rho_t, \sigma_t)dw_t$$
$$d\rho_t = h(\rho_t, \sigma_t)dt + s(\rho_t, \sigma_t)dw_t$$

- This model is short-hand for a more complete model of crash risk which arguably should contain common asymmetric jump-risk, a more sensible way to produce increases both in correlations as well as in measured volatility

Correlation Modeling 2

- There is a more difficult relation linking vols/correlation to flows and positions and the nature of the market participants
- Feedback effect: it is a general principle in derivatives trading: If party A sells and delta-hedge an option to party B who does not hedge, actual return volatility will be dampened
- This is true also for correlation riss: the existence of correlation books, on the back of structures placed with retail investors who do not hedge, imparted downward pressures on realized correlations
- A model without flow information is incomplete

Rationale for the Trade: A Demand & Supply Perspective

- Why has index vol traded at a premium?
  - Index vol is (relatively) rich:
    - Portfolio insurance (makes puts expensive)
    - Structurers
  - Individual vol is (relatively) cheap/fair
    - Reverse convertibles
    - call-overwriting (indexers)

Opportunities

- Equity correlation vs. credit correlation
- Equity bespoke correlation
- Hybrid-basket bespoke correlation: baskets of commodities and equities, or commodities and FX, etc.
- Asset-class correlation
  - Pension plans exposure to fixed income and equity
  - Counterparty risk (bank's counterparty credit risk, by positions)

General Notes

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